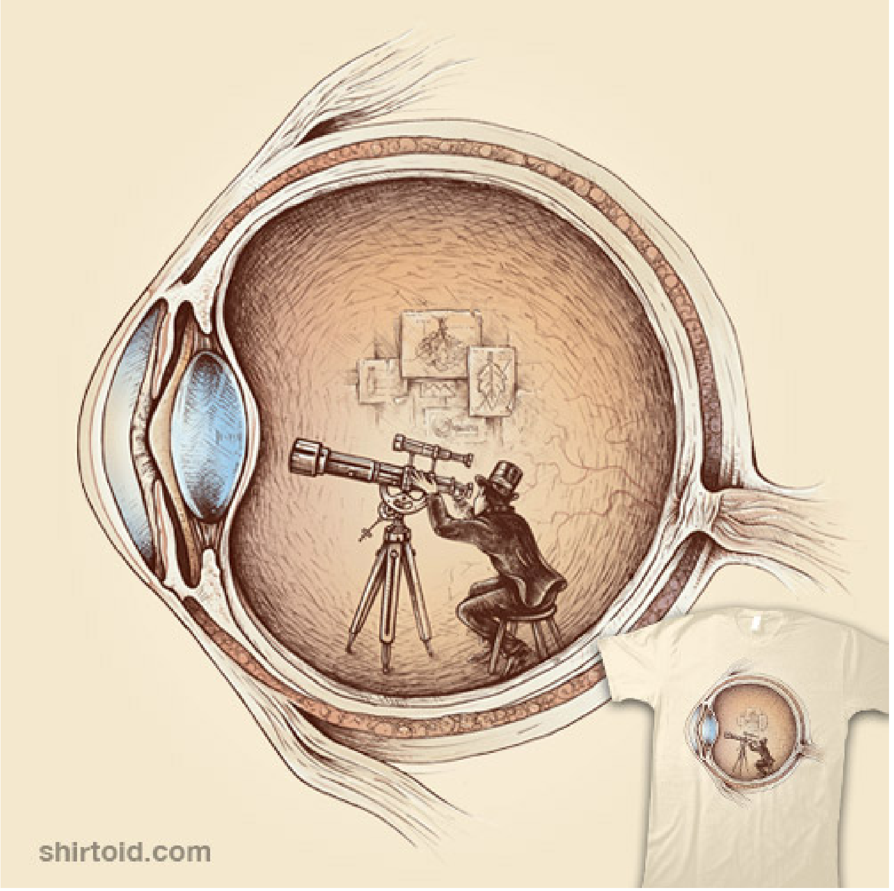


Prof **François Bochud**

Institute of Radiation physics (IRA)

UNIL - CHUV

Master of Science EPF-ETH degree in **Nuclear Engineering**
Medical Radiation Physics

Mathematical model observers

https://shirtoid.com/12257/extraordinary-observer/

Learning objectives

- List the **four properties** that an **objective image quality** must satisfy

- Explain why **model** observers are **useful in medical imaging**

- Explain the meaning of the **ideal observer** and express it in different simple situations

- Explain the meaning of an **anthropomorphic** observer

Mathematical model observers

1.

Image quality

Low-dose

NI = 70

($mA_{min/max}$ = 40/100)

100 kVp

15-year old patient

Dose report:

Type	Scan Range (mm)	CTDIvol (mGy)	DLP (mGy-cm)	Phantom cm
Helical	S8.250-I335.500	1.37	56.14	Body 32

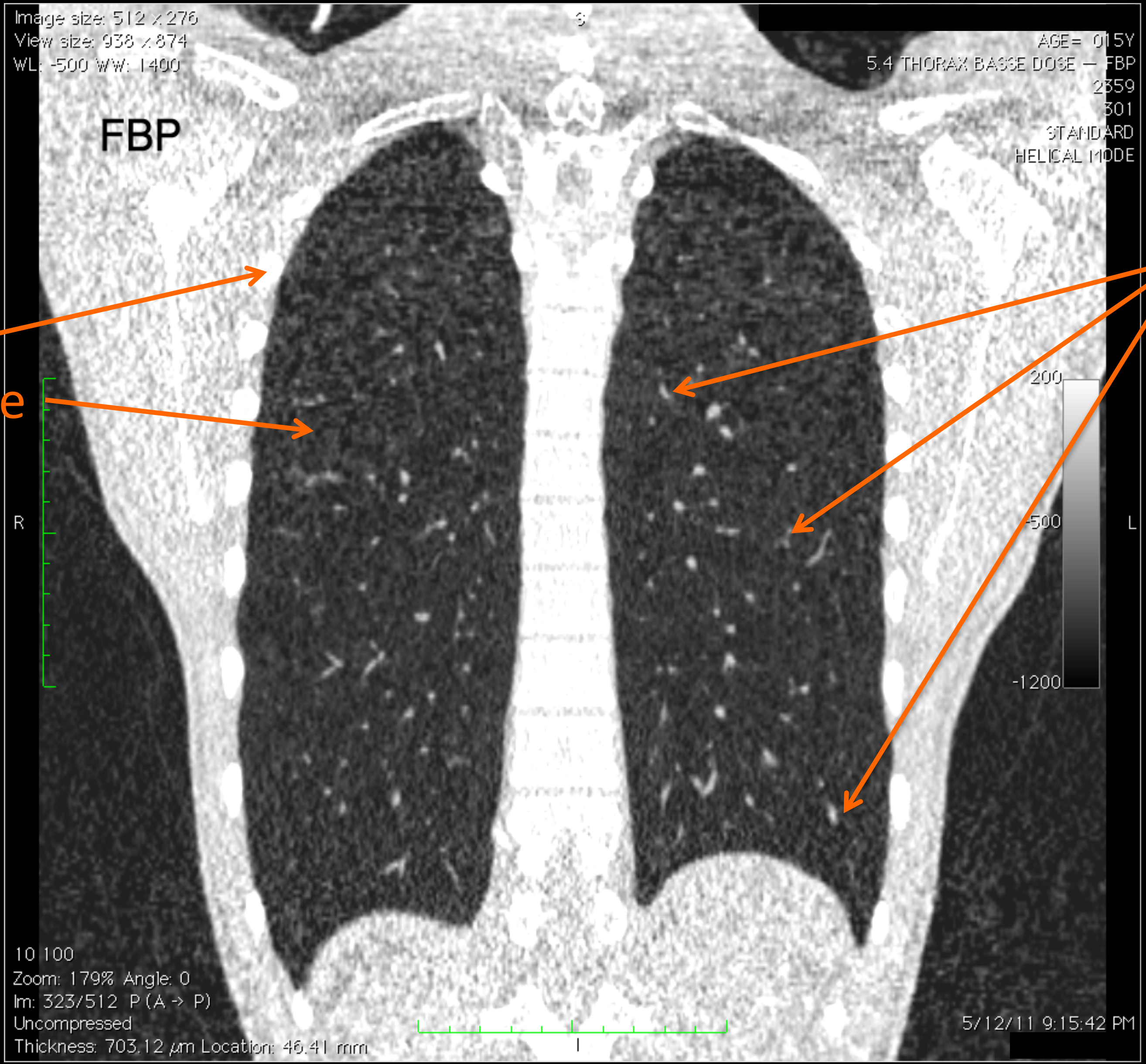
Ultra-low-Dose

10 mA

100 kVp

Ribs

Lung fissure



15-year old patient

Vessels

Dose report:

Type	Scan Range (mm)	CTDIvol (mGy)	DLP (mGy-cm)	Phantom cm
Helical	S8.250-I335.500	0.19	7.78	Body 32

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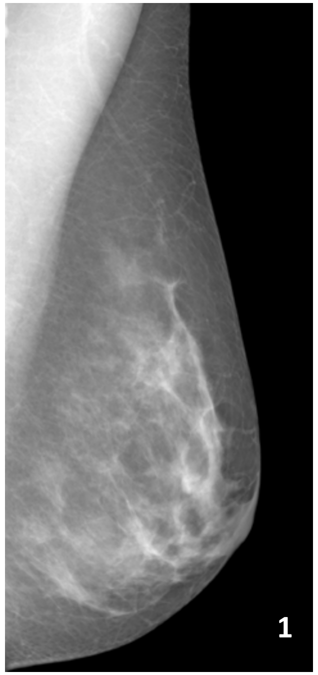
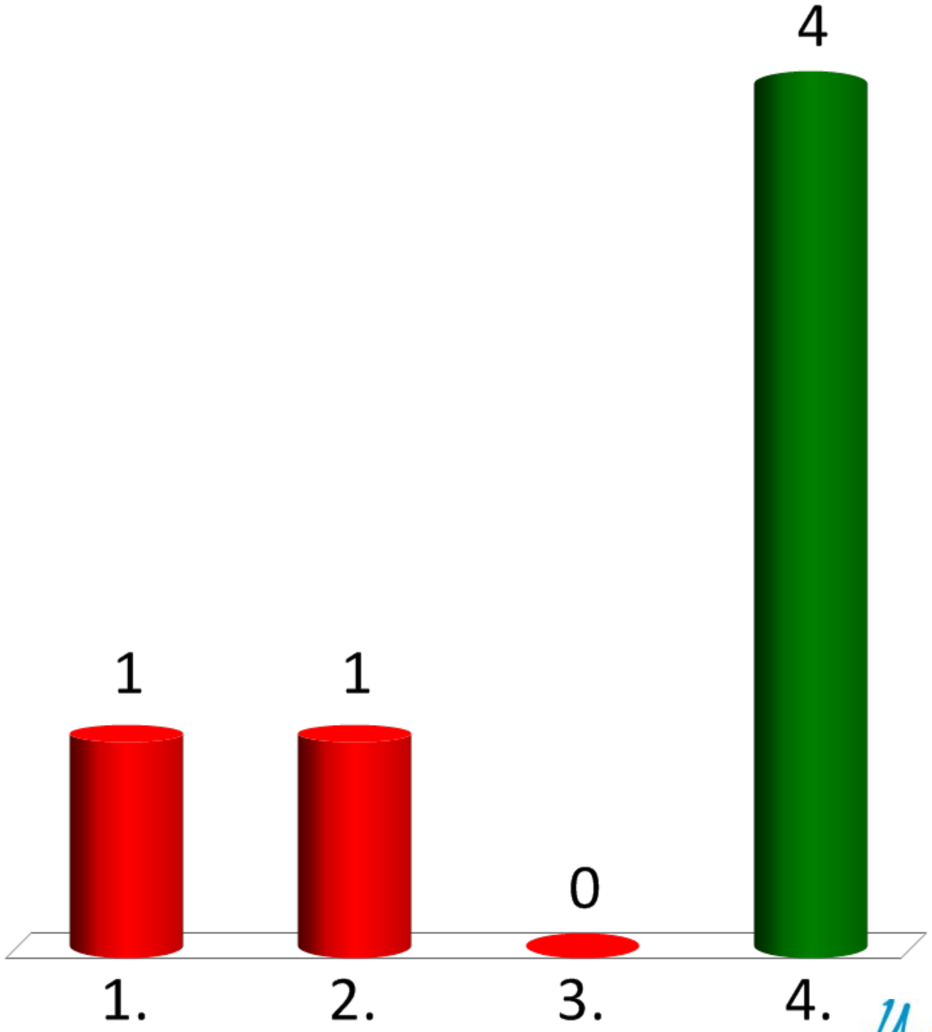
Why do you prefer image 1?

(choose the most important reason)

1. better contrast
2. better resolution
3. less noise
4. easier to find a pathology





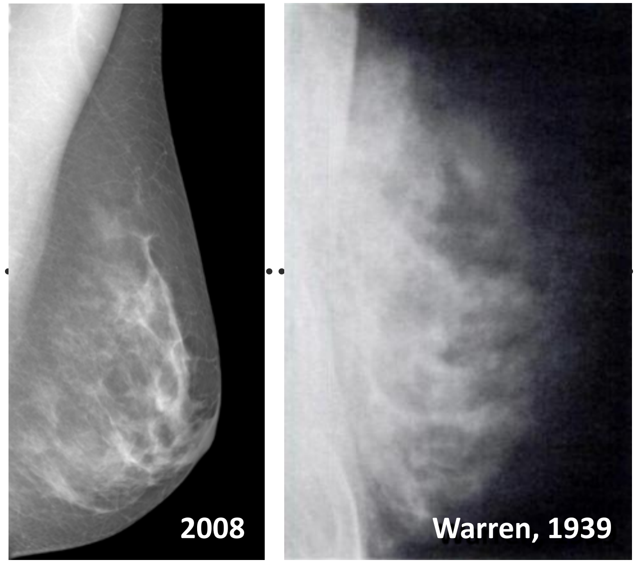
Diagnostic feasible?

Task?

