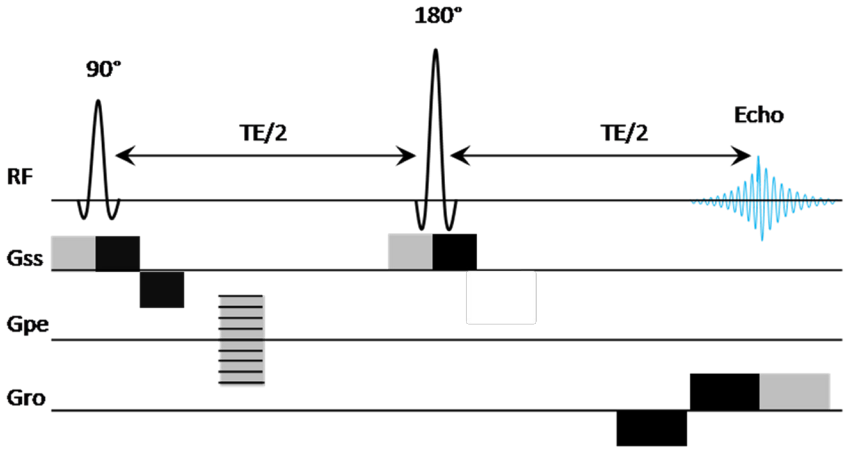
Solution 1: The Spin-Echo Sequence

a)



RF: Radio frequency

Gss : Slice selection gradients

Gpe : Phase encoding gradients

Gro: Read-out gradients

The black half of each gradient corresponds to its corresponding rephasing pair and has the same area.

b) T_1: Spin-lattice relaxation describes how fast the spin system loses energy to its surroundings and comes back to equilibrium. Seen as an increase of longitudinal magnetization

T_2: Spin-spin relaxation, describes how individual spins dephase each other and make the ensemble lose coherence. This is seen as a decrease in transverse magnetization. Not to be confused with T_2*.

c) T1-w: having TR in the order of T_1 of the sample so that differences in T_1 cause a different degree of return to steady-state of the longitudinal magnetization. Choose TE as short as possible to minimise the effect of differences in transverse relaxation on the resulting image contrast.

T2-w: having TE in the order of T_2 of the sample so that the differences in T_2 cause a different degree of magnetization to be refocused in the spin-echo. Choose TR long enough to maximize longitudinal relaxation.

d) Choose TE as short as possible (e.g. TE = 5 ms)

Choose TR as the average of the T_1's (i.e. TR = 1000 ms)

It would formally more correct to calculate the derivative to TR of the difference of the spin echo equation for the two tissues.

Solution 2: Contrast Agents

a) A T_1-contrast agent will have an r_1 (almost) as big as its r_2 (it can never be bigger, for the same reasons as T_1 can never be shorter than T_2), and when used in a T_1-weighted sequence it will lighten up areas where it is present. This is because these areas now have shorter T_1s and thus return to steady-state (=maximum signal) faster.

b) The ratio becomes:

$$\frac{1 - e^{-\frac{TR}{T_{1a}}}}{1 - e^{-\frac{TR}{T_{1b}}}} = 1.4$$

$$1 - e^{-\frac{TR}{T_{1a}}} = 1.4 - 1.4e^{-\frac{TR}{T_{1b}}}$$

$$e^{-\frac{TR}{T_{1a}}} = -0.4 + 1.4e^{-\frac{TR}{T_{1b}}}$$

$$\frac{1}{T_{1b}} + r_1[CA] = -\frac{\ln\left(-0.4 + 1.4e^{-\frac{TR}{T_{1b}}}\right)}{TR}$$

$$[CA] = \frac{-\frac{\ln\left(-0.4 + 1.4e^{-\frac{TR}{T_{1b}}}\right)}{TR} - \frac{1}{T_{1b}}}{r_1}$$

