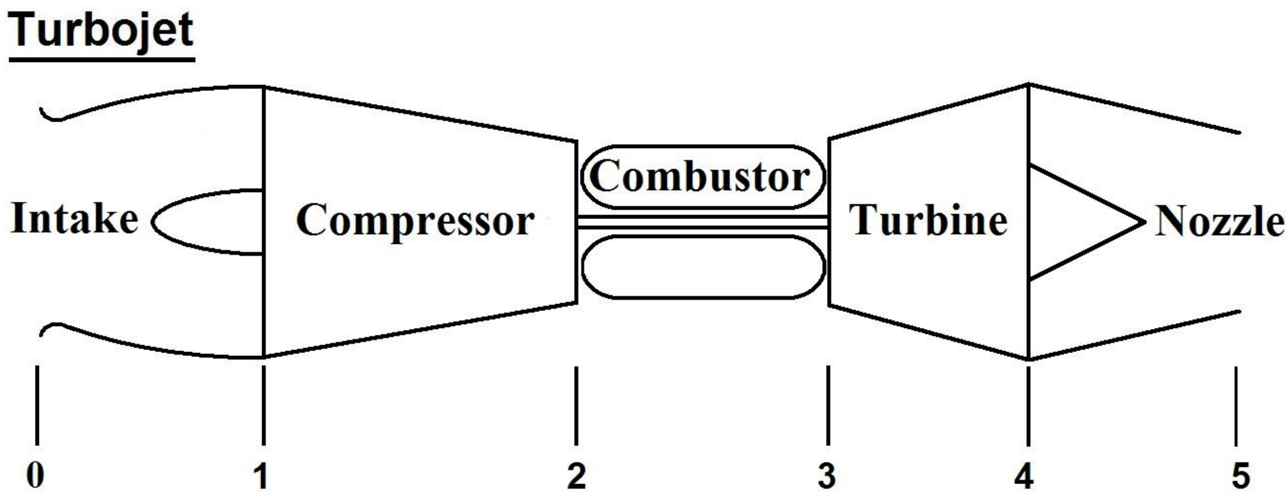
# ENGINE PERFORMANCE EXEERCISE SOLUTION

The station numbers of the engine are illustrated in the figure below. It is a single-spool turbojet engine and the main parts of the engine are the intake (0-1), the compressor (1-2), the combustion chamber (2-3), the turbine (3-4) and the exit nozzle (4-5).




| | | | | | | |
|---|---|---|---|---|---|---|
| **(P)** | **101** | **101** | **2020** | **1919** | **718.9** | **718.9** |
| **(T)** | **288** | **288** | **711.7** | **1800** | **1441.9** | **1441.9** |

### $\mathbf{0 \rightarrow 1}$ *INTAKE*

The intake is a critical part of an engine installation having a significant effect on both engine performance and aircraft safety. The primary role of an intake is to minimize the pressure loss up to the compressor face while ensuring that air enters the compressor with a uniform pressure and velocity distribution at all flight conditions. Given that there is no Turbomachinery component to change the fluid energy, it is assumed that the total pressure and temperature remain constant through the intake. Of course, this is an approximation. Usually there is a pressure recovery. However,

$\boldsymbol{P_0 = P_1 = 101\ KPa}$

$\boldsymbol{T_0 = T_1 = 288\ K}$

### $\mathbf{1 \rightarrow 2}$ *COMPRESSOR*

The overall pressure ratio (PR) is 20. As a result:

$$P_2 = PR \times P_1 = 20 \times 101 \Rightarrow \boldsymbol{P_2 = 2020K}$$

From the isentropic process:

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma} \Rightarrow T_{2s} = 288 \times (20)^{0.285} \Rightarrow T_{2s} = 677.82\ K$$

However, the compressor has an isentropic efficiency 0.92. Therefore:

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} \Rightarrow T_2 = \frac{T_{2s} - T_1}{\eta_c} + T_1 = \frac{677.82 - 288}{0.92} + 288 \Rightarrow \boldsymbol{T_2 = 711.71\ K}$$