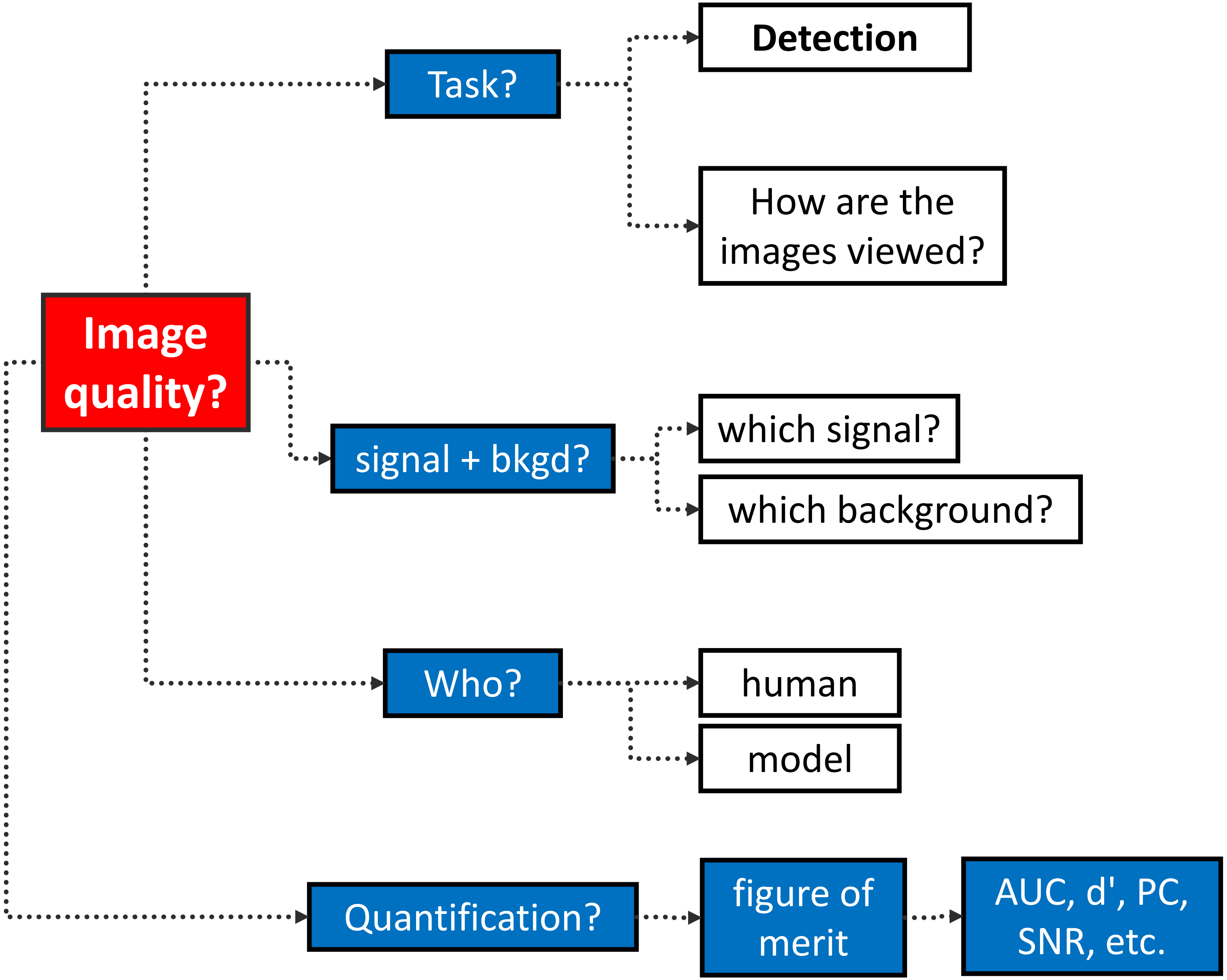
Detection

How are the images viewed?

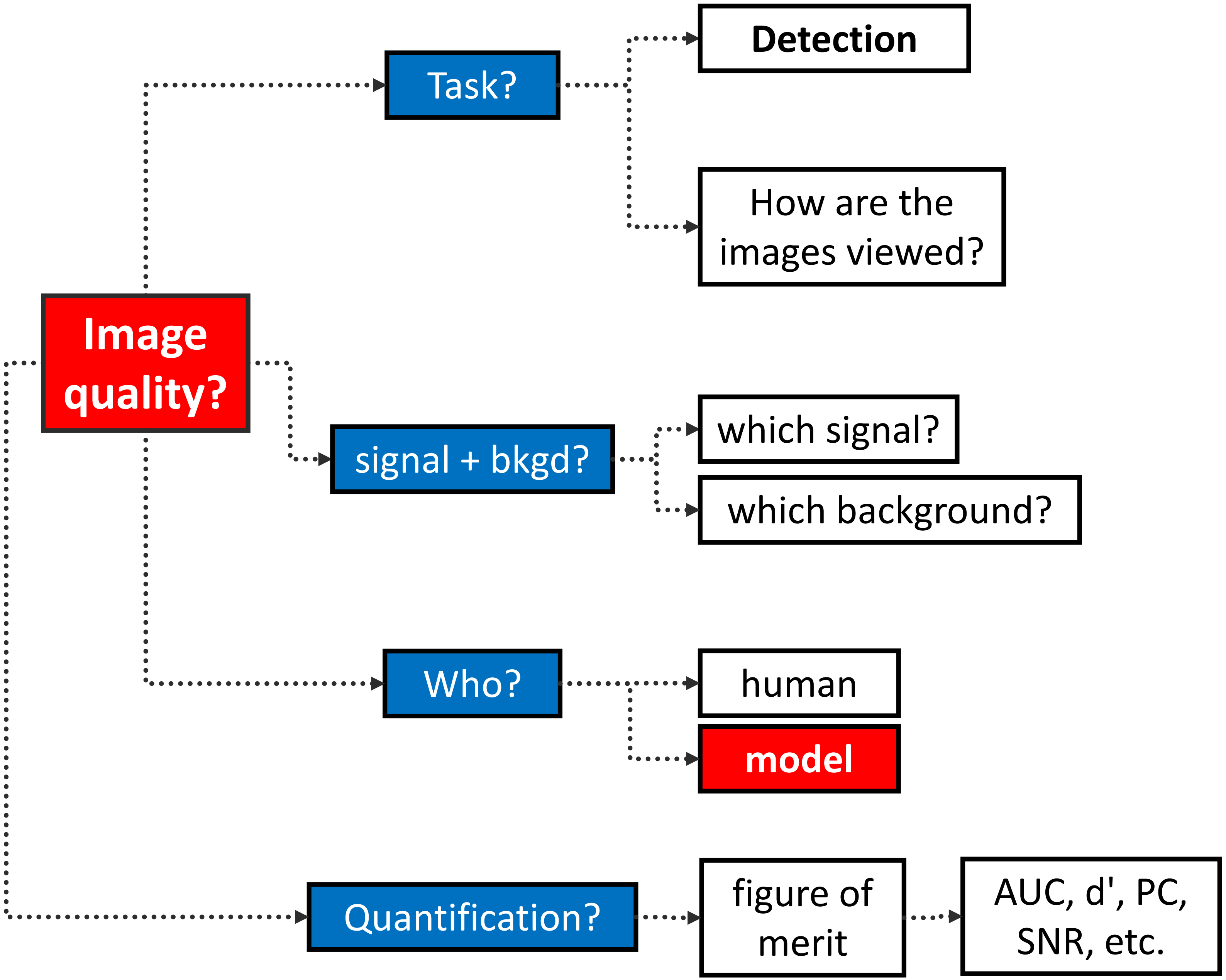
Image quality?



I like it



have to be cautiously chosen in order to be able to do ***comparisons***



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Humans are subject to (large) **variations**

Humans' time is **costly**

(especially if one wants test many image parameters)

Mathematical models could provide a **measuring instrument** for image quality

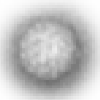
$$\lambda = \mathbf{w}^{\mathrm{T}}\mathbf{g}$$

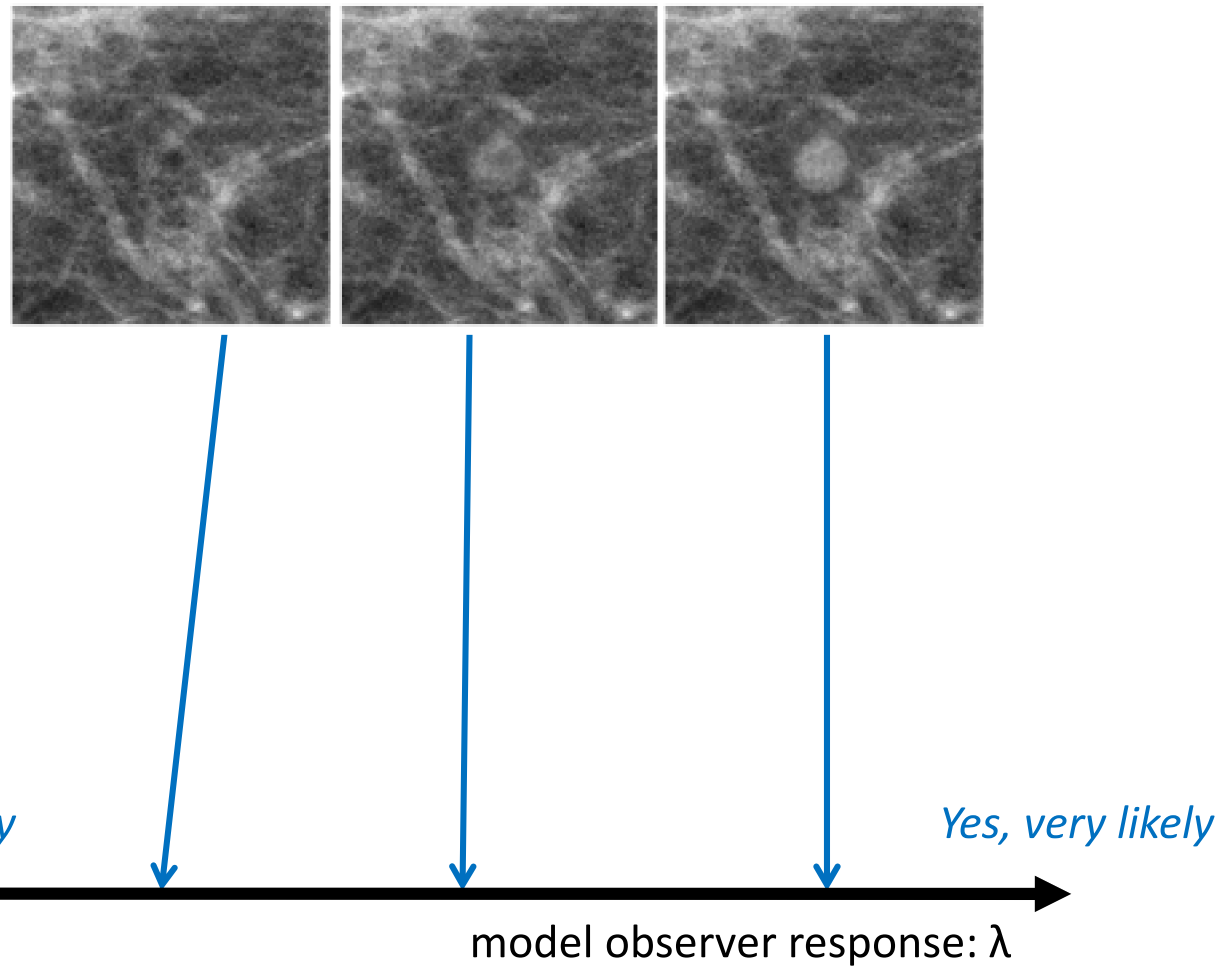
Mathematical model observers

2.

Linear model observers

is this **signal present**?





is this **signal present**?

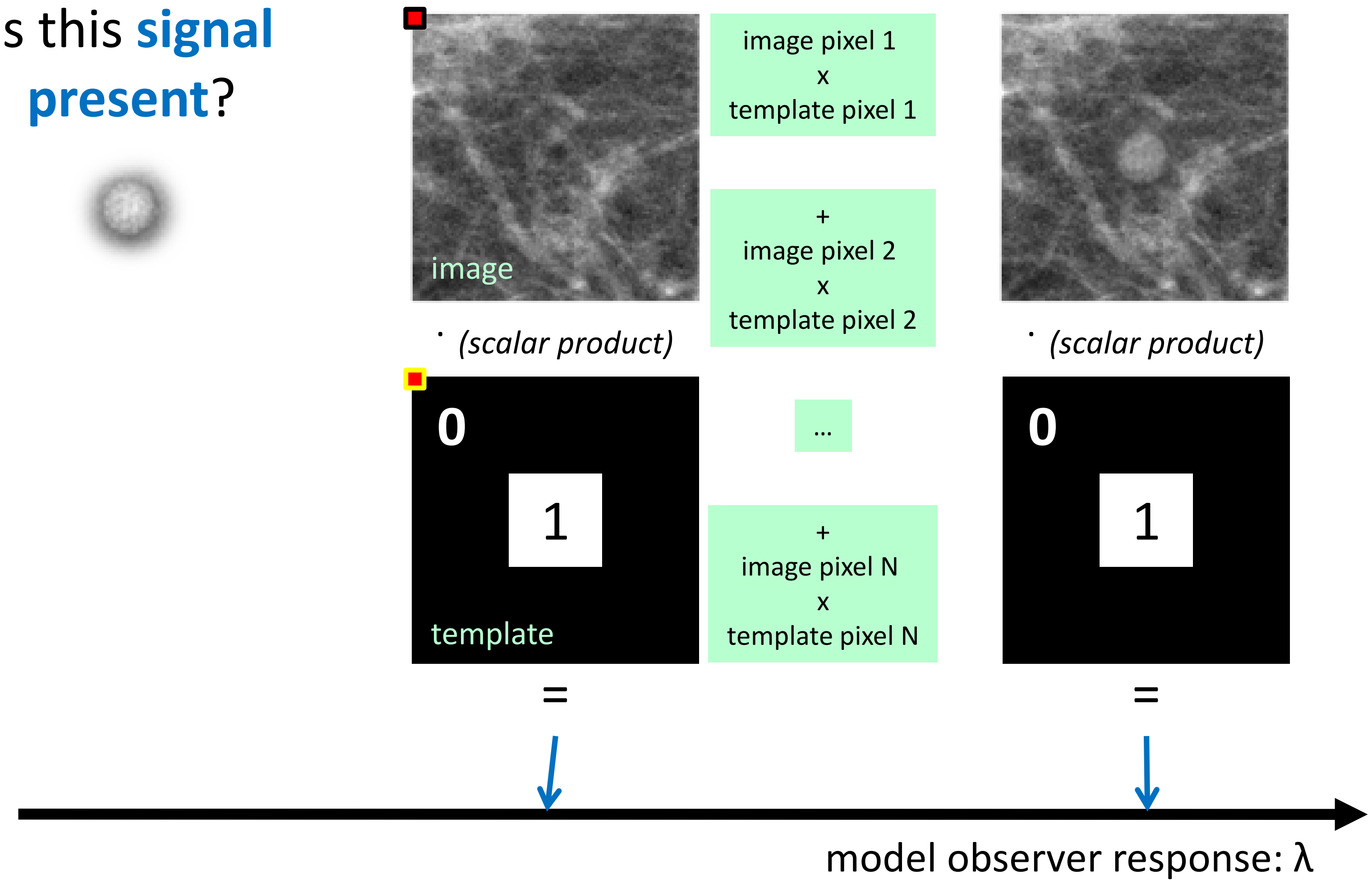
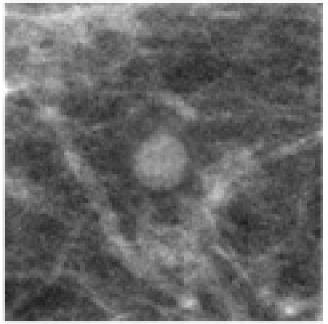


