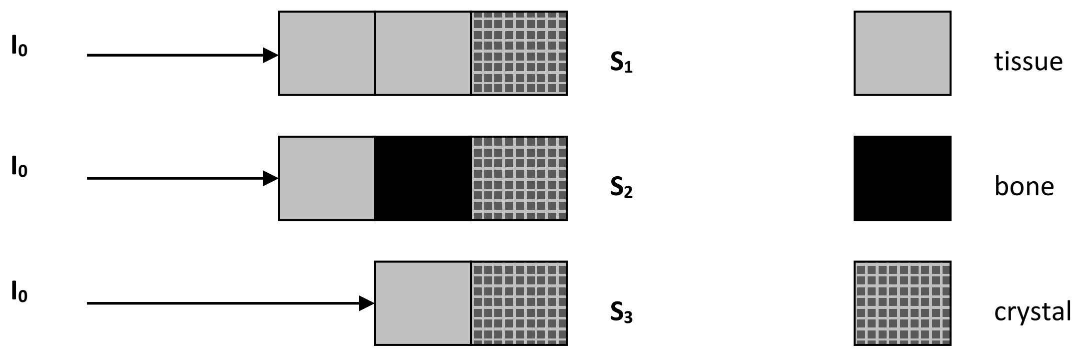
Solution 1 – Crystal Sensors

The value of μ_{tissue} is 0.5 cm$^{-1,}$ μ_{bone} is 1 cm^{-1}, and $\mu_{crystal}$ is 2 cm^{-1}



The intensity of signal (light) produced by the crystal is proportional to the attenuation of X-rays within the crystal.

The intensity of signal coming into the crystal is equal to:

$$I_{in_1} = I_0 e^{-0.5*1} e^{-0.5*1} = I_0 e^{-0.5*2}$$
$$I_{in_2} = I_0 e^{-0.5*1} e^{-1*1}$$
$$I_{in_3} = I_0 e^{-0.5*1}$$

The intensity of signal coming out of the crystal is equal to:

$$I_{out_1} = I_{in_1} e^{-2*1}$$
$$I_{out_2} = I_{in_2} e^{-2*1}$$
$$I_{out_3} = I_{in_3} e^{-2*1}$$

Therefore the intensity of signal (light) produced by the crystal is given by:

$$S \propto crystal\ absorption = I_{in} - I_{out}$$

$$S_1 \propto I_{in_1} - I_{out_1} = I_{in_1} - I_{in_1} e^{-2*1} = I_0 e^{-0.5*2}(1 - e^{-2*1}) = 0.318\, I_0$$
$$S_2 \propto I_{in_2} - I_{out_2} = I_{in_2} - I_{in_2} e^{-2*1} = I_0 e^{-0.5*1} e^{-1*1}(1 - e^{-2*1}) = 0.193\, I_0$$
$$S_3 \propto I_{in_3} - I_{out_3} = I_{in_3} - I_{in_3} e^{-2*1} = I_0 e^{-0.5*1}(1 - e^{-2*1}) = 0.524\, I_0$$

Solution 2 – Radiation Detection

With all the conversion efficiency coefficients given for the different physical processes involved in the radiation detection, the total efficiency of the detection can be obtained by simply multiplying the efficiencies of the subprocesses :

Nb of scintillation photons produced in the NaI(T1) crystal : $60[keV] * 30 \left[\frac{\gamma}{keV}\right] = 1800\, \gamma$
Nb of photons absorbed by the photocathode: $1800\, \gamma * 80\% = 1440\, \gamma$
Nb of electrons produced in the photocathode: $1440\, \gamma * 0.05 = 72\ electrons$

Nb of electrons produced after multiplication in the dynodes: $72 * 3^{10} = \mathbf{4.25\ 10^6}\ \boldsymbol{electrons}$

Solution 3 – SNR Considerations

a) Since the SNR is proportional to the square root of the number of counts, the doubled injected dose increases the SNR by the square root of 2 to give a value of 71:1.

b) The activity decay is described by $\mathrm{A(t) = A_0 e^{-\lambda t}}$.

λ can be derived from the half-life time with the following relation : $\lambda = \frac{\ln(2)}{T_{1/2}} = 3.21\ 10^{-5} s^{-1}$

The number of counts (C) in the experiment is assumed to be proportional to the number of disintegrations (D): $\mathrm{C = \alpha\ D}$ where α is the efficiency of the detection.