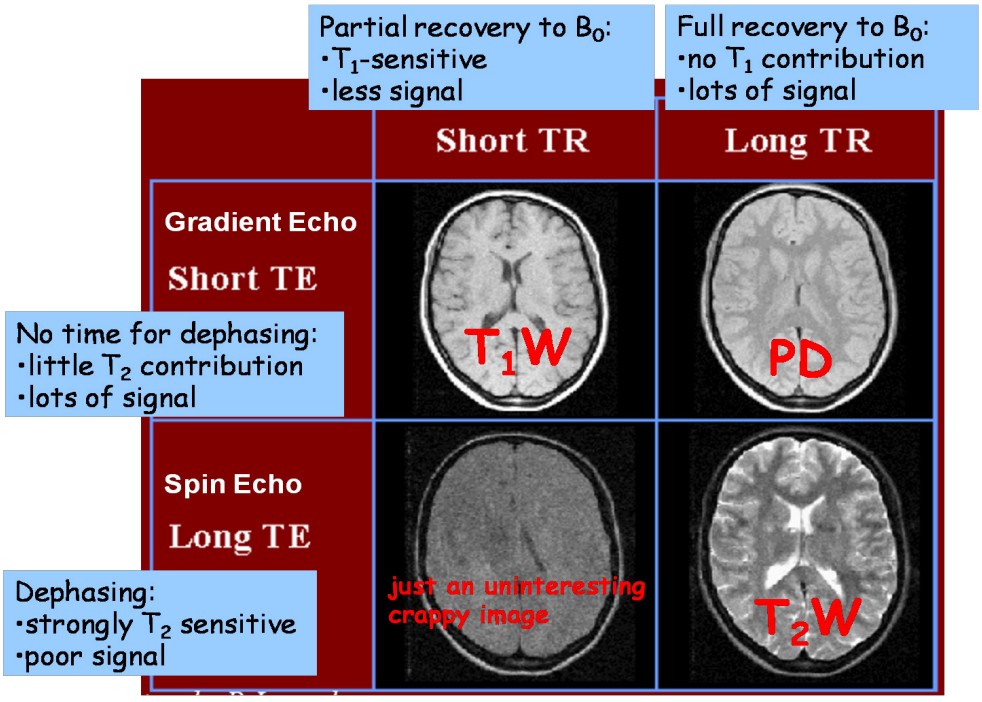
This is the concentration of the individual contrast agent components. Divide this by 50000 to get the number of complexes, and multiply it by 2000 to arrive at the concentration of the medicine CM.

$[CA] = 216\ \mu M$

$\Rightarrow [CM] = 8.6\ \mu M$

So, not enough medicine has arrived at the plaque.

Solution 3: Image Contrast Optimisation



$$Signal \approx \rho \cdot \left(1 - e^{-TR/T_1}\right) \cdot e^{-TE/T_2}$$

a) Based on the signal intensity of the CSF is easy to distinguish the 3 types of weighting of the 3 images.
   a. CSF has the longest T_2, so it will be the brighter signal on T_2-W image
   b. CSF has the longest T_1, so it will be the darker signal on T_1-W images
   c. Proton density (PD) of the different type of tissue are very similar, thus proton density give a poor contrast

b) T_2-W:
   a. Reduce T_1 effect → long TR. $\left(1 - e^{-TR/T_1}\right) \xrightarrow{TR \gg T_1} 1$
   b. Accentuate T_2 effect → longer TE ≈ T_2 → spin echo sequence

   T_1-W :
   a. Reduce T2 effect → short TE. $\left(e^{-TE/T_2}\right) \xrightarrow{TE \ll T_2} 1$ → gradient echo sequence
   b. Accentuate T_1 effect → saturation : TR ≈ T_1

   PD :
   a. Reduce T2 effect → short TE. $\left(e^{-TE/T_2}\right) \xrightarrow{TE \ll T_2} 1$ → gradient echo sequence
   b. Reduce T_1 effect → long TR. $\left(1 - e^{-\frac{TR}{T_1}}\right) \xrightarrow{TR \gg T_1} 1$