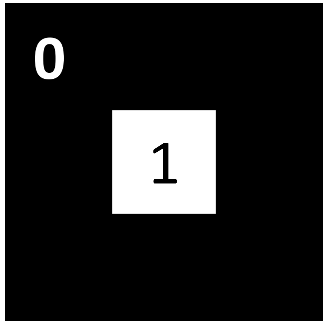
image **g**
(written as a column vector)



g
(image)

· *(scalar product)*

"template" $\mathbf{w}^T$
(written as line vector)



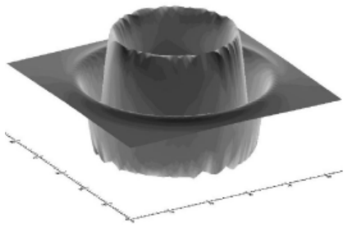
w
(template)

response λ
(scalar)

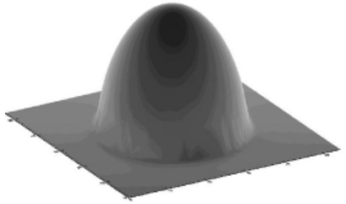
=

λ

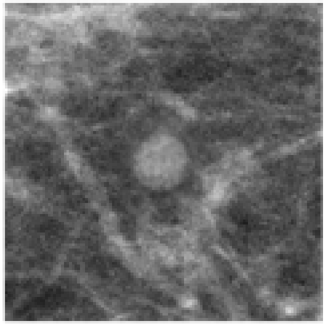
linear model observers $\lambda = \mathbf{w}^T\mathbf{g}$



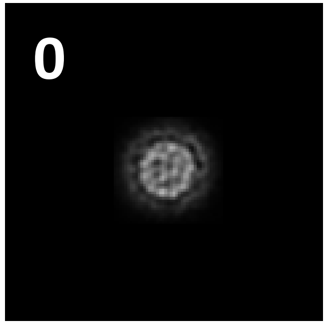
humans look
for borders



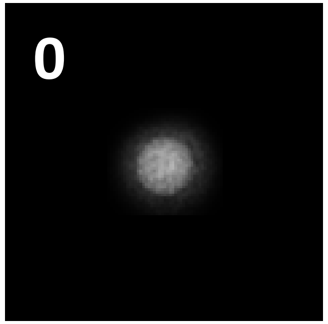
looking for the
signal



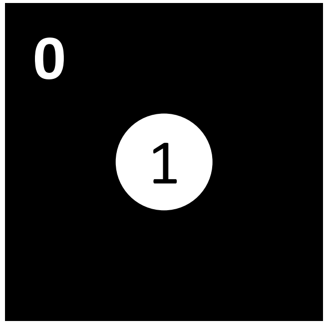
g
(image)



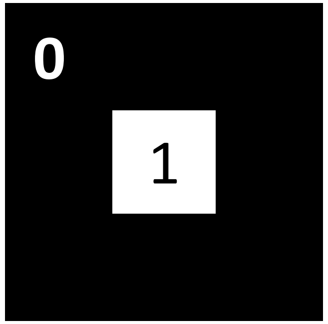
matched-filter with eye-response model



matched-filter model



region of interest model

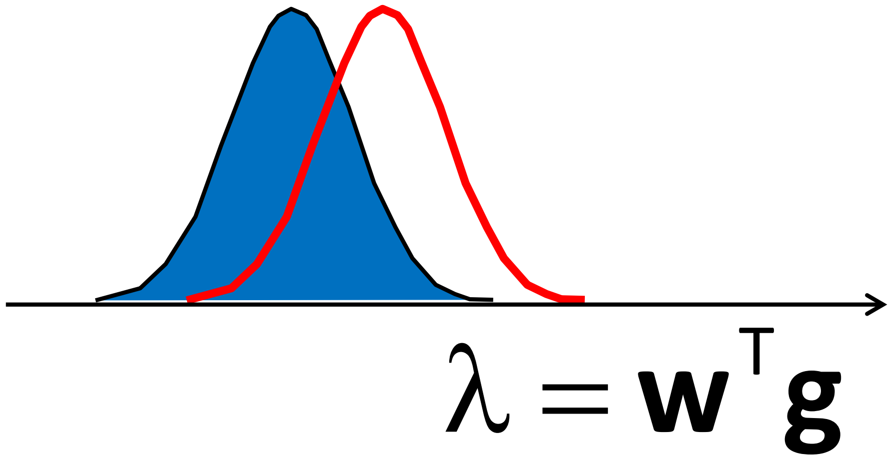


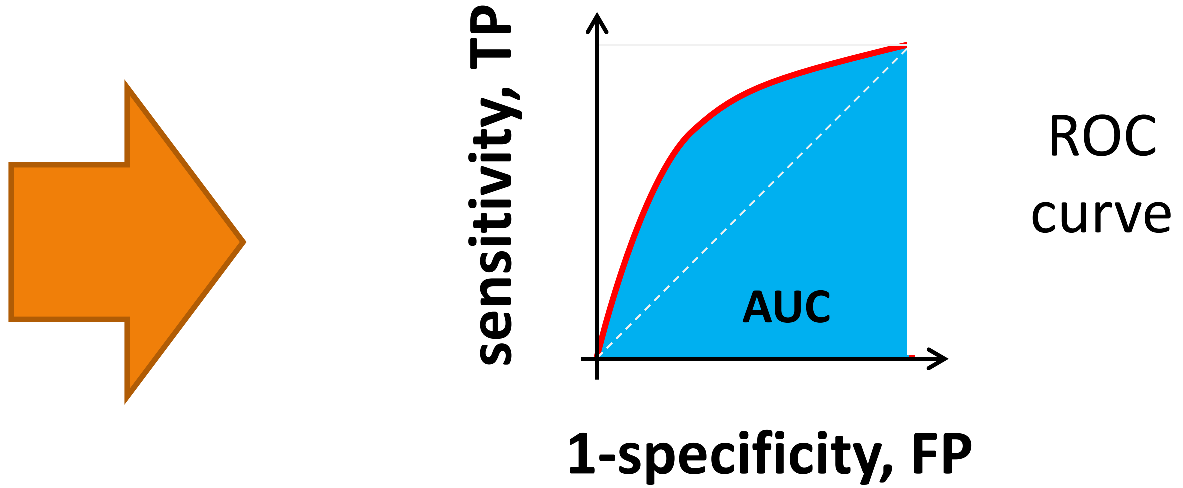
region of interest model

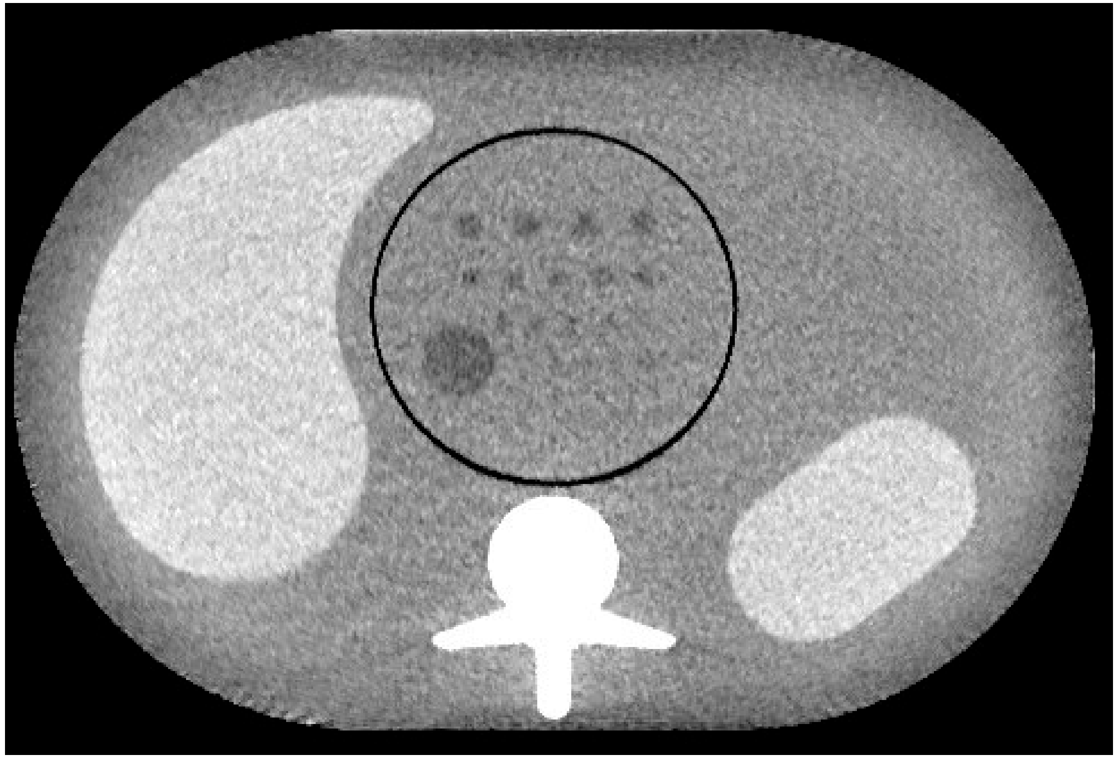
w
(template)

linear model observers $\lambda = \mathbf{w}^T \mathbf{g}$

ROC curves can be derived from model observer responses





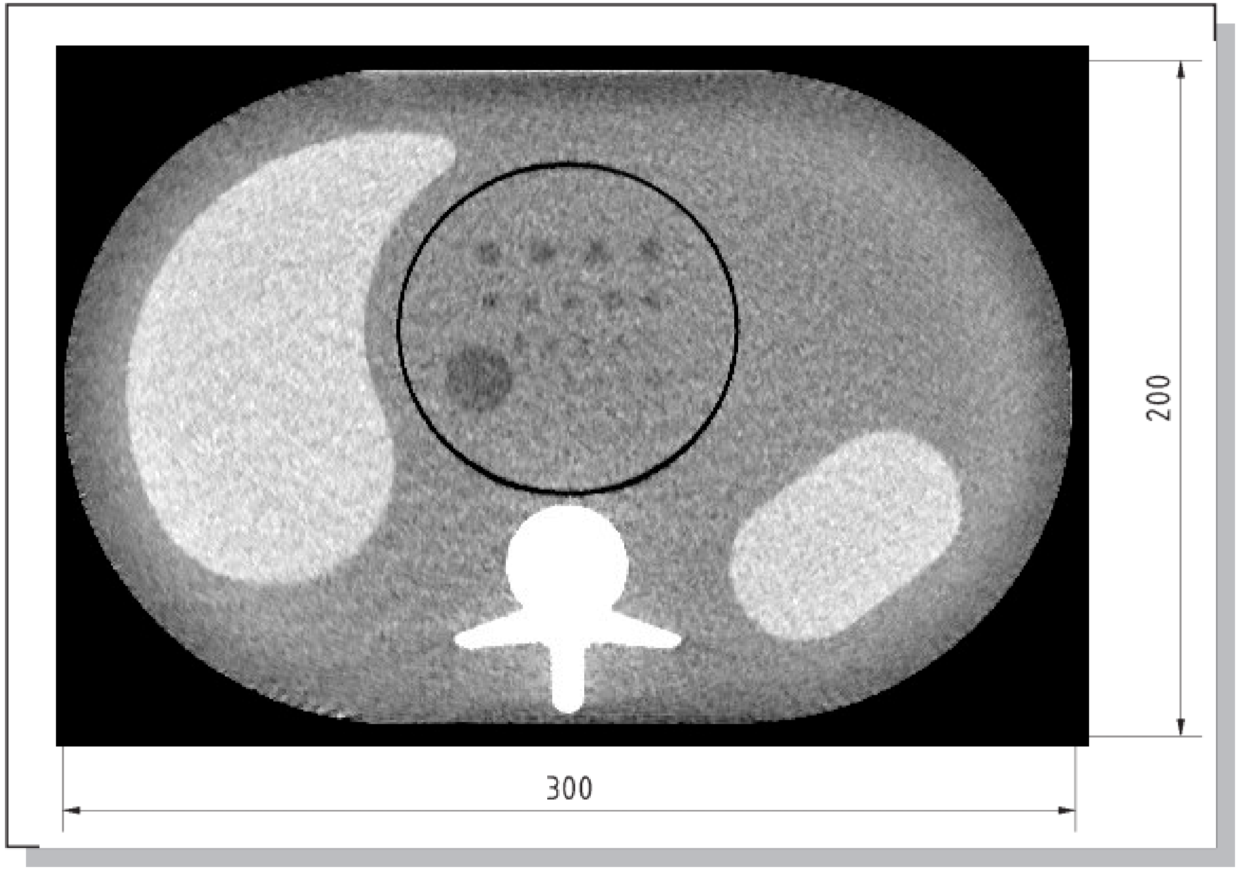


Mathematical model observers

3.