Calculation of Compressor Work (CW). It is assumed constant specific heat of air over the temperature range in the compressor. This is a big assumption. $C_p$ highly depends on temperature.

$$CW = \dot{m}C_{P,cold}(T_2 - T_1) = 75 \times 1000 \times (711.71 - 288) \Rightarrow \boldsymbol{CW = 31.78\ MW}$$

### $\mathbf{2 \rightarrow 3}$ *COMBUTOR*

The combustion process at gas turbines takes place at constant pressure. However, there are always some pressure losses. Pressure drop is in the order of 5% and it is beneficial for the supply of cooling air. Think of film cooling…if the pressure of the coolant is lower than the pressure of the mainstream, the hot gas flow would go inside the blade. We do like this pressure drop because we can inject *cold* air on the blade surface. As a result:

$P_3 = (1 - 0.05) \times P_2 = 0.95 \times 2020 \Rightarrow \boldsymbol{P_3 = 1919\ KPa}$

Turbine inlet temperature (TET) = $\boldsymbol{T_3}$ **= TET=1800K**
Note that typical material limits are in the order of 1250K. We can achieve this temperature only with turbine blade cooling.

Calculation of fuel flow. How much fuel do we need in order to reach this turbine inlet temperature level?

$$\dot{m}C_{P,hot}(TET - T_2) = \dot{F} \times FCV \Rightarrow \dot{F} = \frac{75 \times 1150 \times (1800 - 711.71)}{43 \times 10^6} \Rightarrow \boldsymbol{\dot{F} = 2.182\ kg/s}$$

The heat input (HI) in the combustor is simply: $\dot{F} \times FCV = 2.182 \times 43 \Rightarrow \boldsymbol{HI = 93.86MW}$

**3 → 4 *TURBINE***

Turbine pressure ratio is unknown. We need to calculate it.

The role of the turbine in aero-engine applications is to drive the compressor. So turbine work equals the compressor work (TW=CW). We need also to include the fuel flow in the calculations and we assume again constant $C_{P,hot}$ within the turbine.

$$CW = TW \Rightarrow (\dot{m} + \dot{F})C_{P,hot}(TET - T_4) = 42.79MW \Rightarrow T_4 = 1800 - \frac{31.78 \times 10^6}{(75 + 2.182) \times 1150} \Rightarrow \boldsymbol{T_4 = 1441.95\ K}$$

The turbine has an isentropic efficiency 92%. Therefore:

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}} \Rightarrow T_{4s} = T_3 - \frac{T_3 - T_4}{\eta_T} = 1800 - \frac{1800 - 1441.96}{0.92} \Rightarrow \boldsymbol{T_{4s} = 1410.82K}$$

From the isentropic process:

$$\left(\frac{P_4}{P_3}\right) = \left(\frac{T_{4s}}{T_3}\right)^{\gamma/\gamma-1} \Rightarrow P_4 = 1919 \times \left(\frac{1410.83}{1800}\right)^{4.03} \Rightarrow \boldsymbol{P_4 = 718.91KPa}$$

and as a result the turbine pressure ratio ($PR_t$) is

$$PR_t = \frac{P_4}{P_3} = \frac{718.91}{1919} = 0.3759$$

**4 → 5 *NOZZLE***

In aero-engine applications a nozzle is mounted after the gas generator of the engine in order to produce *thrust* from the hot gases. Given that there is no component to change the fluid energy, it is assumed that the total pressure and temperature remain constant through the nozzle.

$\boldsymbol{P_4 = P_5 = 718.91KPa}$

$\boldsymbol{T_4 = T_5 = 1441.95\ K}$

First of all we need to check if the nozzle is choked, where Mach number at the throat equals 1.

$$\frac{P_t}{p_{st}} = \left(1 + \frac{\gamma - 1}{2}M^2\right)^{\gamma/\gamma-1} = \left(1 + \frac{1.33 - 1}{2} \times 1^2\right)^{1.33/0.33} = 1.825\ \ while\ \frac{P_4}{p_a} = \frac{718.91}{101} = 7.11\ \ it\ is\ choked!$$

