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Competitive Learning

Exercise 1

1.1 Show for competitive learning that every prototype is at the center of its group/cluster after convergence (i.e. in the steady state).

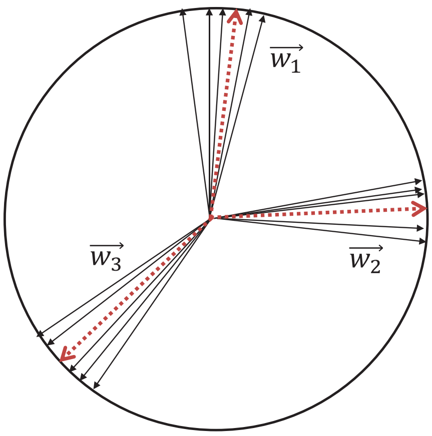
Use the (batch) learning rule: $\Delta \vec{w}_i = \eta \sum_{\mu \in C_i} (\vec{x}^\mu - \vec{w}_i)$.

1.2 Consider now the online version of the learning rule: $\Delta \vec{w}_i = \eta(\vec{x}^\mu - \vec{w}_i)$. Calculate the fluctuation of the weight update in the steady state, i. e. its variance $\langle \Delta \vec{w}_i^2 \rangle$.

Exercise 2

2.1 Show that if all prototypes/weight vectors $\vec{w}_k$ are normalized, choosing the nearest prototype $\vec{w}_i$ (i.e., the prototype for which $||\vec{x} - \vec{w}_i||^2 \leq ||\vec{x} - \vec{w}_k||^2 \; \forall k \neq i$) is equivalent to choosing the neuron i with the strongest input $\vec{w}_i^T \vec{x}$.

2.2 Does the result still hold true if it is the data that is normalized (i. e. $||\vec{x}^\mu||^2 = 1 \, \forall \mu$), but the weight vectors are not? Consider the case when the clustering algorithm is close to convergence. (Hint: See image.)



Exercise 3

Consider two neurons with three input synapses each. The initial weights are:

$$w_1 = (1.5, \; 0, \; 0.5) \quad \text{and} \quad w_2 = (0, \; 0.5, \; 1.5).$$

3.1 Perform competitive learning by hand. Use again the rule $\Delta w = \eta(x - w)$ for the weights of the winning unit with a learning rate of $\eta = 0.5$. The winning unit is the one with the highest activation (maximal scalar product $\vec{x}^T \vec{w}_i$). To do the competitive learning, present the inputs $x_1 = (0,\ 1,\ 0)$, $x_2 = (1,\ 0,\ 1)$ and $x_3 = (0,\ 1,\ 1)$ in the order $x_1, x_2, x_3, x_1, x_2, x_3$ and observe the evolution of the weights of both neurons.

3.2 Without explicit calculation, what would happen if you only applied two input vectors, i. e. the sequence x_1, x_2, x_1, x_2?