How are they used in practice

Anthropomorphic phantom
(low-contrast spheres)



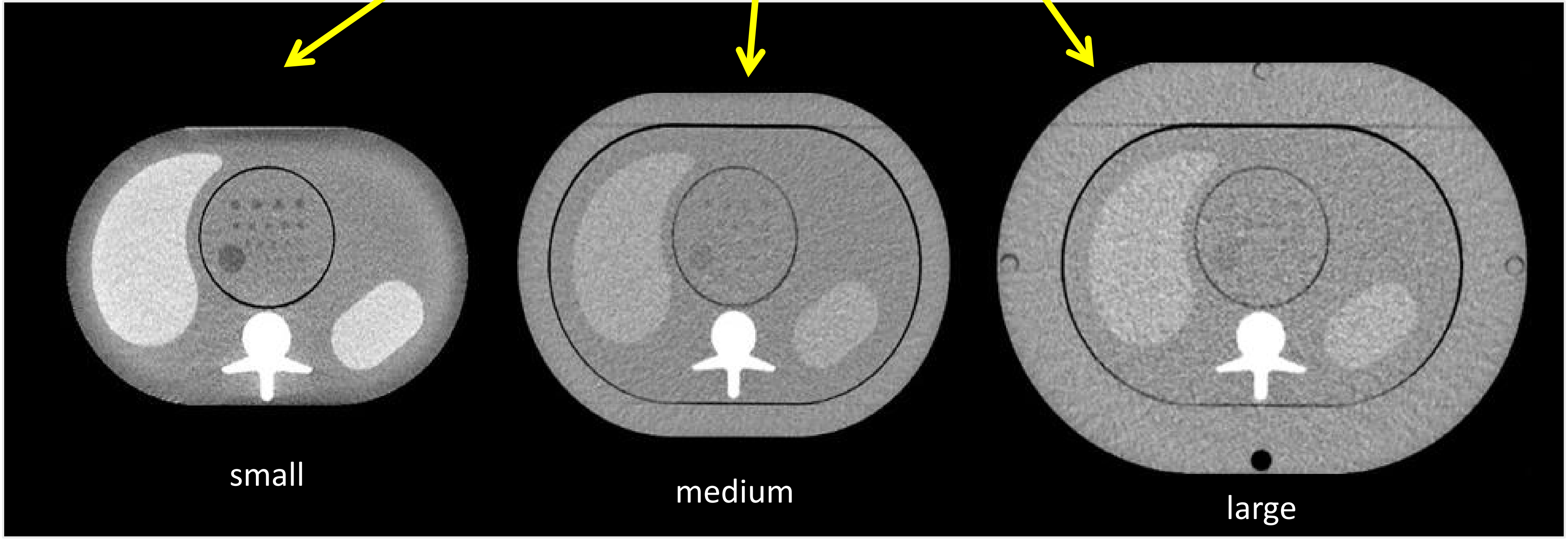
Sketch of the complete anthropomorphic QRM-Abdomen (height 100 mm).

QRM-Abdomen

Source: http://www.qrm.de/content/pdf/QRM-Abdomen.pdf

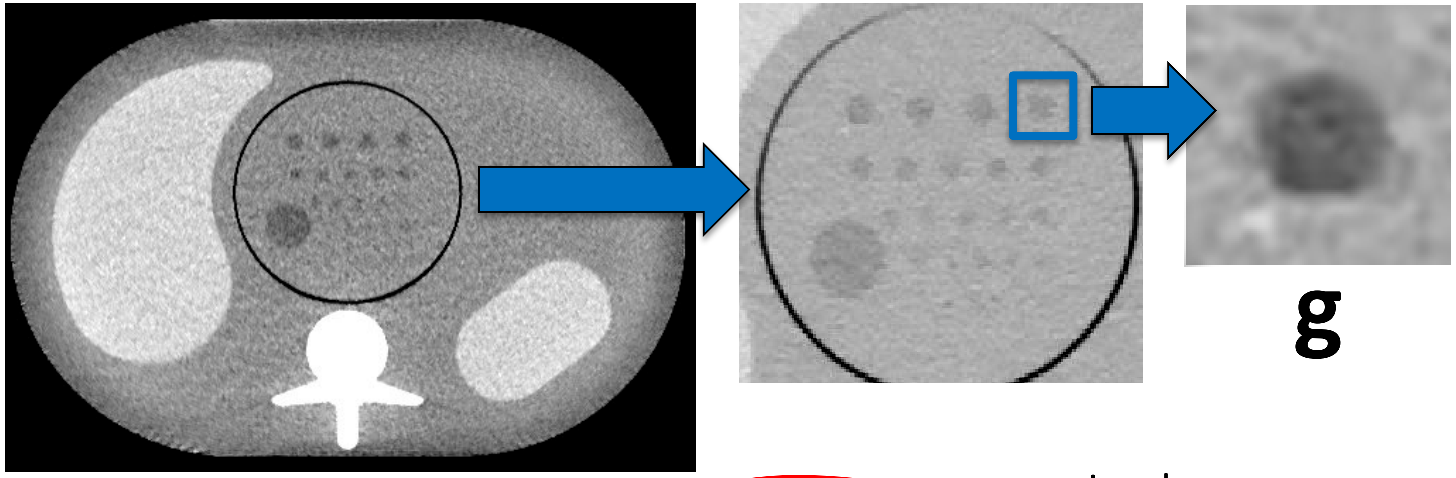
Dose *(CTDI)* and **image quality** *(model observer AUC)* are systematically measured for different modalities

model performance
evaluation

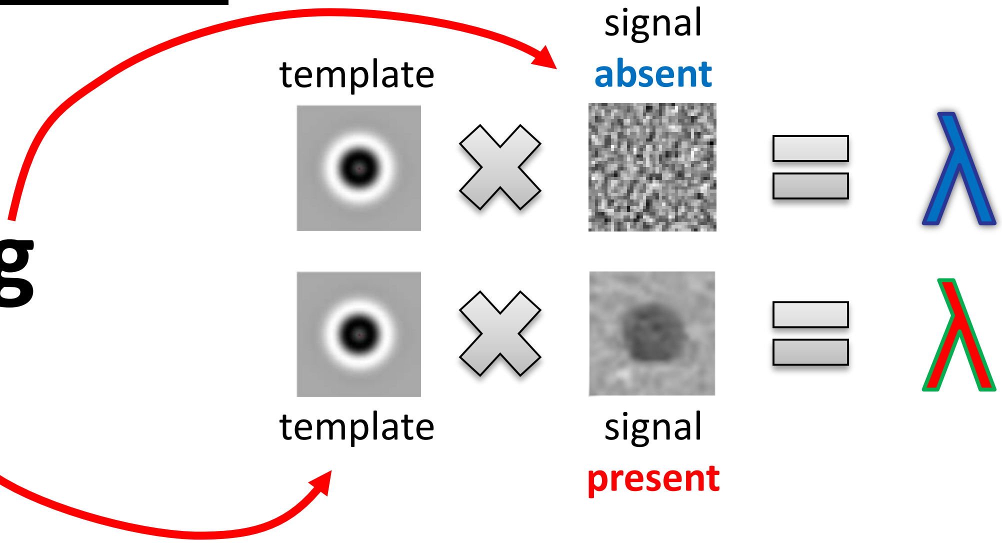


Data: IRA, Francis R. Verdun, Damien Racine, *et al* (2017)

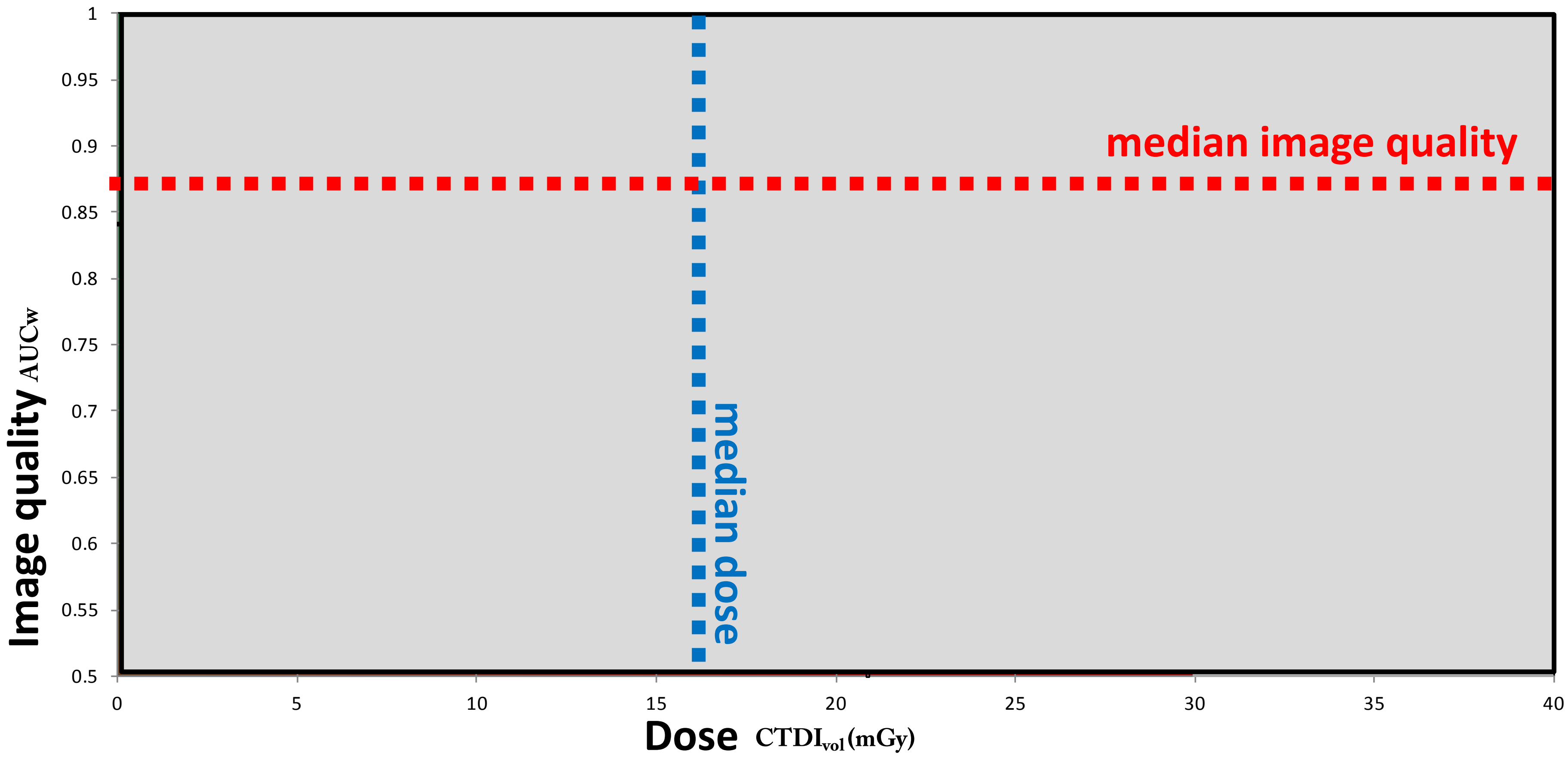
Computation of the **model response** λ



$$\lambda = \mathbf{w}^{T}\mathbf{g}$$



Example of result



Data: IRA, Francis R. Verdun, Damien Racine, *et al* (2017)

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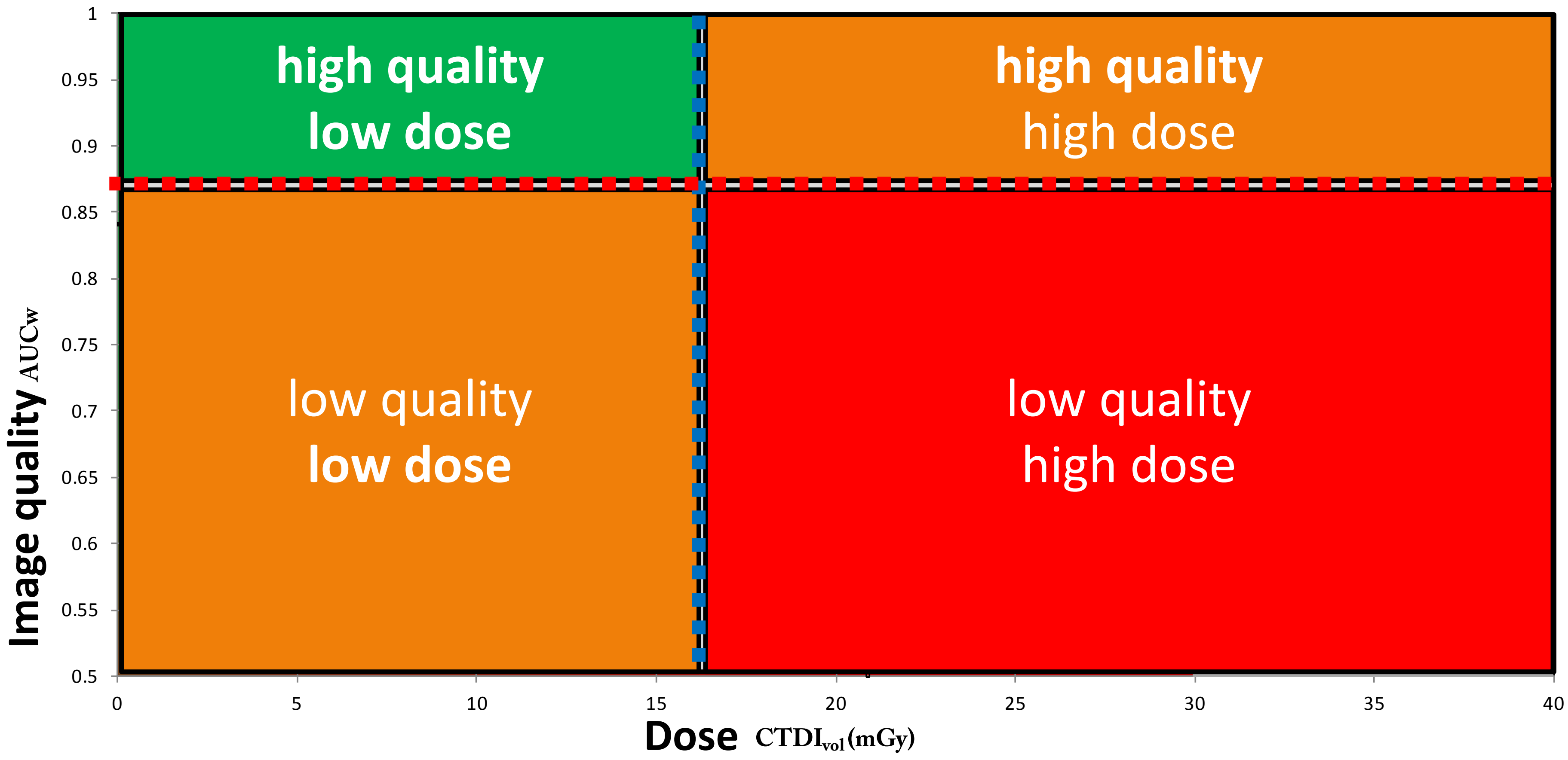
Example of result



Data: IRA, Francis R. Verdun, Damien Racine, *et al* (2017)