As a result we get thrust from pressure difference.
We are able to calculate the static pressure and static temperature:

$$p_{5,st} = \frac{P_5}{1.825} = \frac{718.91}{1.825} \Rightarrow \boldsymbol{p_{5,st} = 393.91\ KPa}$$

$$\frac{T_{5,t}}{t_{5,st}} = \left(1 + \frac{\gamma - 1}{2}M^2\right) \Rightarrow t_{5,st} = \frac{1441.95}{\left(1 + \frac{0.33}{2} \times 1\right)} \Rightarrow \boldsymbol{t_{5,st} = 1237.37\ K}$$

Calculation of air density at nozzle exit (5):

$$\rho_5 = \frac{p_{5,st}}{Rt_{5,st}} = \frac{393.91 \times 1000}{287 \times 1237.73} \Rightarrow \boldsymbol{\rho_5 = 1.108 Kg/m^3}$$

Calculation of air-gas jet velocity at nozzle exit (5) (M=1, choked nozzle)

$$V_{j,5} = \sqrt{\gamma R T_{5,st}} = \sqrt{1.33 \times 287 \times 1237.73} \Rightarrow \boldsymbol{V_5 = 687.35\ m/s}$$

Calculation of nozzle exit area (5):

$$A_5 = \frac{\dot{m}}{V_{j,5} \times \rho_5} = \frac{(75 + 2{,}182)}{687.35 \times 1.108} \Rightarrow \boldsymbol{A_5 = 0.101 m^2}$$

And therefore the nozzle exit diameter can be estimated as:

$$A_5 = \frac{\pi D_n^2}{4} \Rightarrow \boldsymbol{D_n = 0.359 m}$$

Calculation of engine thrust. We assume take-off conditions where the aircraft velocity is zero ($V_o$=0m/s):

$$F_{total} = F_N + F_A = (\dot{m} + \dot{F}) \times (V_{j,5} - V_o) + A_5 \times (p_{5,st} - p_a)$$

$$= (75 + 2.182) \times (687.35 - 0) + 0.101 \times (393.92 - 101)$$

$$= 53.05 kN + 29.58 kN = \boldsymbol{82.63 kN}$$

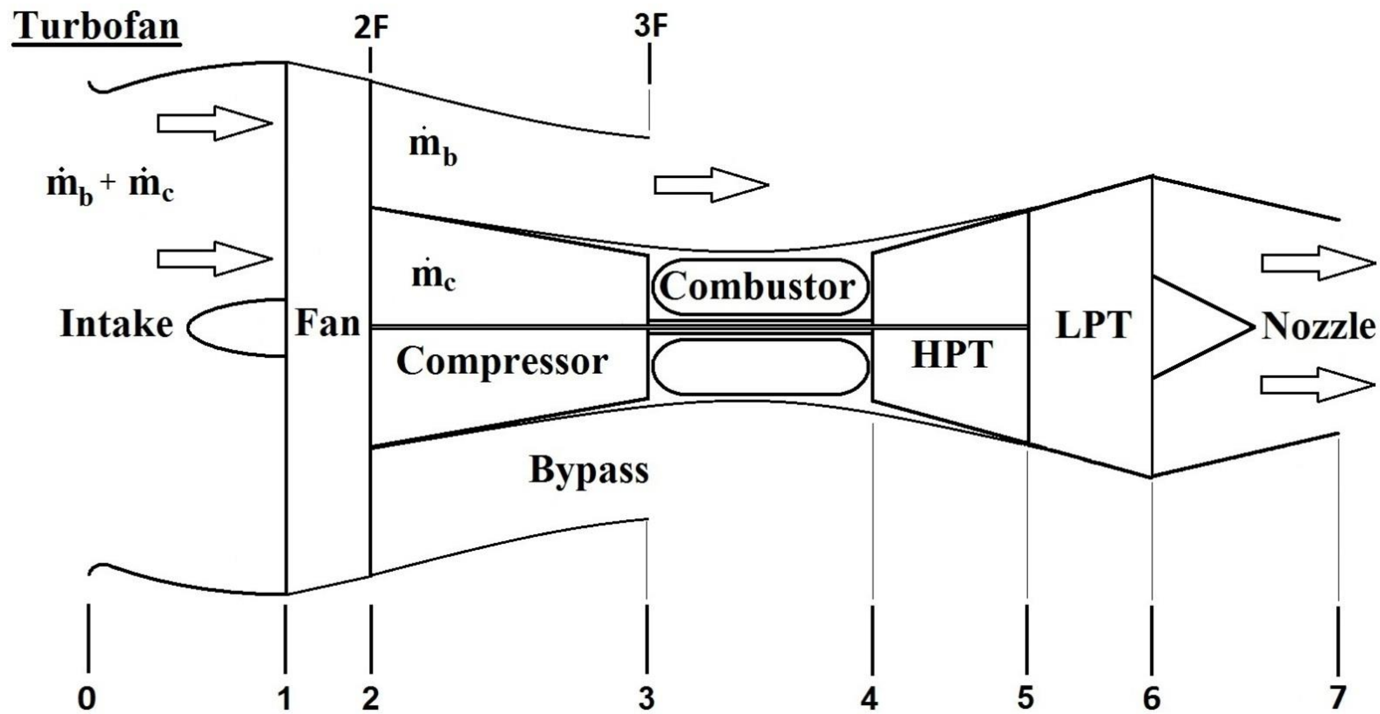
Calculation of Specific fuel consumption (SFC) and Specific Thrust (ST):

