Solution 1 – PET vs SPECT characteristics

	CT	SPECT	PET
Method of projection data collection	Directionality is defined by the incident x-ray, but collimation is used in order to reduce scattering effect.	Collimation is essential to know where the signal comes from.	Annihilation coincidence detection gives the signal directionality.
Transverse image reconstruction	Filtered backprojection or iterative methods		
X-rays emission	Incident x-ray beam (attenuated through the sample)	Any radionucleides emitting x-rays, gamma rays, or annihilation photons; optimal performance for photon energies of 100 to 200 keV	Positron emitters only
Spatial resolution	Depends basically on camera orbit. Typically between μm and mm.	Depends on collimator and camera orbit Within a transaxial image, the resolution in the radial direction is relatively uniform, but the tangential resolution is degraded toward the center Typically about 10 mm FWHM at center for a 30 cm diameter orbit and Tc-99m Larger camera orbits produce worse resolution	Relatively constant across transaxial image, best at center Typically 4.5 to 5 mm FWHM at center
Attenuation	CT measures x-ray attenuation, so NO attenuation correction. However absorption depends on x-ray energy and creates contrast changes with large objects.	As x-rays emitted by tracers in body are measured, attenuation is an issue. No severe attenuation. Attenuation correction sources available, but utility not yet established.	Severe attenuation. However, accurate attenuation correction is possible, with a transmission source.

Solution 2 – Coincidence Detection

$$\varepsilon_{geom} = \frac{\pi d^2/4}{4\pi R^2} = \frac{0.25\cdot 3.14\cdot 5^2}{4\cdot 3.14\cdot 20^2} = 3.91\cdot 10^{-3}.$$

a) The geometric efficiency is given by the relative spatial angle at which the source 'sees' the detector:

b) Two 511 keV gammas are created per decay. Therefore the activity appears to be 2A for a single detector. If a photon reaches a detector, the chance of a photo peak is
$\varepsilon_{fotopeak} = 0.7 \cdot 0.5 = 0.35$. The singles count rate of photo peak pulses is then $n_{singles} = 2 \cdot A \cdot \varepsilon_{geom} \cdot \varepsilon_{fotopeak} = 2 \cdot 10^7 \cdot 3.91\ 10^{-3} \cdot 0.35 = 2.74\ 10^4$ cnts/s.

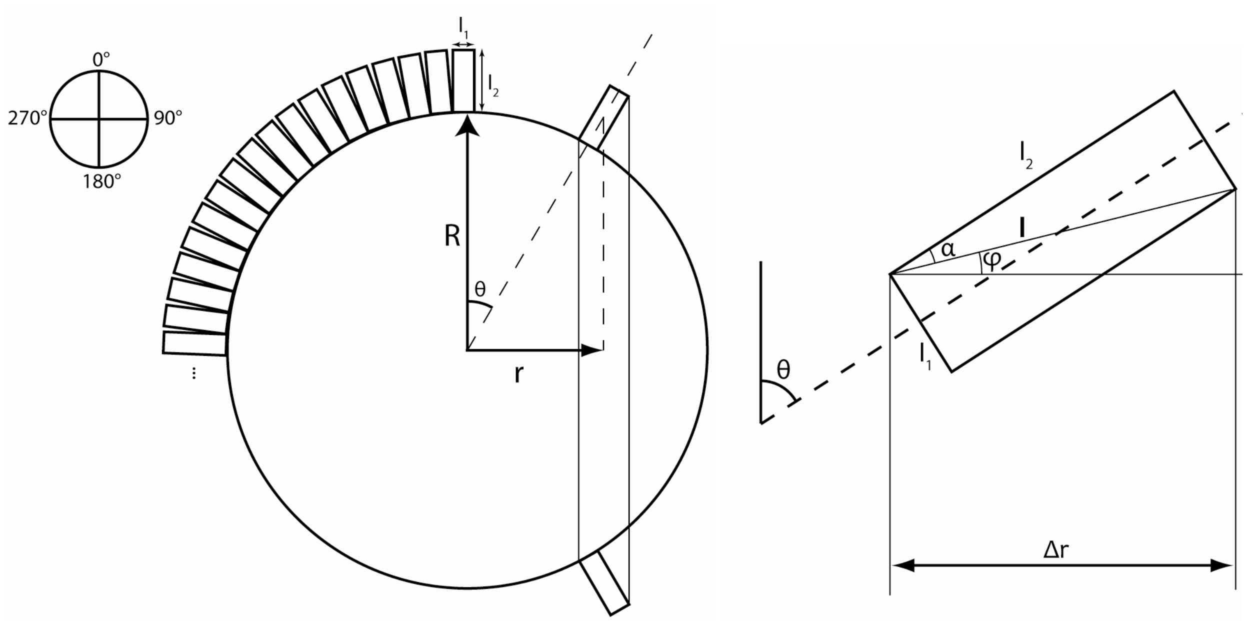
c) The coincidence count rate is given by $n_{coinc} = n_{singles} \cdot \varepsilon_{fotopeak} = 2.74\ 10^4 \cdot 0.35 = 9.60$ kcnts/s. This expression lacks ε_{geom} because a photon that reaches the surface of a detector has a partner that must reach the opposing detector because of the symmetry of the setup.