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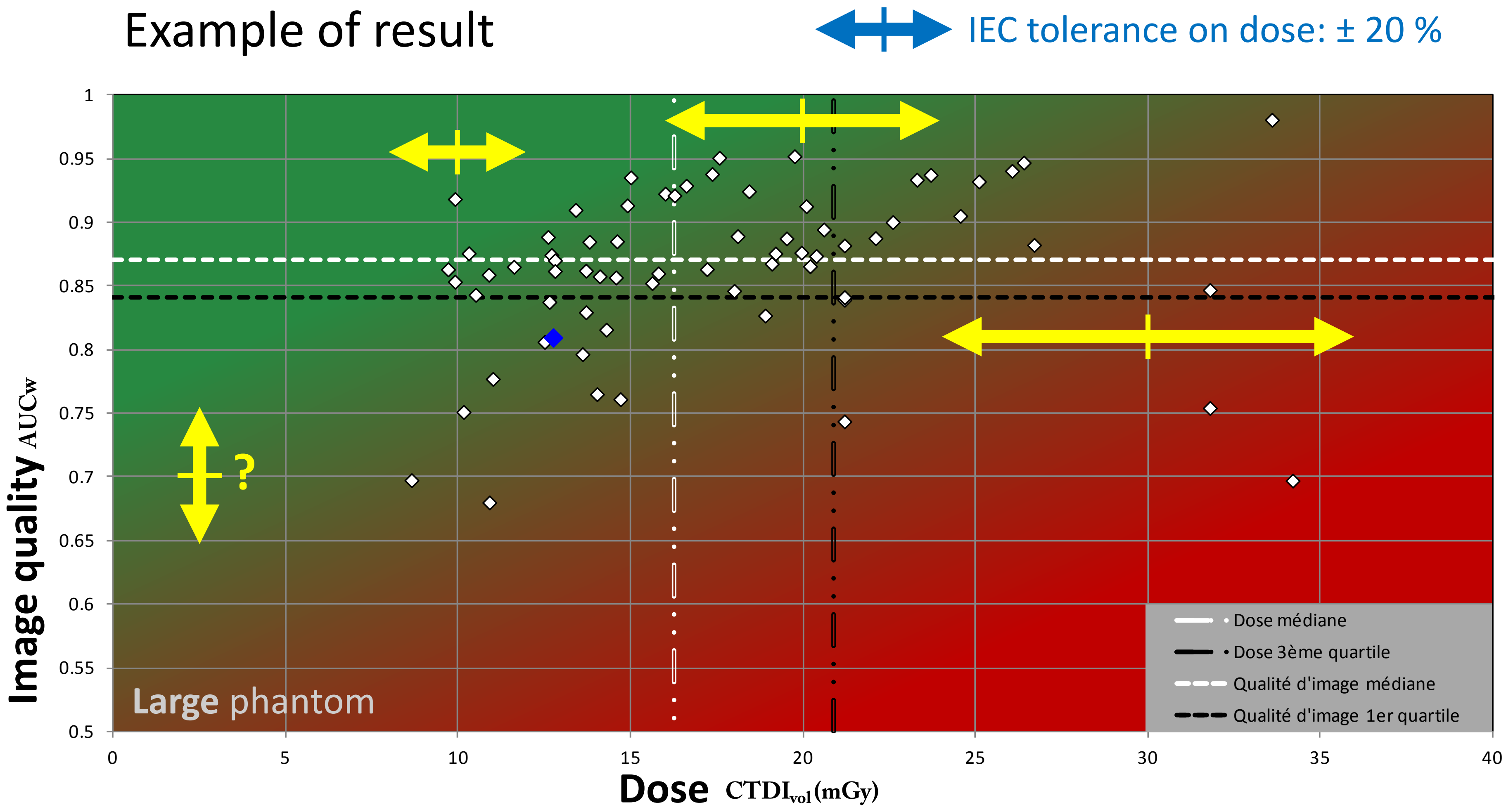
Example of result



Data: IRA, Francis R. Verdun, Damien Racine, *et al* (2017)

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Mathematical model observers

4.

Vision characteristics that need to be incorporated into model observers

Spatial frequencies

depending on the **visual angle** with which we are looking at this image we see the **low** or **high** frequency component, or **both**

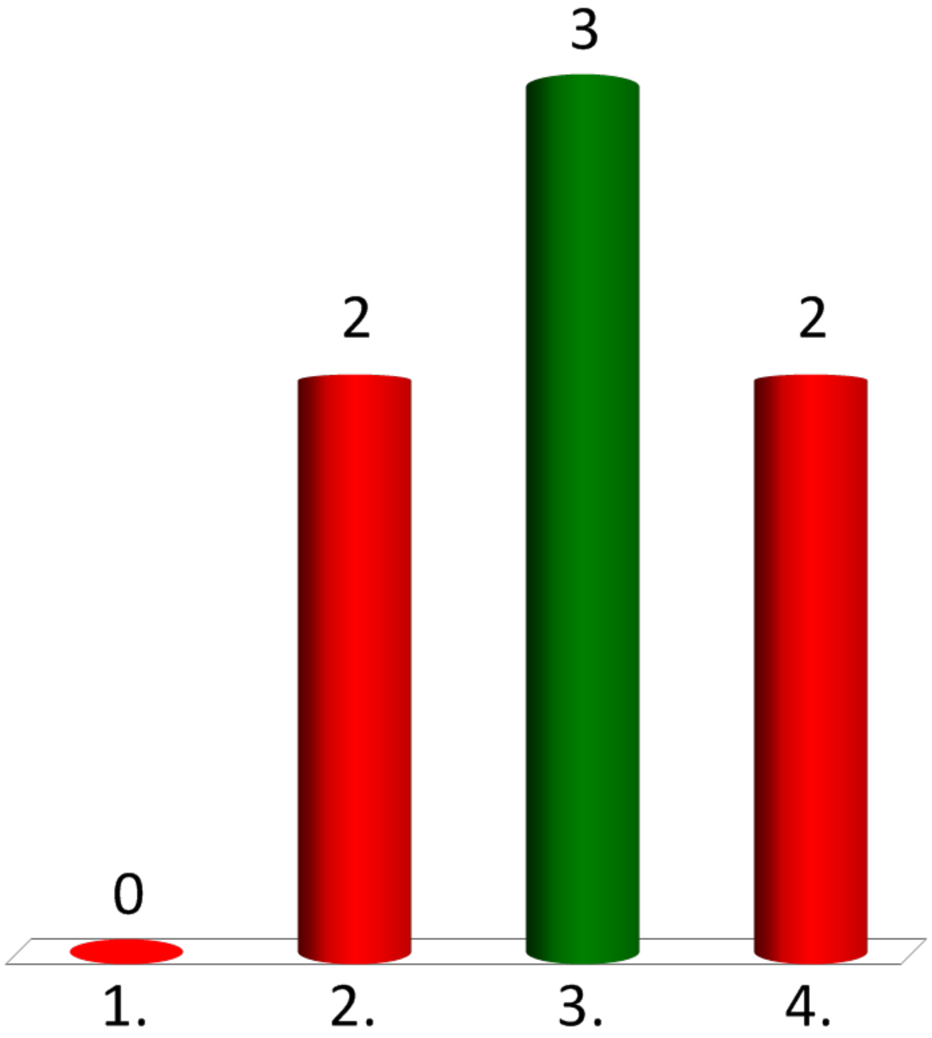
image containing only **low** spatial **frequencies**

image containing only **high** spatial **frequencies**

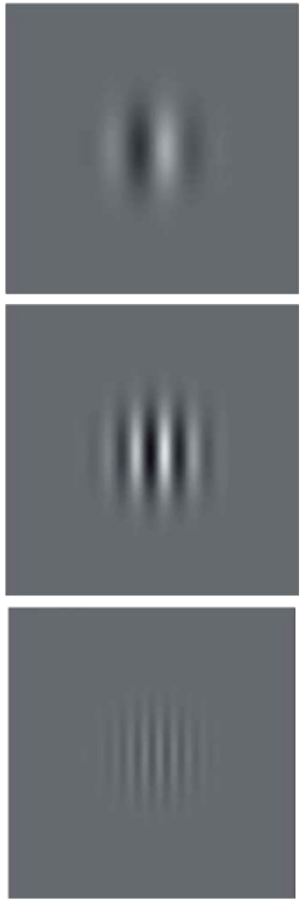
At which **angular frequency** is our visual system the **most effective** from the detection point of view?

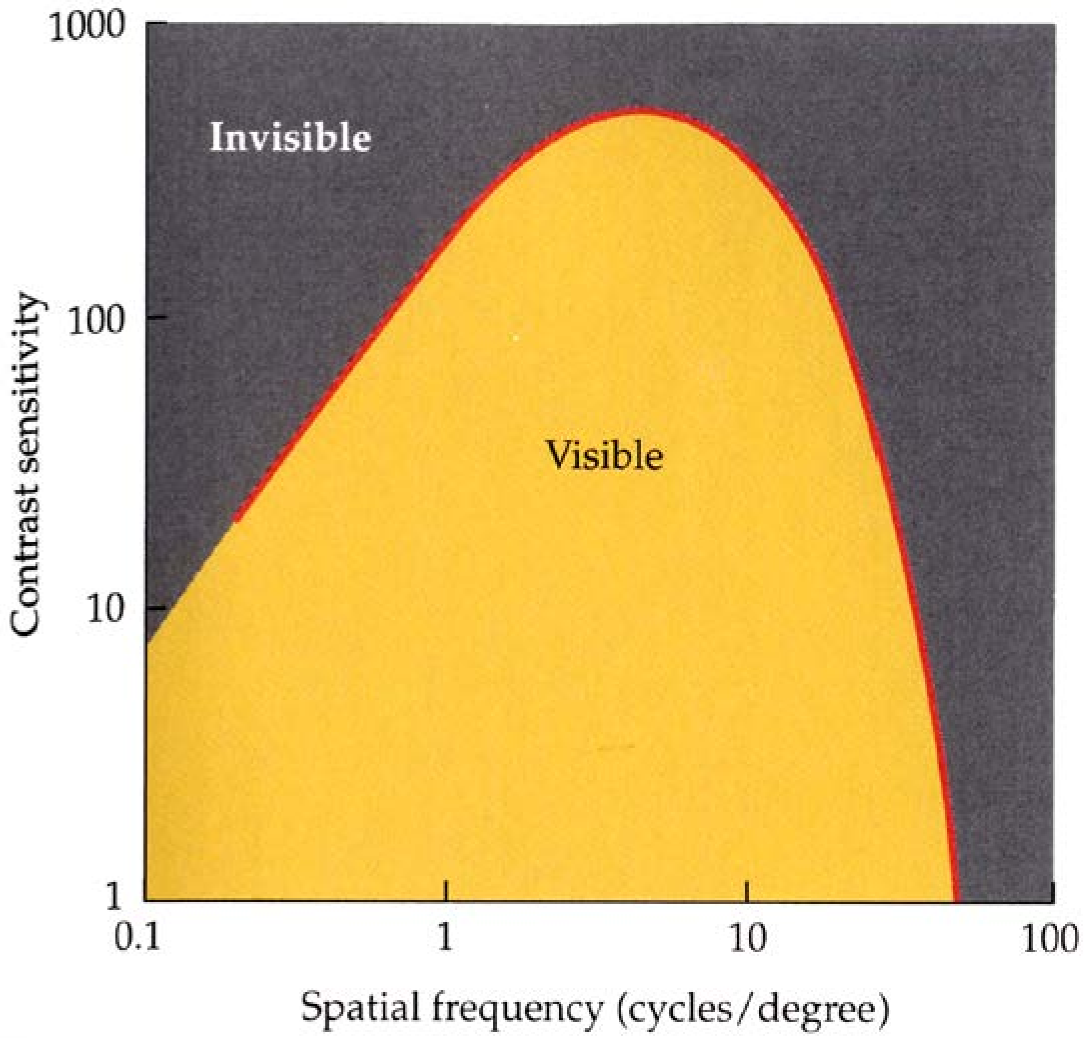
1. 0.04 cycle/deg
2. 0.4 cycle/deg
3. 4 cycles/deg
4. 40 cycles/deg

7

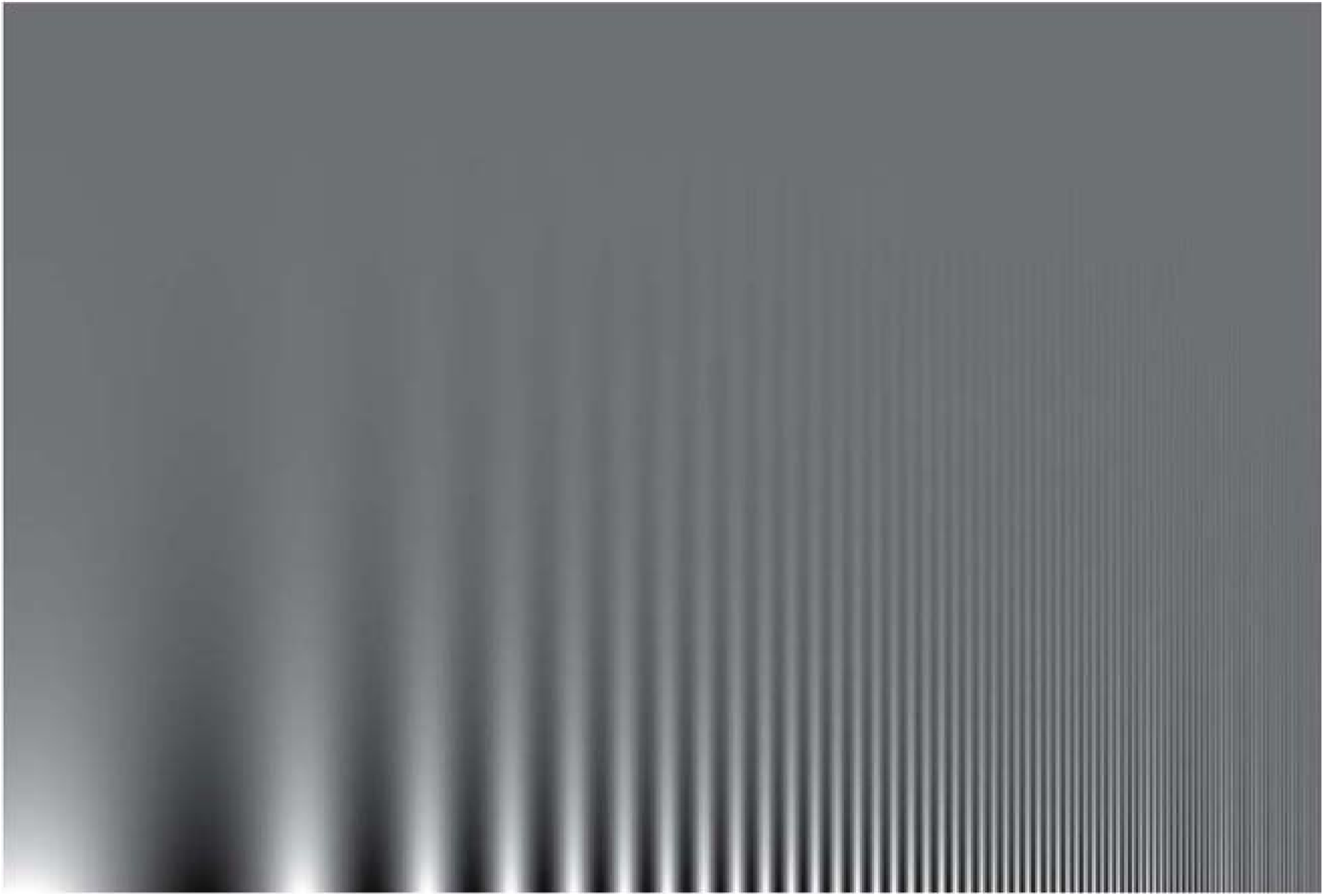


Contrast sensitivity function of the human visual system. The threshold is the contrast required to detect a stimulus with a give precision (e.g. 75% of correct response in a yes/no experiment). The **contrast sensitivity** is defined by 1/threshold contrast: the lower the threshold, the higher the sensitivity.

