ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Unsupervised and reinforcement learning in neural networks

Week 8

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1 Exponential STDP Window

1.1

Since there is only a single presynaptic spike we can assume that initially $x_j^{\text{pre}} = y_i^{\text{post}} = 0$. Integrating x_j^{pre} gives

$$\begin{aligned}\tau_+ x_j^{\text{pre}} &= \Theta(t - t_j^{\text{pre}}) \int -x_j^{\text{pre}} + \delta(t - t_j^{\text{pre}}) \quad dt \\ &= \Theta(t - t_j^{\text{pre}}) \tau_+ \exp\left(-\frac{t - t_j^{\text{pre}}}{\tau_+}\right)\end{aligned}$$

Now we use this expression in the weight update equation and integrate again

$$\begin{aligned}\Delta w_{ij} &= a_+(w_{ij}) \int_{-\infty}^{\infty} x_j^{\text{pre}} \delta(t - t_i^{\text{post}}) \quad dt \\ &= a_+(w_{ij}) \int_{t_j^{pre}}^{\infty} \exp\left(-\frac{t - t_j^{\text{pre}}}{\tau_+}\right) \delta(t - t_i^{\text{post}}) \quad dt \\ &= a_+(w_{ij}) \exp\left(-\frac{t_i^{\text{post}} - t_j^{\text{pre}}}{\tau_+}\right)\end{aligned}$$

1.2

In analogy to above we get a weight update

$$\Delta w_{ij} = a_-(w_{ij}) \exp\left(\frac{1}{\tau_-}(t_i^{\text{post}} - t_j^{\text{pre}})\right)$$

2 STDP and Hebbian Rate Model

2.1

The temporal average is defined as follows

$$\langle f(t) \rangle_T \equiv \frac{1}{T} \int_t^{t+T} f(s) ds \tag{1}$$

We define further $S_i(t) = \sum_k \delta(t - t_i^k)$ the spiketrain emitted by neuron i, where the sum runs over all firing times t^k. We are interested in the averaged weight change per unit time during the interval T

$$\begin{aligned}\langle \frac{d}{dt} w_{ij}(t) \rangle_T &= \int_{-\infty}^{\infty} W(s) \langle S_i(t-s) S_j(t) \rangle_T \quad ds \\ &= \nu_i \nu_j \int_{-\infty}^{\infty} W(s) \quad ds\end{aligned}$$

where we used

$$\langle S_i(t-s)S_j(t)\rangle_T = \nu_i\nu_j \tag{2}$$

We therefore see the Hebbian nature of the learning rule. Here the integral over the learning window W corresponds to a_2^{corr}.

2.2

In a more realistic scenario presynaptic spikes are the cause of postsynaptic spiking. i.e. the two spike trains are correlated.
We define the response kernel $\epsilon(x)$ is the response kernel $\epsilon(x) = \mu\Theta(x)\Theta(\Delta - x)S_j(x)$ with the Heaviside function Θ and the time window Δ that describes the effect of a presynaptic spike on the postsynaptic neuron. We also define $\mu = 10\,\text{Hz}$. Temporal averaging now gives

$$\begin{aligned}
\langle \frac{d}{dt} w_{ij}\rangle_T &= \nu_i\nu_j \int_{-\infty}^{\infty} W(s)\ ds + \int_{-\infty}^{\infty} \langle W(s)\epsilon(t-s)\rangle_T\ ds \\
&= \nu_i\nu_j \int_{-\infty}^{\infty} W(s)\ ds + \mu\nu_j \int_{-\infty}^{\infty} W(s)\Theta(s)\Theta(\Delta - s)\ ds \\
&= \nu_i\nu_j \int_{-\infty}^{\infty} W(s)\ ds + \mu\nu_j \int_{0}^{\Delta} W(s)\ ds
\end{aligned}$$