Problem 1: Quiz of the Week

Remark on Notation

We try to use the symbol ***γ*** (gamma) for the gyromagnetic ratio in ***rad/s*** throughout the course (if we did not, please let us know).

Answer the following questions :

a) What changes in the image if the k-space is sampled more densely (i.e. the distance between two k-space lines gets smaller)?

b) What condition has to be fulfilled so that a gradient echo occurs?

c) Explain why gradient-encoding cannot be used simultaneously in all three dimensions.

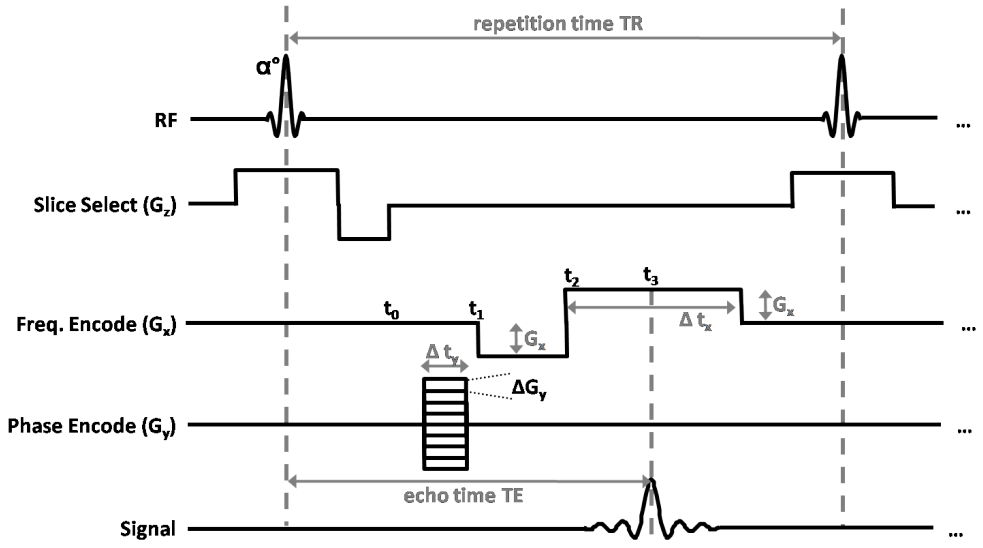
d) Why is it interesting to have strong gradient coils in MRI?

e) Why is an echo needed and the signal is not simply sampled with a reading gradient directly after a 90° pulse, for example?

Problem 2: The 2D FLASH Sequence

The development of the fast gradient echo sequence using small flip angles (which was baptised "fast low angle shot = FLASH") in the mid-eighties has greatly helped to improve and speed up MR imaging. In this Problem, we will try to analyse and understand this sequence in more detail.

Here is the basic gradient echo sequence you learned about in the course:



I. Slice Selection

In order to image a three-dimensional object with our 2D imaging gradient echo sequence, we first have to excite only a thin layer (i.e. a slice) of the object. This allows us to consecutively acquire one slice after the other, eventually yielding the whole volume image of the object.

a) Suppose the RF bandwidth is 1.0 kHz and a 3-mm-thick slice is desired. What gradient amplitude should be used for the slice selection?

b) At 1.5 T, water resonates approximately $_{fcs}$=210 Hz higher than lipids (i.e. fat). If the amplitude of the slice-selection gradient is +11mT/m, what is the slice-selection offset caused by chemical shift?

c) After slice selection, the phases of the excited spins are de-phased in the direction orthogonal to the slice. This so-called "phase dispersion" can be rewound using a gradient opposite to the one used for selecting the slice. Assume the spins are excited exactly at time point 0.5*RF_duration. The RF pulse is 1 ms long. How big is the maximal phase dispersion of the slice in Problem a) above?

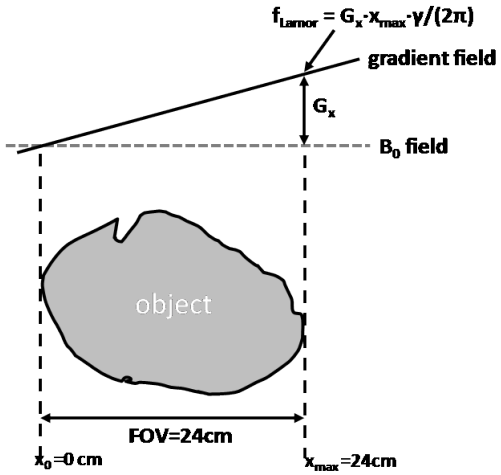
II. Gradient Echo

We are now at time point t_0, having excited only the spins of the desired slice; this reduced our imaging problem to only two dimensions. In the following, we analyse this two-dimensional image acquisition.

a) For the sake of simplicity, the gradients in the diagram are drawn as rectangles (instead of trapezoids like in the course). Why is this (i.e. the rectangular case) unrealistic? Give a physical explanation.

b) Let $\varphi(t, x, y)$ be the phase of a spin at time t and position (x,y) of the excited slice. We assume that after slice selection, the phase is zero ($\varphi(t_0, x, y) = 0\ rad$). Give a mathematical description of the phase at time points t_1-t_3 of the N^{th} phase encoding step with respect to the imaging gradients amplitudes G_x and ΔG_y.

c) Suppose our scanner has a maximal gradient amplitude of G_{max}=40 mT/m. A volume of 80 slices, each having a matrix size of 128x128 pixels on a field-of-view (FOV) of 24x24 cm^2 is acquired from this scanner with a sampling bandwidth f_s=128 kHz. Calculate the necessary gradient amplitude G_x using the diagram below.



Using this result, you can calculate the minimal time needed for the phase-encoding gradient. Given that your slice selection gradient is 1 ms long, what is the minimal TE and TR of this sequence? Using these values, what is the time it takes to acquire the whole volume?

d) In the sequence as shown in the diagram, the slice rephase gradient, the phase encode gradient and the frequency encode dephasing gradient are played out one after the other. This prolongs the echo time TE and the repetition time TR considerably. To speed up the sequence, the three gradients can be played out at the same time. Prove (taking the considerations from b)) that this still creates the same gradient echo at time point t_3. Sketch also the new sequence diagram and calculate the minimal TE with the parameters from c).

e) What further possibilities do you see to speed up the sequence?

f) Sketch the k-space trajectory of this sequence.

g) Usually, only one k-space line per TR is acquired. Why is that? If we acquired more than one line per excitation, what would we have to do and what problems would we encounter?

Problem 3: Creatine Bottles

3 bottles of diluted Creatine in water (having a singlet at 3.02 ppm) are positioned in a 3T magnetic field. After excitation, the signal is acquired using a gradient echo 40ms after excitation. A gradient is applied along + x with strength of $5*10^{-2}$ mT/m.

a) Explain what the effect of the gradient is on the Creatine resonance frequency.

b) How far the bottles should be from each other and which concentration ratio would they have in order that the acquired Creatine signal (of the singlet) would look like an ethanol (CH_3-CH_2-OH) spectrum? At low resolution, the ethanol spectrum is composed of three peaks at 1.23 ppm (CH_2), 2.61 ppm (OH) and 3.79 ppm (CH_3).

c) The 3 bottles are replaced by 3 pure water bottles and are randomly positioned in a 2D space. A 2D imaging experiment is performed using a phase encoding gradient along the x direction and a readout gradient along the y direction.
How will the k-space look like? Is it true that only three points of the k-space will be non zero? Explain why.