*SFC* is the one of the most important engine performance parameters. Describes the fuel efficiency of an engine design with respect to thrust output and it is used to compare different engines:

$$SFC = \frac{\dot{F}}{F_{total}} = \frac{2.182 \times 1000}{82.63} \Rightarrow \boldsymbol{SFC = 26.4\ {}^{g}/_{kN \times sec}}$$

$$ST = \frac{F_{total}}{\dot{m}} = \frac{82.63}{75 + 2.182} \Rightarrow \boldsymbol{ST = 1.07\ {}^{kN\ s}/_{(kg)}}$$

**(2)**

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|---|---|---|---|---|---|---|---|---|
| **(P)** | **101** | **101** | **141.4** | **2828** | **2686.6** | **1006.4** | **426.4** | **426.4** |
| **(T)** | **288** | **288** | **319.5** | **789.7** | **1997.2** | **1600** | **1334.5** | **1334.5** |

- We follow again the procedure of the design point calculation (similar to the turbojet of question (a)).

**$0 \rightarrow 1$ *INTAKE***

$\boldsymbol{P_0 = P_1 = 101\ KPa}$

$\boldsymbol{T_0 = T_1 = 288\ K}$

**$1 \rightarrow 2/2F$ *FAN***

The fan pressure ratio (FPR) is 1.4. As a result:

$\Rightarrow P_2 = FPR \times P_1 = 1.4 \times 101 \Rightarrow \boldsymbol{P_2 = 141.4K}$

From the isentropic process:

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma} \Rightarrow T_{2s} = 288 \times (1.4)^{0.285} \Rightarrow T_{2s} = 317.06\ K$$

Isentropic efficiency of the fan is 92%. Therefore:

$$\eta_{Fan} = \frac{T_{2s} - T_1}{T_2 - T_1} \Rightarrow T_2 = \frac{T_{2s} - T_1}{\eta_{Fan}} + T_1 = \frac{317.06 - 288}{0.92} + 288 \Rightarrow \boldsymbol{T_2 = T_{2F} = 319.58\ K}$$

We DON'T know the massflow! We are NOT able to calculate the Fan Work (FW). We need to calculate the massflow.
The gas generator (core of the engine) continues to operate at the <u>same non-dimensional power setting</u>. This means that all the pressure and temperature ratios at the gas generator remain constant. As a result:

$$\left(\frac{T_3}{T_2}\right)_{TF} = \left(\frac{T_2}{T_1}\right)_{TJ} \Rightarrow T_3 = 319.58 \times \left(\frac{711.71}{288}\right) \Rightarrow \boldsymbol{T_3 = 789.7}$$

$$\left(\frac{P_3}{P_2}\right)_{TF} = (PR)_{TJ} \Rightarrow P_3 = (PR)_{TJ} \times P_2 = 20 \times 141.4 \Rightarrow \boldsymbol{P_3 = 2828 KPa}$$