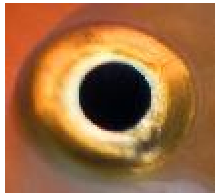




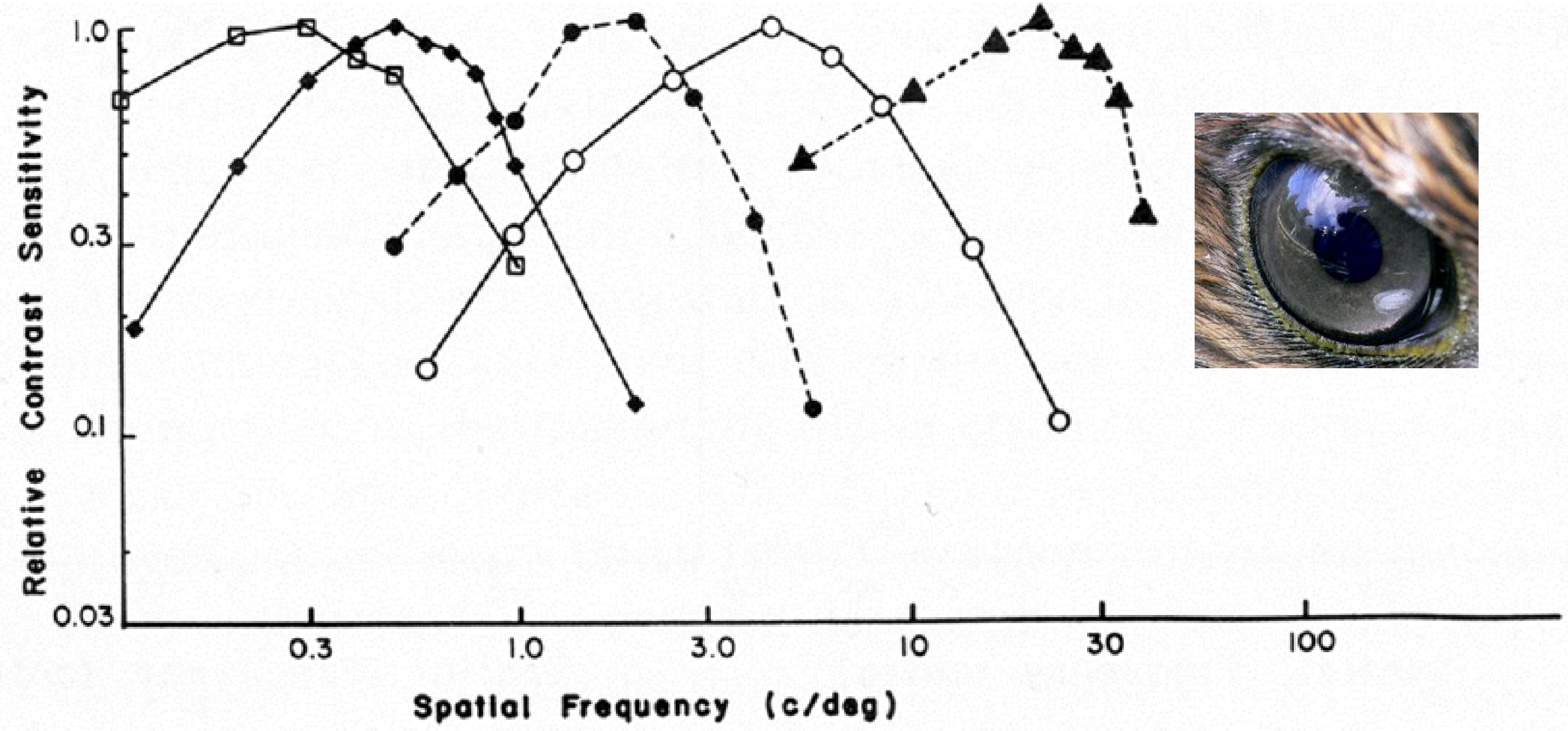
http://homepage.psy.utexas.edu/homepage/class/Psy380E/VS_8_retina.pdf



http://homepage.psy.utexas.edu/homepage/class/Psy380E/VS_8_retina.pdf

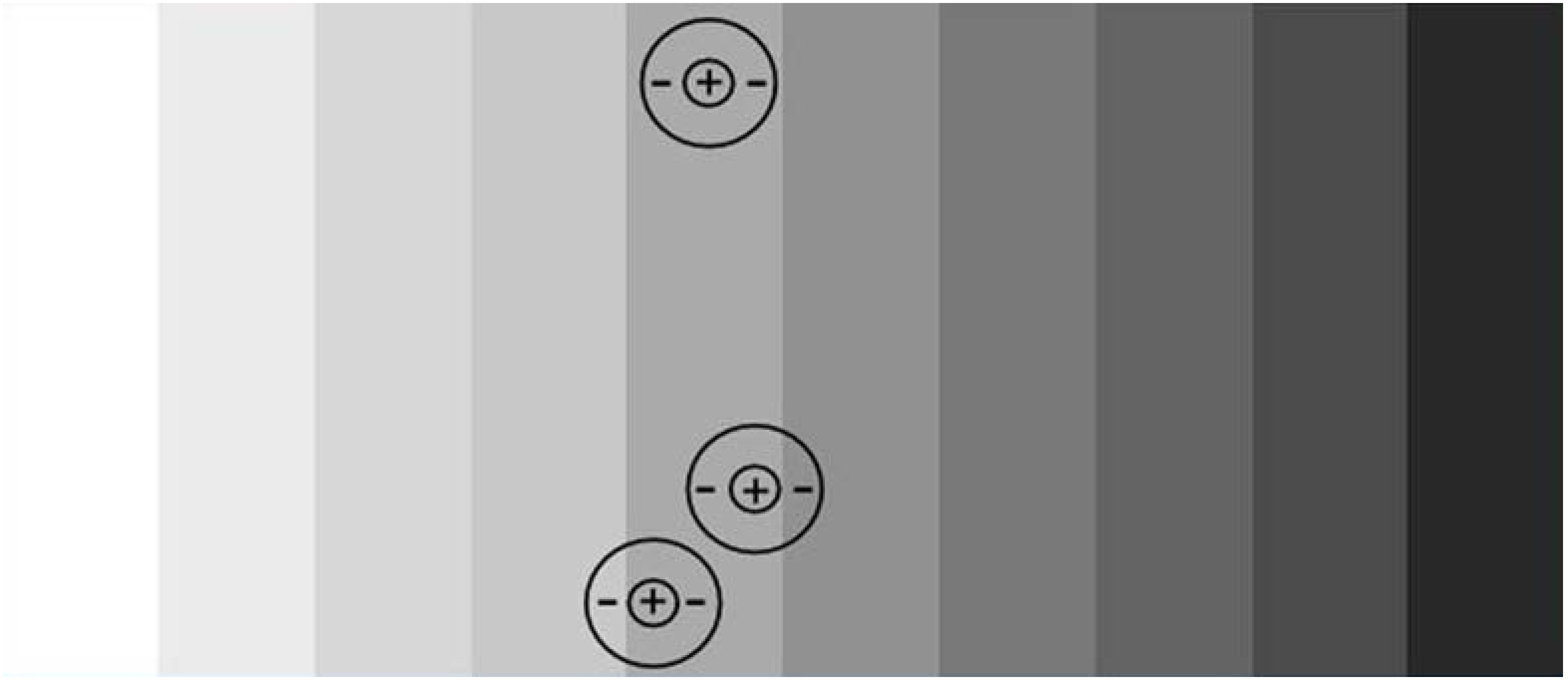


human


falcon
macaque
owl monkey
cat
goldfish
Relative Contrast Sensitivity
1.0
0.3
0.1
0.03
0.3
1.0
3.0
10
30
100
Spatial Frequency (c/deg)

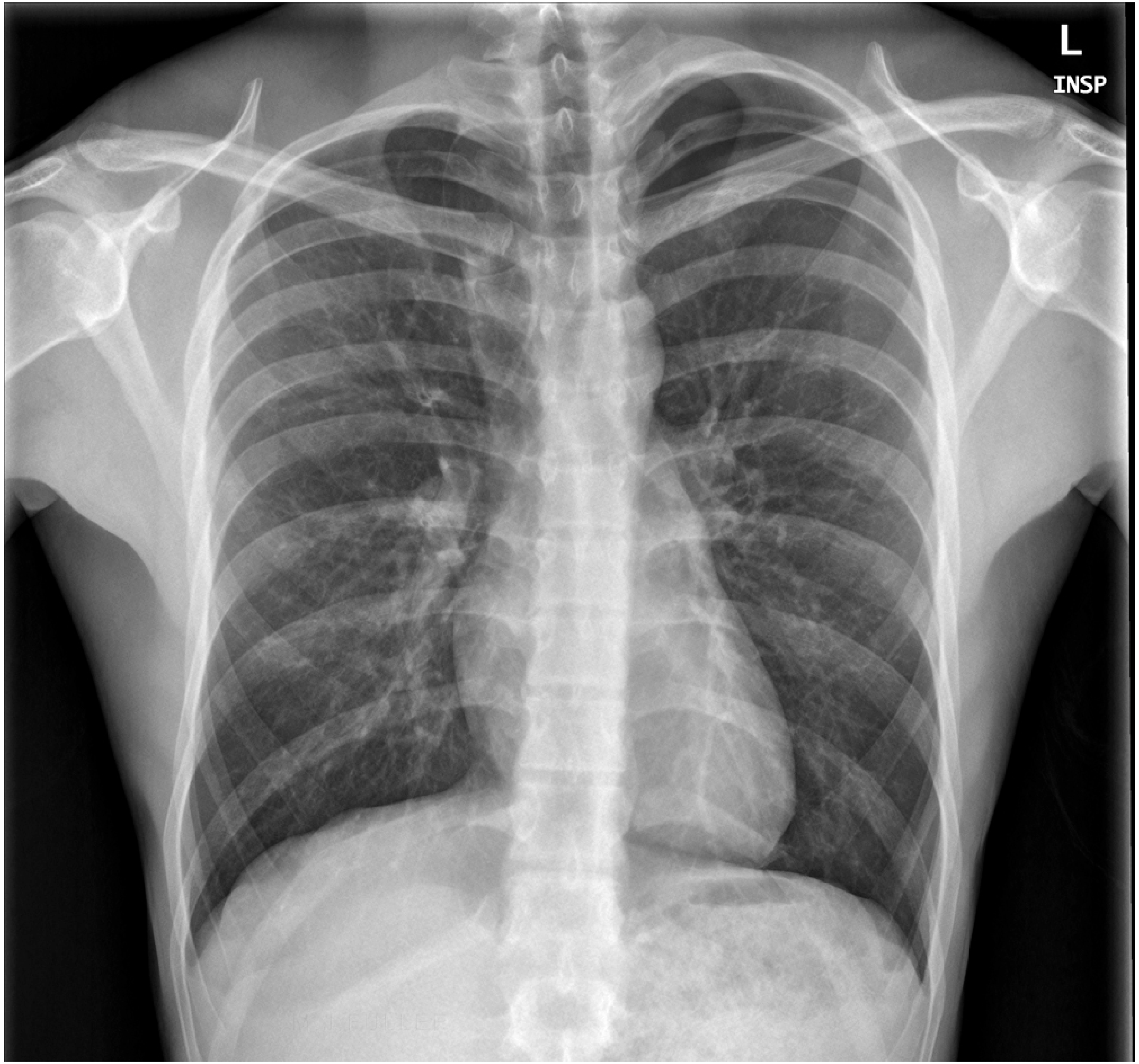
Edge enhancement effect

Herring ladder and **Mach band**



Each stripe is **uniform gray**, but they appear **darker** near the **light boundary** and **lighter** near the **dark boundary**

The receptive field properties the **retina cells** contribute to this perceptual effect



