Fundamentals of Bioimaging (Prof. Gruetter)
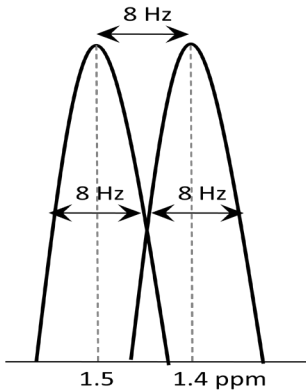*Solutions to Problem Set No. 10*

Lifmet CBM

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Solution 1: spectroscopy basics

a)   The ppm (parts per million) is defined as the difference in frequency with a reference signal (in ^{1}H usually TMS, tetramethylsilane, $Si(CH_3)_4$), divided by that reference signal, times one million:

$\delta = (\nu - \nu_{REF})$ x10^6 / ν_{REF}. It is used since this unit makes the chemical shift magnetic-field independent, thus making studies at different fields more comparable.

b)   The two peaks are 0.1 ppm apart. This 0.1 ppm must be at least 8 Hz to be able to separate the peaks. Therefore, the question translates to 'At what field strength is 8 Hz equal to 0.1 ppm?'

Since $\Delta\omega/\omega_a$*10^6 = 0.1 ppm, 8 = ω_a*10^{-6}*0.1 = γB_0*10^{-7} = 42.58*10^6 * B_0*10^{-7}

B_0 = 80/42.58 = 1.88 T.

c)   Solution 1: Since the T2 of molecule a and b are much longer than those of the oil, we can employ a spin echo sequence with a long TE (eg. 300ms), and the transverse magnetization of the oil will mostly decay while most transverse magnetization of molecule a and b will be reserved.

        Solution 2: Since the T1 of the oil is much shorter than those of the molecules a and b, we can also use spin echo combined with inversion recovery. The TI has to be chosen such that the oil magnetization is zeroed.

$$M_z = M_0(1 - 2\exp(-\frac{TI}{T_1})) = 0$$

$$T_1 = 360(ms)$$
$$TI = 360 \cdot \ln 2(ms)$$

TE should be as short as possible. The oil signal will be suppressed by magnetization preparation before the spin echo sequence.

Solution 2: MRS sensitivity

a) The energy difference between the two spin states of ^{1}H in a magnetic field of 3T:

$$\Delta E_{3T} = \frac{\gamma h B_0}{2\pi} = \frac{(267.512 \times 10^6 \, rad \cdot T^{-1} \cdot s^{-1})(6.63 \times 10^{-34} J \cdot s)(3T)}{2 \cdot 3.14 \, rad} = 0.85 \times 10^{-25} J$$

The population for each state is given by the Boltzmann equation:

$$\frac{P_{(m=-1/2)}}{P_{(m=+1/2)}} = \exp(-\frac{\Delta E}{kT}) = \exp(-\frac{0.85 \times 10^{-25} J}{1.381 \times 10^{-23} JK^{-1} \times 310K}) = 0.99998$$

Since there are only two spin states,

$$P_{(m=+1/2)} + P_{(m=-1/2)} = 1$$

$$SNR \propto \left| P_{(m=+1/2)} - P_{(m=-1/2)} \right| = \left| \frac{\exp(-\frac{\Delta E}{kT}) - 1}{\exp(-\frac{\Delta E}{kT}) + 1} \right| = 9.9 \cdot 10^{-6}$$

The same calculation is done at 14T :

$$\Delta E_{14T} = \frac{\gamma h B_0}{2\pi} = \frac{(267.51 \times 10^6 \, rad \cdot T^{-1} \cdot s^{-1})(6.63 \times 10^{-34} J \cdot s)(14T)}{2 \cdot 3.14 \, rad} = 3.96 \times 10^{-25} J$$

$$\frac{P_{(m=-1/2)}}{P_{(m=+1/2)}} = \exp(-\frac{\Delta E}{kT}) = \exp(-\frac{3.96 \times 10^{-25} J}{1.381 \times 10^{-23} JK^{-1} \times 310K}) = 0.99991$$

$$P_{(m=+1/2)} + P_{(m=-1/2)} = 1$$

$$SNR_{14T} \propto \left| P_{(m=+1/2)} - P_{(m=-1/2)} \right| = \left| \frac{\exp(-\frac{\Delta E}{kT}) - 1}{\exp(-\frac{\Delta E}{kT}) + 1} \right| = 4.6 \cdot 10^{-5}$$

As result at 14T the signal intensity is stronger: $\dfrac{SNR_{14T}}{SNR_{3T}} = 4.6 \approx \dfrac{14}{3}$

b) The larmor frequency of 1H at 3T and 14T is equal to:

$$\omega_0(3T) = \gamma B_0 = 127 MHz$$
$$\omega_0(14T) = 600 MHz$$

So the difference in hertz between 2 peaks separated of 0.1ppm is given by:

Δf(3T)=0.1ppm ω_0(3T) 10^{-6}=12.7Hz

Δf(14T)=0.1ppm ω_0(14T) 10^{-6}=60Hz

As result at 14T the difference is bigger: $\dfrac{\Delta f(14T)}{\Delta f(3T)} \cong 4.7 \approx \dfrac{14}{3}$

c) In conclusion as shown in part a) at high magnetic field strength the sensitivity is improved proportionally to the magnetic field strength. Secondly the spectral dispersion (separation between resonances in Hertz) is increased with increasing field strength as well (solution b).

d) The signal intensity at 14T from proton has been already calculated in a):

$SNR_{14T}(^1H) \propto 4.6 \cdot 10^{-5}$

For 13C the gyromagnetic ratio change:

$$\Delta E_{14T} = \frac{\gamma(^{13}C)hB_0}{2\pi} = \frac{(67.2615 \times 10^6 \, rad \cdot T^{-1} \cdot s^{-1})(6.63 \times 10^{-34} J \cdot s)(14T)}{2 \cdot 3.14 \, rad} = 0.994 \times 10^{-25} J$$

$$\frac{P_{(m=-1/2)}}{P_{(m=+1/2)}} = \exp(-\frac{\Delta E}{kT}) = \exp(-\frac{0.994 \times 10^{-25} J}{1.381 \times 10^{-23} JK^{-1} \times 310K}) = 0.99998$$

$$P_{(m=+1/2)} + P_{(m=-1/2)} = 1$$

$$SNR_{14T}(^{13}C) \propto |P_{(m=+1/2)} - P_{(m=-1/2)}| = \left| \frac{\exp(-\frac{\Delta E}{kT}) - 1}{\exp(-\frac{\Delta E}{kT}) + 1} \right| = 1.16 \cdot 10^{-5}$$

$$\frac{SNR_{14T}(^1H)}{SNR_{14T}(^{13}C)} \cong 4 \approx \frac{\gamma(^1H)}{\gamma(^{13}C)}$$

e) 2 solutions: First increase the voxel size 4 times bigger for the ^{13}C acquisition (increased 4x the number of spins). Secondly increased 16 times the acquisition time (SNR ~ $\sqrt{\#scans}$). However, there is a third possibility by lowering the temperature but this solution is not used in vivo (77 K or -196 °C).

Solution 3: TR optimization

The optimized signal is given by the Ernst angle, which is defined by the formula :

$$cos\alpha_E = e^{-TR/T_1}$$

The optimized angle is: For Taurine, $\alpha_E = \arccos\left(e^{-\frac{TR}{T_1}}\right) = 66°$

For tCr, $\alpha_E = \arccos\left(e^{-\frac{TR}{T_1}}\right) = 82°$

Consequently the optimized flip angle would be $\alpha_E = 74°$.

Solution 4: Water and lipid suppression

a) For water protons, after the 45 degree pulse applied along the x axis:

$$M_{z,water} = M_{0,water} \cos 45, \quad M_{y,water} = M_{0,water} \sin 45$$

During the time τ, since the water protons are on-resonance, the nuclei do not precess in the rotating reference frame. The effect of the 45 degree pulse along the −x axis is to return the net magnetization to the z-axis, thus giving no observable signal.

For the fat proton, the situation after the first 45 degree pulse is exactly as before:

$$M_{z,fat} = M_{0,fat} \cos 45, \quad M_{y,fat} = M_{0,fat} \sin 45$$

However, during the time τ, the fat protons precess around B₀. Since ω=π/τ, in time τ they precess π radians, and so the situation after time τ is:

$$M_{z,fat} = M_{0,fat} \cos 45, \quad M_{y,fat} = -M_{0,fat} \sin 45$$

Now the application of a 45 degree pulse applied about the −x axis gives:

$$M_{z,fat} = 0, \quad M_{y,fat} = -M_{0,fat}$$

So there is maximum signal from fat, and zero from water. This is a good way to maximize contrast in an MR image. It is also used in spectroscopy where the water signal dominates that of metabolites. Notice that if we change the reference frequency we can eliminate the signal from fat, which is often done in fat-suppressed MR imaging.

b) As seen in serie 9, the pulse length of a hard pulse is inversely proportional to its bandwidth.
At a B_0 field, a bandwidth of 3.6ppm is equal to $3.6*\gamma B_0/(2\pi)*10^{-6}$ Hz
It means that, at 14.1T (resp.9.4T), the excited bandwidth is 2160Hz (1440Hz) and the needed pulse length is 0.5ms (0.7ms).