The SNR is proportional to the square root of the number of counts. The activity is the infinitesimal number of disintegrations per unit of time. Thus, the number of disintegrations during a given period is the integral of the activity in this period:

$$\mathrm{D_{1,2}} = \int_{t1}^{t2} \mathrm{A(t)dt} = \int_{t1}^{t2} \mathrm{A_0 e^{-\lambda t} dt} = \frac{-1}{\lambda} \mathrm{A_0} \left[\mathrm{e^{-\lambda t}}\right]_{t_1}^{t_2} = \frac{\mathrm{A_0}}{\lambda}\left(\mathrm{e^{-\lambda t_1} - e^{-\lambda t_2}}\right) = \frac{1}{\lambda}\left(\mathrm{A(t_1) - A(t_2)}\right)$$

The detected counts during this period are thereby:

$$\mathrm{C_{1,2}} = \frac{\alpha}{\lambda}\left(\mathrm{A(t_1) - A(t_2)}\right) = \frac{\alpha\,\mathrm{A_0}}{\lambda}\left(\mathrm{e^{-\lambda t_1} - e^{-\lambda t_2}}\right)$$

We know now that the SNR is proportional to the number of counts in the measurement. In the case of the 30 min scan, we have:

$$\mathrm{SNR_1} \propto \sqrt{\mathrm{C_{0,30min}}} = \sqrt{\frac{\alpha}{\lambda}\left(\mathrm{A(0) - A(30min)}\right)}$$

In the case of the 60 min scan, we have:

$$\mathrm{SNR_2} \propto \sqrt{\mathrm{C_{0,60min}}} = \sqrt{\frac{\alpha}{\lambda}\left(\mathrm{A(0) - A(60min)}\right)}$$

In both cases, λ is the same since the radiotracer is the same and α is also the same in both measurements, assuming same detection geometry. Thus, since we are working with proportionalities, we can write:

$$\mathrm{SNR_1} \propto \sqrt{\left(\mathrm{A(0) - A(30min)}\right)}$$
$$\mathrm{SNR_2} \propto \sqrt{\left(\mathrm{A(0) - A(60min)}\right)}$$

$\mathrm{A(0)}$ is in both cases 1mCi.

$\mathrm{A(30min) = A(1800\ seconds) = A(0)e^{-\lambda(t=1800s)} = A(0)e^{-3.21*10^{-5}(t=1800s)}}$

Similarly, $\mathrm{A(60min) = A(3600\ seconds) = A(0)e^{-\lambda(t=3600s)} = A(0)e^{-3.21*10^{-5}(t=3600s)}}$

We know that the $\mathrm{SNR_1 = 50{:}\,1}$. Let's calculate the ratio between $\mathrm{SNR_1}$ and $\mathrm{SNR_2}$ to get free from the proportionalities:

$$\frac{\mathrm{SNR_2}}{\mathrm{SNR_1}} = \sqrt{\frac{\left(\mathrm{A(0) - A(60min)}\right)}{\left(\mathrm{A(0) - A(30min)}\right)}} = \sqrt{\frac{\mathrm{A(0)}\left(1 - \mathrm{e^{-3.21*10^{-5}*3600}}\right)}{\mathrm{A(0)}\left(1 - \mathrm{e^{-3.21*10^{-5}*1800}}\right)}} = 1.394$$

So, since $\mathrm{SNR_1}$ is 50:1, then $\mathrm{SNR_2}$ is $1.394 * \mathrm{SNR_1} = \boxed{69.7{:}\,1}$

c) We calculate first the energy of the emitted photon :

$$E = h\nu = h\frac{c}{\lambda} = 2.26\,J = 141\ keV$$

In the table at the end of the series 5, we see that the mass attenuation coefficient of iron at this energy is :

$$\mu/\rho = 1.96\ 10^{-1}\ cm^2/g$$

The linear attenuation coefficient can then be calculated using the density of iron:

$$\mu = \mu/\rho * \rho = 1.54\ cm^{-1}$$

The measured intensity behind 2 cm of iron will be :

$$I_1 = I_0 e^{-\mu x} = I_0 e^{-1.54*2} = 0.046\ I_0$$

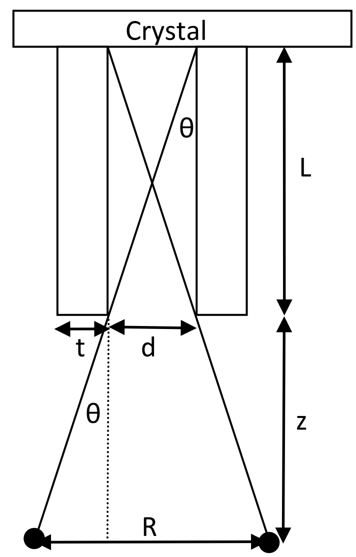
We know that the SNR goes with the square root of the number of counts (or intensity).
For I_0, we had a SNR of 50:1. We have then :