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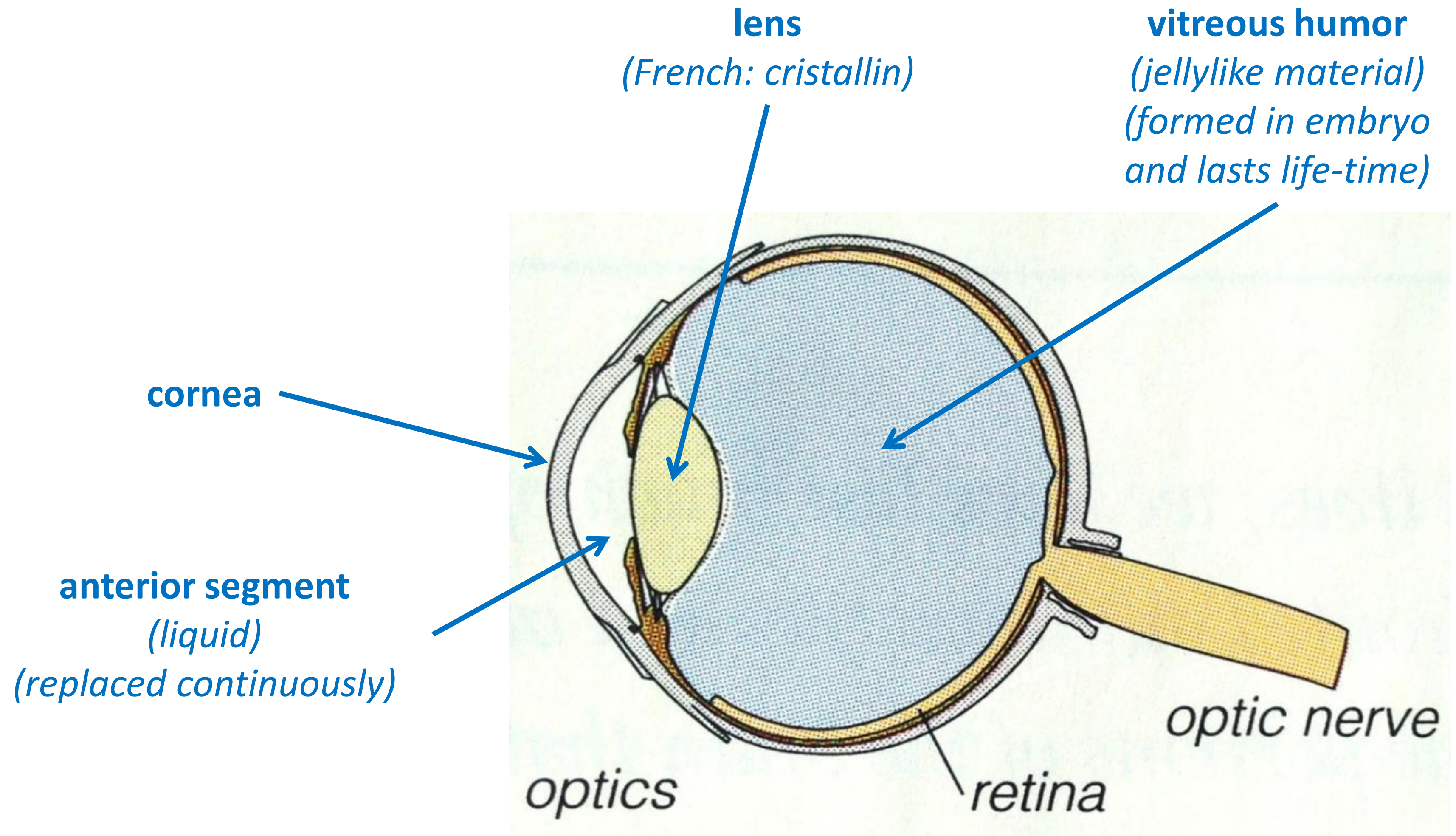
Human eye (top view)



From which direction do the photons enter the retina?

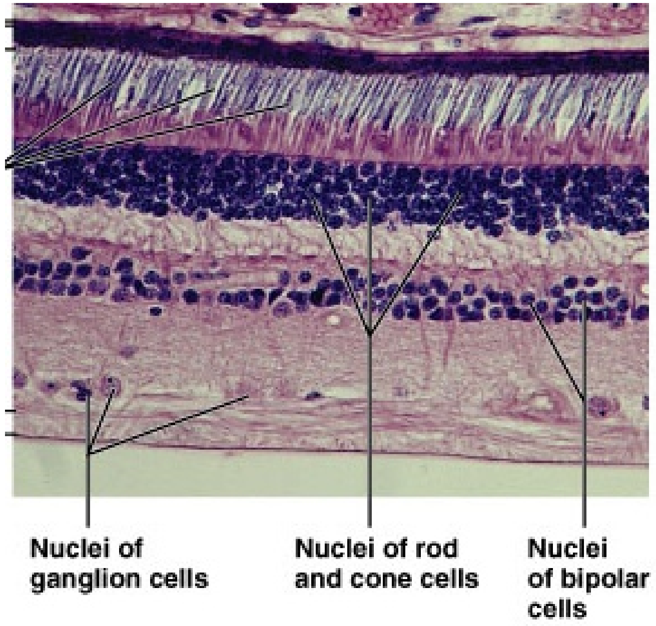
1. top
2. bottom
3. no idea

from the top

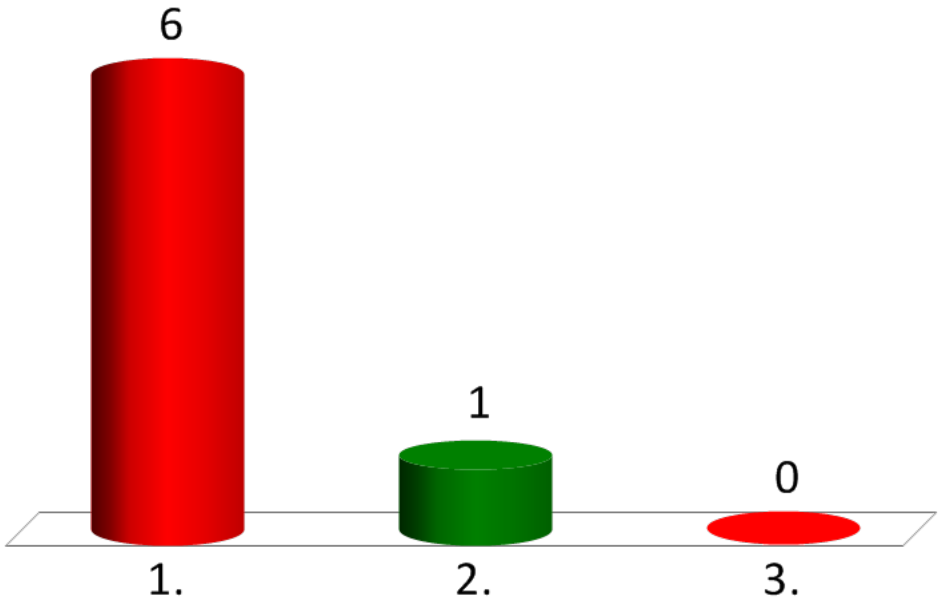
photodetectors

"electronics"

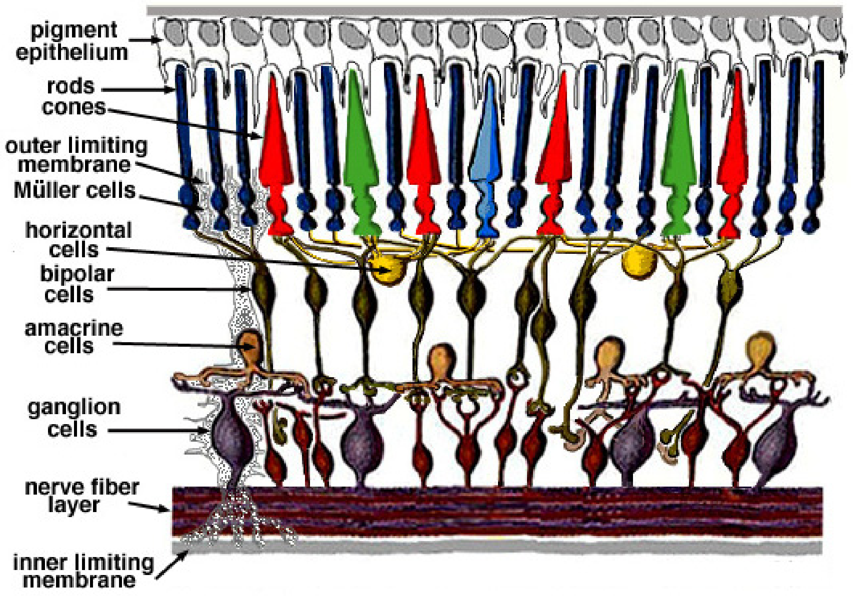
retina

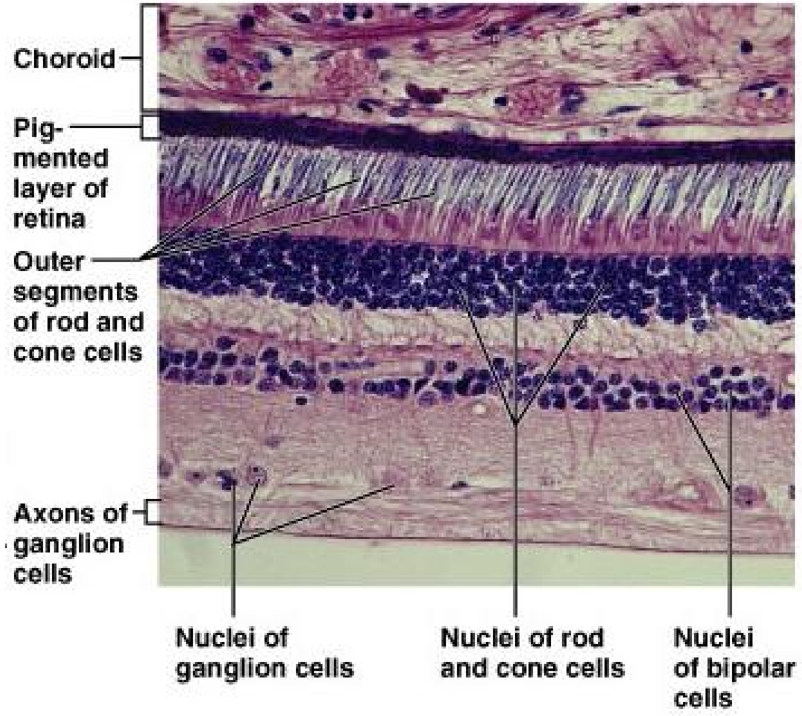


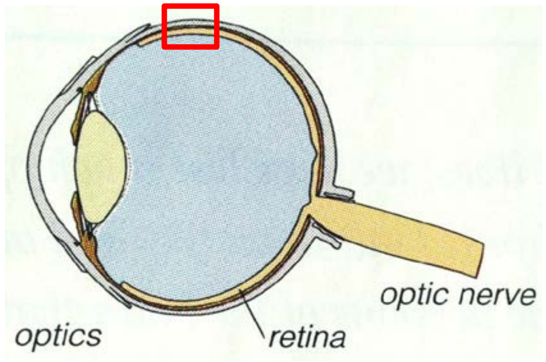
from the bottom



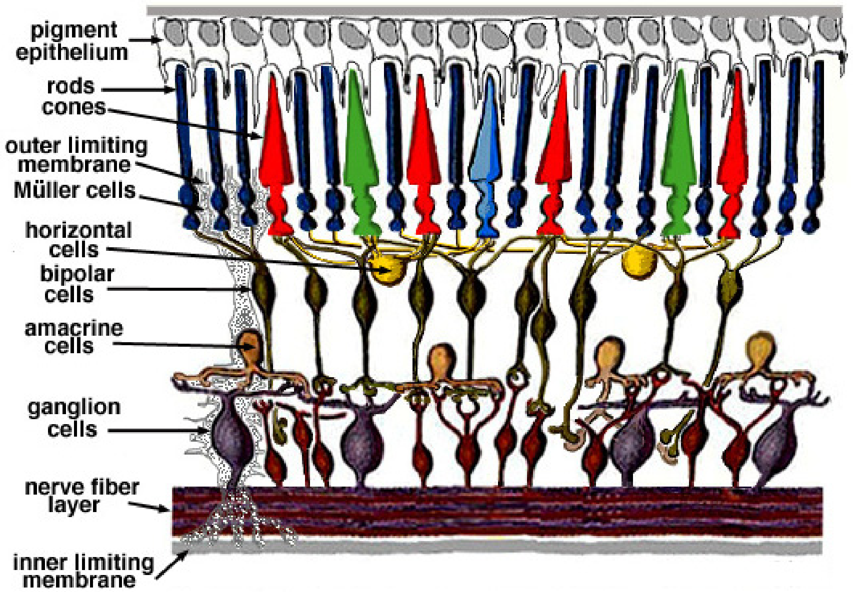
7


pigment epithelium
rods
cones
outer limiting membrane
Müller cells
horizontal cells
bipolar cells
amacrine cells
ganglion cells
nerve fiber layer
inner limiting membrane

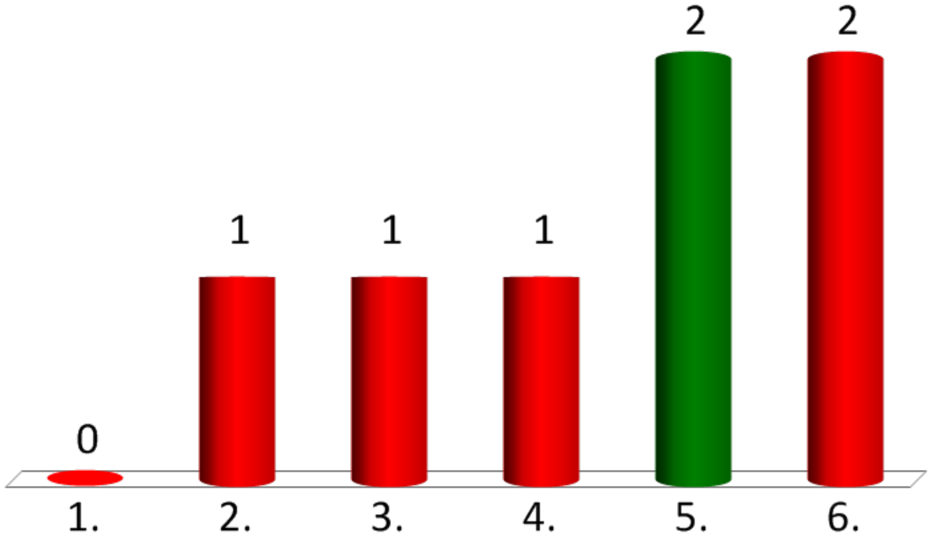

Choroid
Pig-mented layer of retina
Outer segments of rod and cone cells
Axons of ganglion cells
Nuclei of ganglion cells
Nuclei of rod and cone cells
Nuclei of bipolar cells


optic nerve
optics
retina

How many **photoreceptors** (rods and cones) do we have in each **human eye**?