Combustor pressure loss is 5%.

$$P_4 = P_5 \times (1 - 0.05) = 0.95 \times 2828 \Rightarrow P_4 = \mathbf{2686.6KPa}$$

$$\left(\frac{T_4(TET)}{T_3}\right)_{TF} = \left(\frac{T_3(TET)}{T_2}\right)_{TF_{TJ}} \Rightarrow T_4(TET) = 789.75 \times \frac{1800}{711.71} \Rightarrow \boldsymbol{T_4(TET) = 1997.2\ K}$$

$$\left(\frac{P_5}{P_4}\right)_{TF} = \left(\frac{P_4}{P_3}\right)_{TJ} \Rightarrow P_5 = 2686.6 \times \frac{718.9}{1919} \Rightarrow \boldsymbol{P_5 = 1006.4\ KPa}$$

$$\left(\frac{T_5}{T_4}\right)_{TF} = \left(\frac{T_4}{T_3}\right)_{TJ} \Rightarrow T_5 = (T_4)_{TF} \times \left(\frac{T_4}{T_3}\right)_{TJ} = 1997.42 \times \frac{1441.96}{1800} \Rightarrow \boldsymbol{T_5 = 1600\ K}$$

We need to find the new massflow rate. How?
The engine operates at the same non-dimensional power setting which also means that the non-dimensional massflow ratio (NDMF) of the core at each component is the same with the turbojet engine.

$$\frac{\dot{m}\sqrt{T}}{P} = C$$

$$\left(\frac{\dot{m}_c\sqrt{T}}{P}\right)_{TF,3} = \left(\frac{\dot{m}\sqrt{T}}{P}\right)_{TJ,2} \Leftrightarrow \dot{m}_c = \left(\frac{P}{\sqrt{T}}\right)_{TF,3} \times \left(\frac{\dot{m}\sqrt{T}}{P}\right)_{TJ,2} = \frac{2828}{\sqrt{789.77}} \times \frac{75 \times \sqrt{711.71}}{2020} \Rightarrow \boldsymbol{\dot{m}_c = 99.67\ Kg/s}$$

The turbofan engine has a bypass ratio of 9. As a result,

$$BR = \frac{\dot{m}_b}{\dot{m}_c} \Rightarrow \dot{m}_b = \dot{m}_c \times BR = 99.67 \times 9 \Rightarrow \boldsymbol{\dot{m}_b = 897\ Kg/s}$$

$$\dot{m}_{total} = \dot{m}_c + \dot{m}_b = 897.12 + 99.68 \Rightarrow \boldsymbol{\dot{m}_{total} = 996.7\ Kg/S}$$

And finally, the fan work can be calculated.

$$FW = \dot{m}_{total} C_{P,cold}(T_2 - T_1) = 996.7 \times 1000 \times (319.58 - 288) \Rightarrow \boldsymbol{FW = 31.47\ MW}$$

**2 → 3 *COMPRESSOR***

No need to calculate anything from the compressor since the core of the engine operates at the same non-dimensional power setting. We have calculated everything previously.

**3 → 4 *COMBUTOR***

- We need to calculate the new fuel flow for the new TET!! These can be calculated from the heat input in the combustor

$$\dot{m}_c C_{P,hot}(TET - T_3) = \dot{F} \times FCV \Rightarrow \dot{F} = \frac{99.67 \times (\frac{1150}{2} + \frac{1000}{2}) \times (1997.42 - 789.77)}{43 \times 10^6} \Rightarrow \boldsymbol{\dot{F} = 3\ kg/s}$$

**4 → 5 *TURBINE (HPT)***

This is the turbine which drives the compressor, known as High Pressure Turbine (HPT). We don't need to calculate anything for this turbine.

