Coupled transport / conversions in Bioreactors

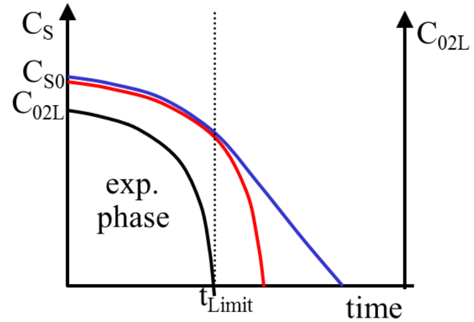
O_2 transfer limitation in **Batch** reactor.
After O_2 limitation, linear phase

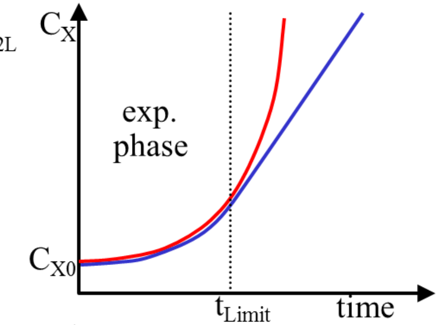
$$C^*_{O2L} - C_{O2L} = \frac{\mu^{max} C_{XO} V}{Y^{max}_{OX} K_L a} \exp\left(\mu^{max} t\right)$$

$$-r_{O2} = K_L a . C^*_{02L} = Cst$$

$$r_X = Y^{max}_{OX} . K_L a . C^*_{02L} = Cst$$

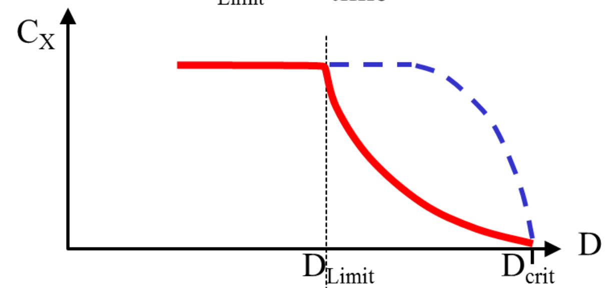
$$\frac{dC_X}{dt} = r_X = Cst$$

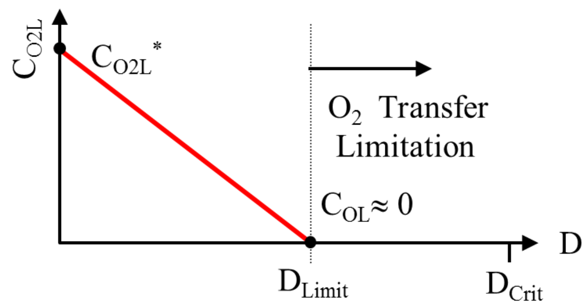


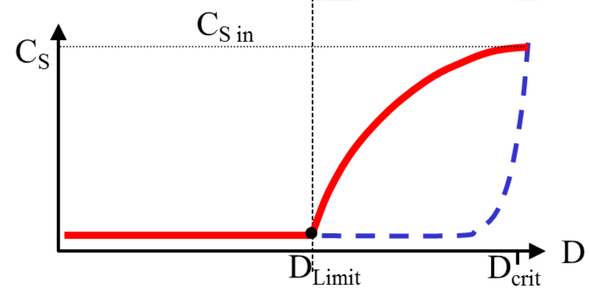


O_2 transfer limitation in **Chemostat** reactor.
Before O_2 limitation, C_{O2L} decrease linearly with D. After O_2 limitation C_{O2L} = 0 C_X decrease and C_S decrease D^{-1}.

$$C^*_{02L} - C_{02L} = \frac{Y^{max}_{SX} . C_{S,in} . V . D}{Y^{max}_{0X} . K_L a}$$







O_2 transfer limitation including **gas depletion**

$\mathbf{X_{O2g} < X_{O2g}{}^{in}}$?

$$-r_{O_2} = \frac{K_L a}{V}\left(C^*_{02L} - C_{02L}\right)$$

$$X_{02g} = \frac{X^{in}_{0g} + \frac{K_L a}{\phi_g} \frac{RT}{P} C_{02L}}{1 + \frac{K_L a}{\phi_g} \frac{1}{m_{02}}}$$

$$\frac{K_L a}{\phi_g} \text{ large} \Rightarrow X_{02g} \approx X^{in}_{02g} + 0 \big/ 1 + 0 \approx X^{in}_{02g}$$

$$\frac{K_L a}{\phi_g} \text{ small} \Rightarrow C_{O2L} \approx X_{O2g} \frac{P}{RT} . \frac{1}{m_{O2}} = \frac{C_{O2g}}{m_{O2}} = C^*_{O2L}$$

O_2 & C_{O2} coupling

$$C_{CO2L} = \frac{\left(1 + \frac{K_L a . V_L}{\phi_g} . \frac{1}{m_{CO2}}\right)}{\left(1 + \frac{K_L a . V_L}{\phi_g} . \frac{1}{m_{02}}\right)} \left(\frac{P}{RT} \frac{X^{in}_{02g}}{m_{02}} - C_{02L} . \left(1 + \frac{K_L a . V_L}{\phi_g} . \frac{1}{m_{02}}\right)\right)$$

→ For constant microenvironment, gas/liquid transfer and microbial growth stoichiometry determine C_{CO2L} and C_{02L} coupling.