ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Unsupervised and reinforcement learning in neural networks

Week 8

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Exercise 1 (in class): Exponential STDP Window

Given the following model:

$$\begin{aligned}
\tau_+ \frac{d}{dt} x_j^{\text{pre}} &= -x_j^{\text{pre}} + \delta(t - t_j^{\text{pre}}) \\
\tau_- \frac{d}{dt} y_i^{\text{post}} &= -y_i^{\text{post}} + \delta(t - t_i^{\text{post}}) \\
\frac{d}{dt} w_{ij} &= a_+(w_{ij}) x_j^{\text{pre}} \delta(t - t_i^{\text{post}}) + a_-(w_{ij}) y_i^{\text{post}} \delta(t - t_j^{\text{pre}})
\end{aligned}$$

assume a single presynaptic spike at time t_j^{pre} and later a single postsynaptic spike at time t_i^{post}.

1. Show that the above model leads to an exponential dependence on different spike times
$$\Delta w_{ij} = a_+(w_{ij}) \exp\left(-\frac{1}{\tau_+}(t_i^{\text{post}} - t_j^{\text{pre}})\right)$$
2. What happens in the case post-before-pre?

Exercise 2: STDP and Hebbian Rate Model

Given a weight update rule which depends on the time difference between pre- and postsynaptic spike
$$\Delta w_{ij} = W(t_j^{\text{pre}} - t_i^{\text{post}})$$
Assume W has a finite support and it is therefore sufficient to integrate W over a finite interval. Furthermore assume a single stochastic neuron presynaptically, which fires in each time step of size $\Delta t = 0.5$ms with a probability $p = \nu_j \Delta t$ where ν_j is a constant firing rate (< 10Hz). The postsynaptic neuron i fires at the constant rate $\nu_i = 10$Hz.

1. Show that the STDP update rule gives an update rule of the Hebbian type. What is the factor a_2^{corr}.
2. Redo the same calculation, but assume that the postsynaptic neuron increases its firing rate for one time step by 10Hz, whenever it receives an input spike.