**5 → 6 *TURBINE (LPT)***

The Low Pressure Turbine (LPT) drives the fan. $TW_{LPT}$ equals the fan work $FW$.

$$FW = TW_{LPT} \Rightarrow (\dot{m}_c + \dot{F})C_{P,hot}(T_5 - T_6) = 31.47MW \Rightarrow T_6 = 1600.11 - \frac{31.47 \times 10^6}{(99.67 + 3) \times 1150} \Rightarrow \boldsymbol{T_6 = 1333.5\ K}$$

Isentropic efficiency 92%. Therefore:

$$\eta_{LPT} = \frac{T_5 - T_6}{T_5 - T_{6s}} \Rightarrow T_{6s} = T_5 - \frac{T_5 - T_6}{\eta_{LPT}} = 1600 - \frac{1600 - 1333.5}{0.92} \Rightarrow \boldsymbol{T_{6s} = 1310.4K}$$

From the isentropic process:

$$\left(\frac{P_6}{P_5}\right) = \left(\frac{T_{6s}}{T_5}\right)^{\gamma/\gamma-1} \Rightarrow P_6 = 1006.4 \times \left(\frac{1310.4}{1600}\right)^{4.3} \Rightarrow \boldsymbol{P_4 = 426.4KPa}$$

***6 → 7 NOZZLE***

It is assumed that the total pressure and temperature in the convergent nozzle remain the same.

$\boldsymbol{P_6 = P_7 = 426.4\ KPa}$

$\boldsymbol{T_6 = T_7 = 1333.5\ K}$

First of all we need to check if the nozzle is choked (M=1 at the throat).

$$\frac{P_t}{p_{st}} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/\gamma-1} = \left(1 + \frac{1.33 - 1}{2} 1^2\right)^{1.33/0.33} = 1.825 \ \ while\ \frac{P_4}{p_a} = \frac{426.4}{101} = 4.22$$

So the nozzle is choked. As a result we are able to calculate the static pressure and temperature:

$$p_{7,st} = \frac{P_5}{1.825} = \frac{426.4}{1.825} \Rightarrow \boldsymbol{p_{5,st} = 233.6\ KPa}$$

$$\frac{T_{7t}}{t_{7,st}} = \left(1 + \frac{\gamma - 1}{2} M^2\right) \Rightarrow t_{7,st} = \frac{2}{2.33} \times 1 \times 1333.5 \Rightarrow \boldsymbol{t_{5,st} = 1144.6\ K}$$

Calculation of air density at nozzle exit (5):

$$\rho_5 = \frac{p_{5,st}}{Rt_{5,st}} = \frac{233.6 \times 1000}{287 \times 1144.6} \Rightarrow \boldsymbol{\rho_5 = 0.71Kg/m^3}$$

Calculation of air-gas jet velocity at nozzle exit (7) (M=1, choked nozzle)

$$V_{j,7} = \sqrt{\gamma R t_{7,st}} = \sqrt{1.33 \times 287 \times 1144.6} \Rightarrow \boldsymbol{V_5 = 661\ m/s}$$

Calculation of nozzle exit area (7):

$$A_5 = \frac{\dot{m}}{V_{j,5} \times \rho_5} = \frac{(99.67 + 3)}{661.16 \times 0.71} \Rightarrow \boldsymbol{A_5 = 0.218m^2}$$

And therefore the nozzle exit diameter can be estimated as:

$$A_5 = \frac{\pi D_n^2}{4} \Rightarrow \boldsymbol{D_n\ = 0.527m}$$

Calculation of engine *core thrust*. We assume take-off conditions where the aircraft flight velocity is zero ($V_o$=0m/s):

$$F_{t,c} = F_N + F_A = (\dot{m} + \dot{F}) \times (V_{j,7} - V_o) + A_5 \times (p_{7,st} - p_a)$$
$$= (99.67 + 3.218) \times (661.16 - 0) + 0.197 \times (426.4 - 101)$$
$$= 67.86kN + 70.93kN = \boldsymbol{138.7kN}$$

Calculation of engine *bypass thrust*

First of all we need to check if the nozzle is choked where Mach number at the throat equals 1.

$$\frac{P_t}{p_{st}} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/\gamma-1} = \left(1 + \frac{1.4 - 1}{2} 1^2\right) = 1.825\ \ while\ \frac{P_{3F}}{p_a} = \frac{141.4}{101} = 1.4$$