Solution 4: BOLD-Effect

a) The difference in magnetic susceptibility of oxyhaemoglobin and deoxyhaemoglobin is the physical effect that influences the BOLD effect. Deoxyhaemoglobin has a magnetic susceptibility which is more paramagnetic than

that of tissue, whereas oxyhaemoglobin has a diamagnetic susceptibility very similar to that of tissue. So deoxyhaemoglobin has more effect on the signal. The decrease in the deoxyhaemoglobin concentration reduces the local magnetic field gradient between the blood in the capillary and the tissue. As a result, the T_2* increases locally in areas of the brain associated and signal increases in the T_2* sensitive pulse sequence.

b) The a-v before the increase of CBF: $C_a - C_v = \frac{MR}{F}$

The a-v after the increase of CBF: $C_a - C_v = \frac{MR}{1{,}5\,F}$

Note: metabolic rate (MR) and arterial oxygen concentration (C_a) are constants. There is thus a 33.3% decrease in the a-v during a 50% increase of CBF.

c) As shown in b, when CBF is increased by 50%, C_v will increase. So the concentration of the deoxyhaemoglobin in the veins will decrease.

d) Due to the decrease of concentration of the deoxyhaemoglobin in the veins, T_2* will increase. Thus, the signal will increase in the image if a T_2* sensitive pulse sequence is used.

e) The sequence used for detecting the BOLD-effect has to be T_2*-sensitive, which is only the case for a GRE sequence. There, the magnetization is dephased and rephased using gradients that act as additional magnetic fields. This affects only the phase of the magnetization. Hence inhomogeneities are not refocused, which renders the sequence sensitive to changes due to T_2* differences. The SE sequence, on the contrary, uses a 180° to refocus the spins; this causes a mirror effect, rephasing both the phase accumulated due to gradients as well as the one due to inhomogeneities effects. Thus, the spin echo sequence is T_2 sensitive and is not affected by inhomogeneities.

f) The visibility of BOLD effect is maximized when the signal difference between the excited (oxygenated blood) and rest (deoxygenated blood) states is maximal. The BOLD signal is given by:

$$S_{oxy} - S_{deoxy}\,(t = TE) = S_{oxy}(0)\,e^{-TE/T_{2\,oxy}^{*}} - S_{deoxy}(0)\,e^{-TE/T_{2\,deoxy}^{*}}$$

Maximization of the BOLD effect:

$$\frac{d\left(S_{oxy} - S_{deoxy}\right)(TE)}{dTE} = -\frac{1}{T_{2\,oxy}^{*}} S_{oxy}(0)\,e^{-\frac{TE}{T_{2\,oxy}^{*}}} + \frac{1}{T_{2\,deoxy}^{*}} S_{deoxy}(0)\,e^{-\frac{TE}{T_{2\,deoxy}^{*}}} = 0$$

NB: Because the magnetic field $B(\vec{r})$ difference in both cases is negligible, the magnetizations are considered as being identical (consider Boltzmann distribution):

$$S_{oxy}(0) = S_{deoxy}(0) = S(0)$$

Thus,

$$\frac{d\left(S_{oxy} - S_{deoxy}\right)(TE)}{dTE} = S(0)\left(-\frac{1}{T_{2\,oxy}^{*}}\,e^{-\frac{TE}{T_{2\,oxy}^{*}}} + \frac{1}{T_{2\,deoxy}^{*}}\,e^{-\frac{TE}{T_{2\,deoxy}^{*}}}\right) = 0$$

$$\frac{T_{2\,deoxy}^{*}}{T_{2\,oxy}^{*}} = e^{-\left(\frac{1}{T_{2\,deoxy}^{*}} - \frac{1}{T_{2\,oxy}^{*}}\right)TE}$$

$$TE = \ln\left(\frac{T_{2\,oxy}^*}{T_{2\,deoxy}^*}\right)\frac{T_{2\,oxy}^* T_{2\,deoxy}^*}{T_{2\,oxy}^* - T_{2\,deoxy}^*} = 36.5\ ms$$