$$\frac{SNR_1}{SNR_0} = \frac{\sqrt{I_1}}{\sqrt{I_0}} = \frac{\sqrt{0.046\ I_0}}{\sqrt{I_0}} = 0.21$$

$$SNR_1 = 0.21 * SNR_0 = 11:1$$

Solution 4 – Collimation I

Sizes are displayed in the figure on the right. The resolution R can be defined as the minimum distance at which two point sources can still be separated. Two triangles with an angle θ can be defined so that tanθ=d/L=½(R-d)/z, so R=(2dz+dL)/L.



Solution 5 – Collimation II

a) The measured sensitivity for ^{99m}Tc is 5.88 10^5/(51.80 10^3·2) = 5.676 counts/(kBq·min). The factory specification is 202 cnts/(μCi·min) = 202/37 cnts/(kBq·min) = 5.459 cnts/(kBq·min). Here μCi = 37 kBq is used. The measurement thus gives a 4% higher sensitivity.

b) The surface of the Petri dish that can be seen from P is circular and has a radius R=0.5 D·(H+L)/L if H is the distance from the dish to the top of the collimator. The surface is then πR^2. Only the activity within this surface can be attributed to the number of counts in P. The chance that radiation from this surface goes through a surface dA around P is equal to the relative spatial angle (ω_r) under which dA is seen: $\omega_r = dA/(4\pi\cdot(H+L)^2)$. The number of counts in dA around P is then proportional to $\pi R^2\cdot\omega_r = \pi\{0.5\cdot D\cdot(H+L)/L\}^2\cdot dA/(4\pi\cdot(H+L)^2) = 0.0625(D/L)^2$ $dA = 0.0625\ (1.45/24.1)^2\ dA = 2.262\ 10^{-4} dA$. This is independent of H. This example can be generalized by looking at a random spatial angle from a random point on the crystal.

c) The number of crystal parts under the Petri dish is given by: $n = \frac{S}{\pi\frac{D^2}{4}}$

The part *a* of the total activity *A* in front of each hole is: $a = \frac{A}{S}\pi\frac{D^2}{4}$

The radiation density *r* seen by each crystal part is given by the spatial angle with which a point on the Petri dish "sees" the crystal part:

$$r = a * \frac{\pi\frac{D^2}{4}}{4\pi L^2}$$

Finally, the total radiation density *R* seen by the complete crystal is:

$$R = n * r = \frac{S}{\pi\frac{D^2}{4}} * a * \frac{\pi\frac{D^2}{4}}{4\pi L^2} = \frac{S}{\pi\frac{D^2}{4}} * \frac{A}{S}\pi\frac{D^2}{4} * \frac{\pi\frac{D^2}{4}}{4\pi L^2} = A * \underbrace{\frac{\pi\frac{D^2}{4}}{4\pi L^2}}_{\substack{geometric\\ efficiency\ \varepsilon}}$$

$$\varepsilon = 2.262 * 10^{-4}$$

d) The factory specification says 202 cnts/(μCi·min) are counted. An activity of 1 μCi corresponds to $60\cdot 3.7\ 10^4$ disintegrations per minute. The fraction of disintegrations that leads to a count (using 1 Ci = 37 GBq, 1 Bq = 1/s) is then:

$$(202\ \text{cnts/min}) / (3.7\cdot 10^4\ \text{Bq}) = (202\ \text{cnts/}\ 60\ \text{s}) / (3.7\cdot 10^4\ \text{s}) = 9.099\ 10^{-5}.$$

e) The geometric sensitivity (c.) is higher because (I) in the real measurement not all gammas are detected (only photons that undergo photo-absorption are counted, but there is also Compton scattering), (II) we neglected the surface of lead which decreases the effective crystal surface, (III) there is attenuation in the fluid and the dish, the coating of the collimator and the protection of the NaI crystal, and (IV) ^{99m}Tc only emits a 140 keV photon in 88% of its disintegrations.

However, detection of scattered photons also takes place, which will slightly compensate for the effects mentioned above. Apparently the effects under (I) to (IV) dominate though.