And the nozzle is NOT choked. As a result there is no thrust produced from pressure difference in the bypass flow. Subsequently,

$$\frac{P_t}{p_{st}} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/_{\gamma-1}} = 1.4 \Rightarrow M = \sqrt{\frac{2}{\gamma - 1} \times \left[\left(\frac{P_t}{p_{st}}\right)^{\gamma-1/_{\gamma}} - 1\right]} = \sqrt{\frac{2}{1.4 - 1} \times 0\left[(1.4)^{0.4/_{1.4}} - 1\right]} \Rightarrow \boldsymbol{M = 0.71}$$

$$\frac{T_{3F}}{t_{3F}} = \left(1 + \frac{\gamma - 1}{2} M^2\right) \Rightarrow t_{3F} = \frac{319.5}{\left(1 + \frac{1.33 - 1}{2} 0.71^2\right)} \Rightarrow \boldsymbol{t_{3F} = 294.96\ K}$$

Calculation of bypass air jet velocity at nozzle exit (3F) (M=0.71, not choked nozzle)

$$V_{j,3} = M \times \sqrt{\gamma R T_{5,st}} = 0.71 \times \sqrt{1.33 \times 287 \times 294.96} \Rightarrow \boldsymbol{V_{3F} = 238.23\ m/s}$$

Calculation of engine bypass thrust. We assume take-off conditions where the aircraft flight velocity is zero ($V_o$=0m/s):

$$F_{t,b} = (\dot{m}_b) \times \left(V_{j,3F} - V_o\right) = 897 \times (238.23 - 0) \Rightarrow \boldsymbol{F_{t,b}} = \textbf{\textit{213.7 kN}}$$

**Total engine thrust = $F_{FAN}$+$F_{CORE}$ = 213.72+132.2 ⇒** $\boldsymbol{T = 345.9\ kN}$

Calculation of Specific fuel consumption (SFC):

$$SFC = \frac{\dot{F}}{F_{total}} = \frac{3 \times 1000}{345.9} \Rightarrow \boldsymbol{SFC = 8.67\ {}^{g}/_{kN \times sec}}$$

Calculation of Specific Thrust (ST):

$$SFC = \frac{\dot{F}}{F_{total}} = \frac{345.9}{997.6} \Rightarrow \boldsymbol{SFC = 0.346\ {}^{g}/_{kN \times sec}}$$

# SOS

FOR THE SOLUTION OF THE ABOVE EXERCISES WE DID A NUMBER OF ASSUMPTIONS!

- **One dimensional flow which means constant air properties in a plane**
- **Inviscid flow. No losses produced by viscous.**
- **Air and heat leakages through the engine are assumed very small and thus ignored**
- **We assumed perfect gases which is not true!**
- **We assumed constant gas properties which is not true!**

Aircraft Engine modeling and simulation requires much better accuracy. This is obtained by using variable gas properties. Constant gas properties can be used ONLY for manual calculation when an additional check of the computed (simulated) values is needed! This procedure (hand calculations) is quite inaccurate and introduces an error in the order of 25%. If you consider also other assumptions the total error could reach the number of 35%!!!

# CONCLUSIONS AND SOME NOTES….

$$\boldsymbol{TurboJet \rightarrow\ ST \sim 1kNs/kg \quad SFC{\sim}25g/(kNs)}$$

$$\boldsymbol{TurboFan \rightarrow\ ST \sim 0.35kNs/kg \quad SFC{\sim}8.5g/(kNs)}$$

**<u>Small Example</u>**: Let assume that we 1000kN to take-off a large aircraft and we decide to use 4 engines.

If we use the **turbo-JET engine**: $1000/4 = 250kN$ per engine.
Which means $\dot{m} = 250kg/s$ and $\dot{F} = 6.25kg/s$ fuel flow per engine. So, total $\times 4 = \boldsymbol{25Kg/sec}$ fuel flow

If we use the **turbo-FAN engine**: $1000/4 = 250kN$ per engine.
Which means $\dot{m} = 714kg/s$ and $\dot{F} = 2.12kg/s$ fuel flow per engine. So, total $\times 4 = \boldsymbol{8.5Kg/sec}$ fuel flow!