d) A random count occurs when detector 1 detects a photon and detector 2 detects one in the interval from 10 ns before to 10 ns after this occurrence. If the singles count rate is $n_{singles}$, then per second during a time $n_{singles} \cdot 2\tau$ in principle random coincidences can be generated by detector 2. Pulses from detector 2 also arrive at random at a count rate of $n_{singles}$. The number of randoms will then be $n_{randoms} = n_{singles}^2 \cdot 2\tau = (2.74\ 10^4)^2 \cdot 20\ 10^{-9} = 15$ cnts/s.

e) Random coincidences can be measured with this setup by making the angle between the detectors 90^o for example. Real coincidences will then no longer be measured. In practice some coincidences will occur due to scattering, but in this problem we assumed there is no scattering.

Solution 3 – PET radial resolution variations

The geometry of the detection in a PET scanner has usually a cylindrical symmetry. It is built in order to obtain the best detection characteristics in the middle of the ring of detectors. However, the resolution and sensitivity might depend significantly from the distance to the center of the scanner.
Let's consider one of these detection rings or radius R with rectangular detectors of size $l_1 * l_2$, ($l_1 << R$) and linear absorption coefficient μ. A point source is placed in the ring (no scatter or random events).



a) The width of the line of response (LOR) *Δr* is the projection of the diagonal of the crystal :

$$\Delta r = l\cos(\varphi)$$
$$= \sqrt{l_1^2 + l_2^2}\cos(\varphi)$$

The angle φ can be derived from the geometry of the setup :

$$\varphi = \frac{\pi}{2} - (\theta + \alpha)$$

where $\alpha = \arctan\left(\frac{l_1}{l_2}\right)$ and $\theta = \arcsin\left(\frac{r}{R+\frac{l_2}{2}}\right)$

Finally, we obtain for Δr :

$$\Delta r = \sqrt{l_1^2 + l_2^2}\ \cos\left(\frac{\pi}{2} - \left(\arcsin\left(\frac{r}{R+\frac{l_2}{2}}\right) + \arctan\left(\frac{l_1}{l_2}\right)\right)\right)$$

$$= \sqrt{l_1^2 + l_2^2}\ \sin\left(\arcsin\left(\frac{r}{R+\frac{l_2}{2}}\right) + \arctan\left(\frac{l_1}{l_2}\right)\right)$$

This relation is only valid for Δr positive. For Δr negative, the other diagonal of the crystal has to be considered. For angles starting from zero, the sine function is a growing function. Arcsin is also a growing function. Δr will thus continuously increase with r.

b) Δr is a sine function multiplied with a scaling factor equal to the diagonal of the crystals $\sqrt{l_1^2 + l_2^2}$. In order to improve the resolution of the center, small square crystals are advantageous. In practice, this is not really possible, because it would reduce significantly the sensitivity of the detection. If the crystals are too small, not enough photons will be absorbed (and thus detected) in the crystals.

We can see that there is an offset in the sine function, equal to $\arctan\left(\frac{l_1}{l_2}\right)$. Reducing this offset would help maintaining Δr smaller. In this case, it would be of advantage to have $l_1 \ll l_2$. However, having a too big l_2

increases drastically the diagonal of the crystal, harming in this case Δr through the scaling factor $\sqrt{l_1^2 + l_2^2}$.

In summary, a compromise has to be found to obtain both good resolution and sensitivity. In practice, small dimensions are also disadvantageous because of the packing of the crystals taking relatively more and more space, thus reducing the effective crystal surface and thus the sensitivity.