# SELECTION OF BYPASS RATIO AND DESING CONSIDERATIONS

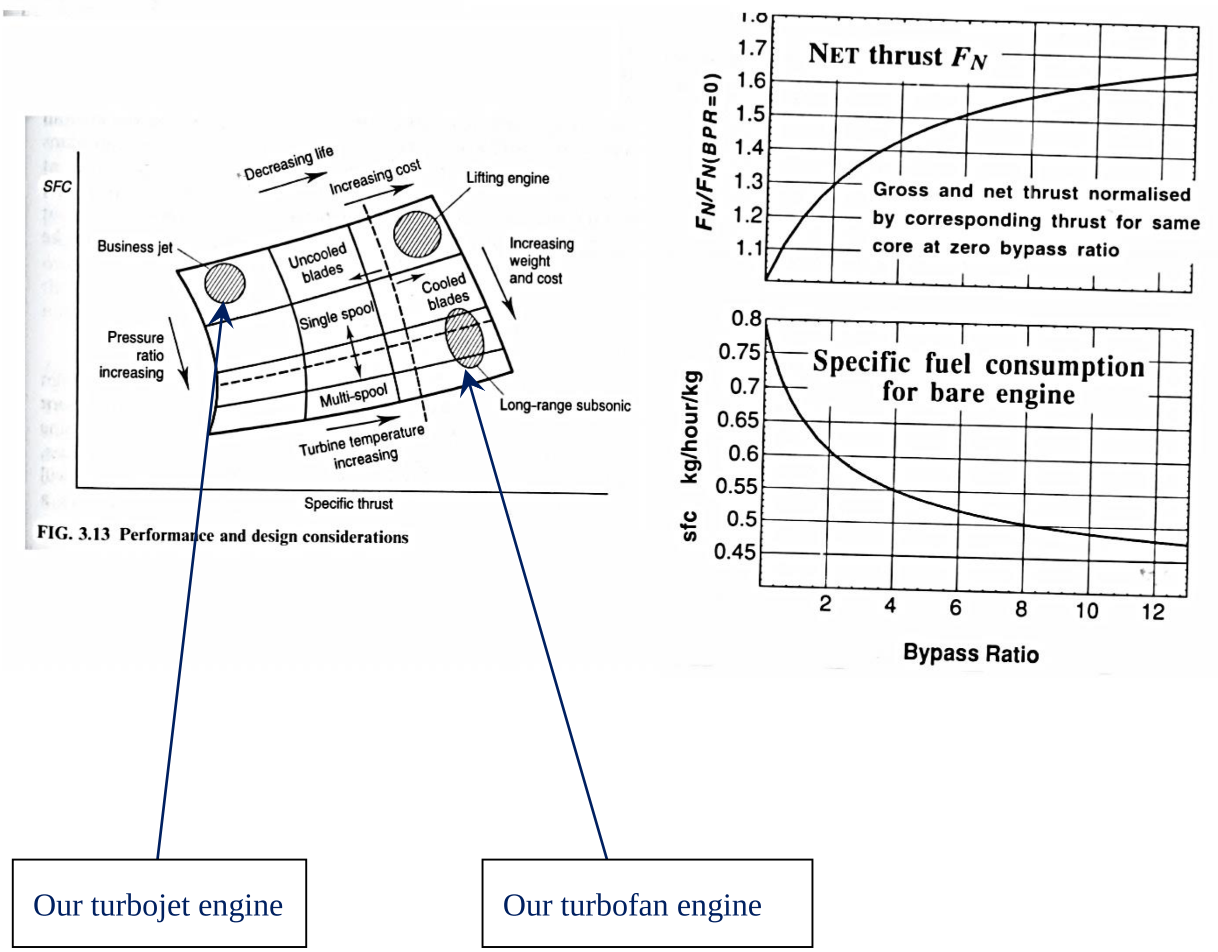

FIG. 3.13 Performance and design considerations

Further questions: alexandros.terzis@epfl.ch