1. about 13 k-receptors
2. about 130 k-receptors
3. about 1.3 M-receptors
4. about 13 M-receptors
5. about 130 M-receptors
6. about 1.3 G-receptors



7

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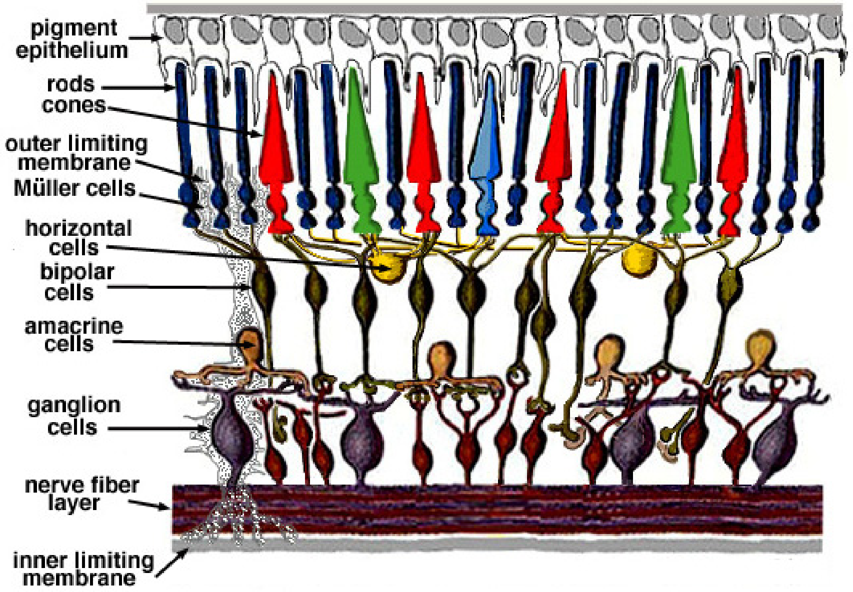
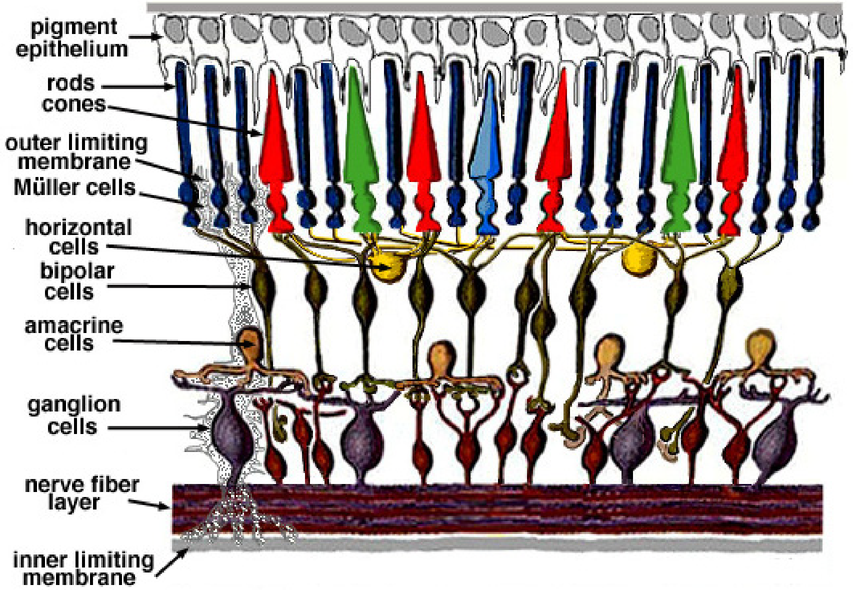
about **130 million** photoreceptors

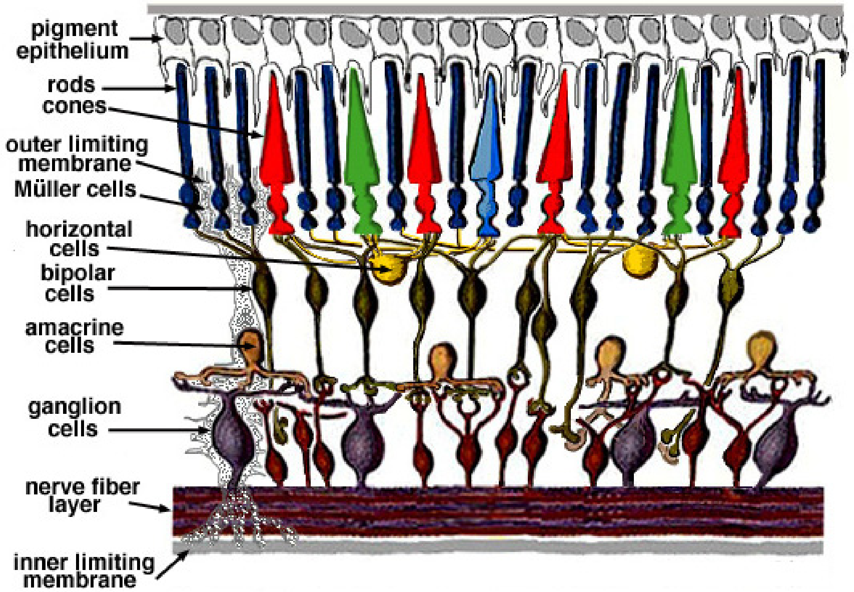


↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

about **10 million** nerve fibers ➡ **to the brain**



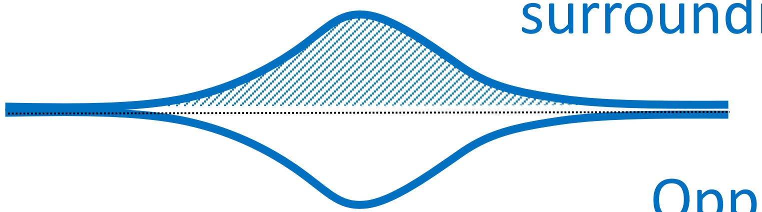
different types of information are collected by the **nerve fibers**

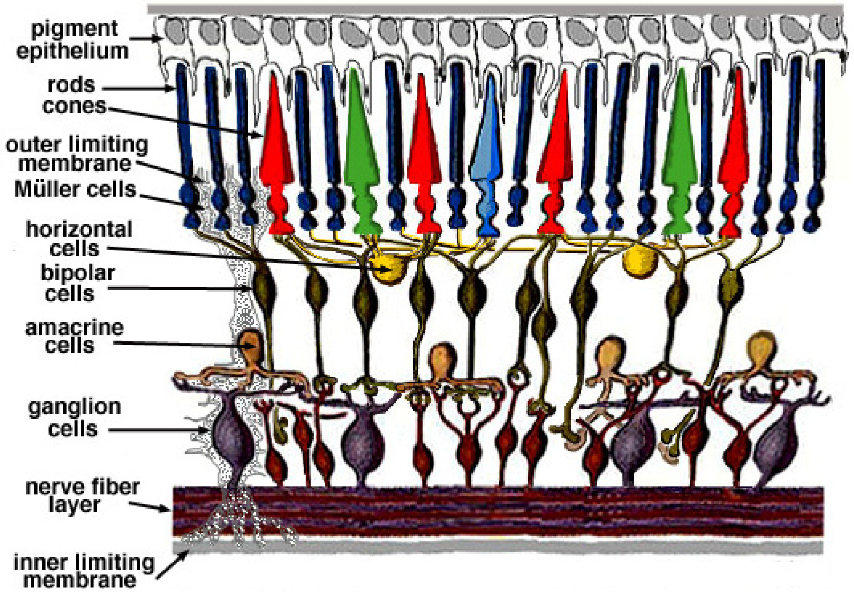


photoreceptors

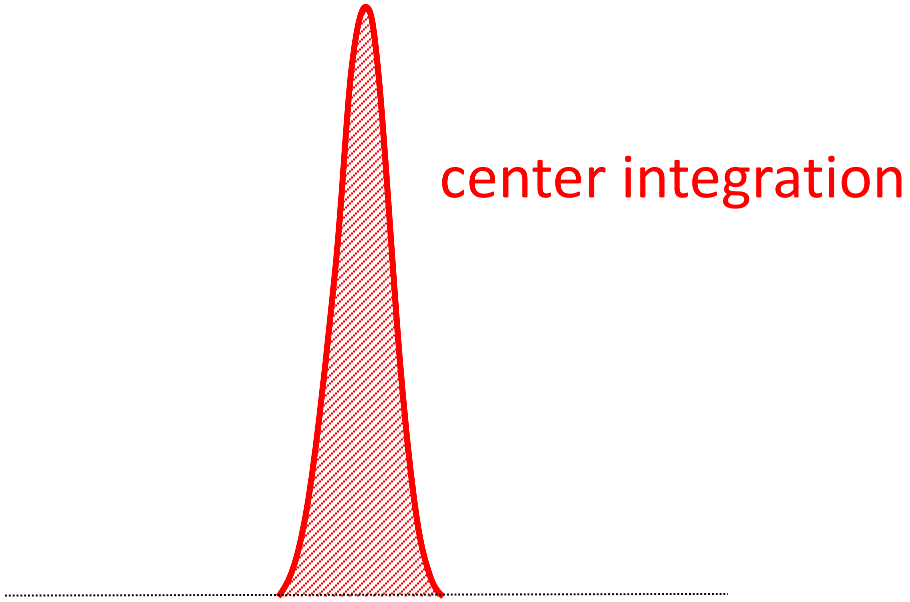
"electronics"

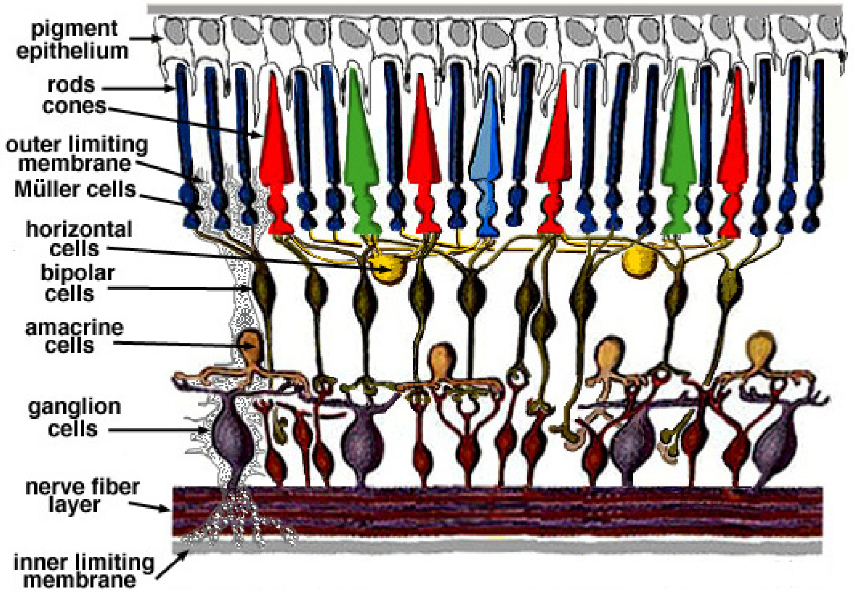
different types of information are collected by the **nerve fibers**

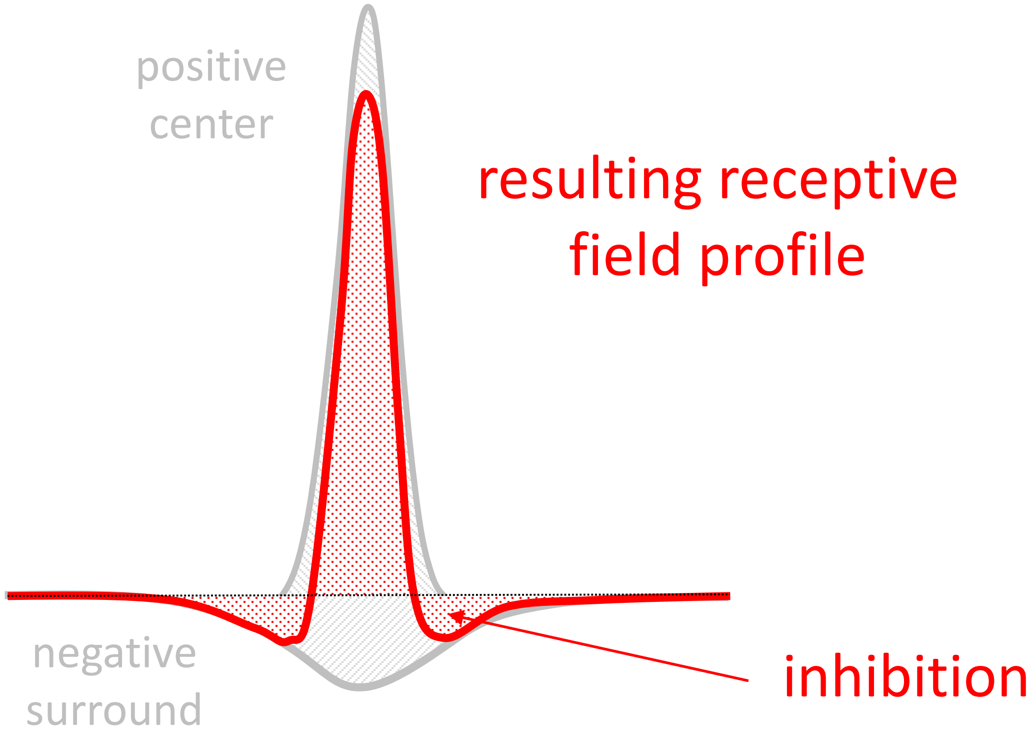




different types of information are collected by the **nerve fibers**







different types of information are collected by the **nerve fibers**

Foveal and peripheral vision

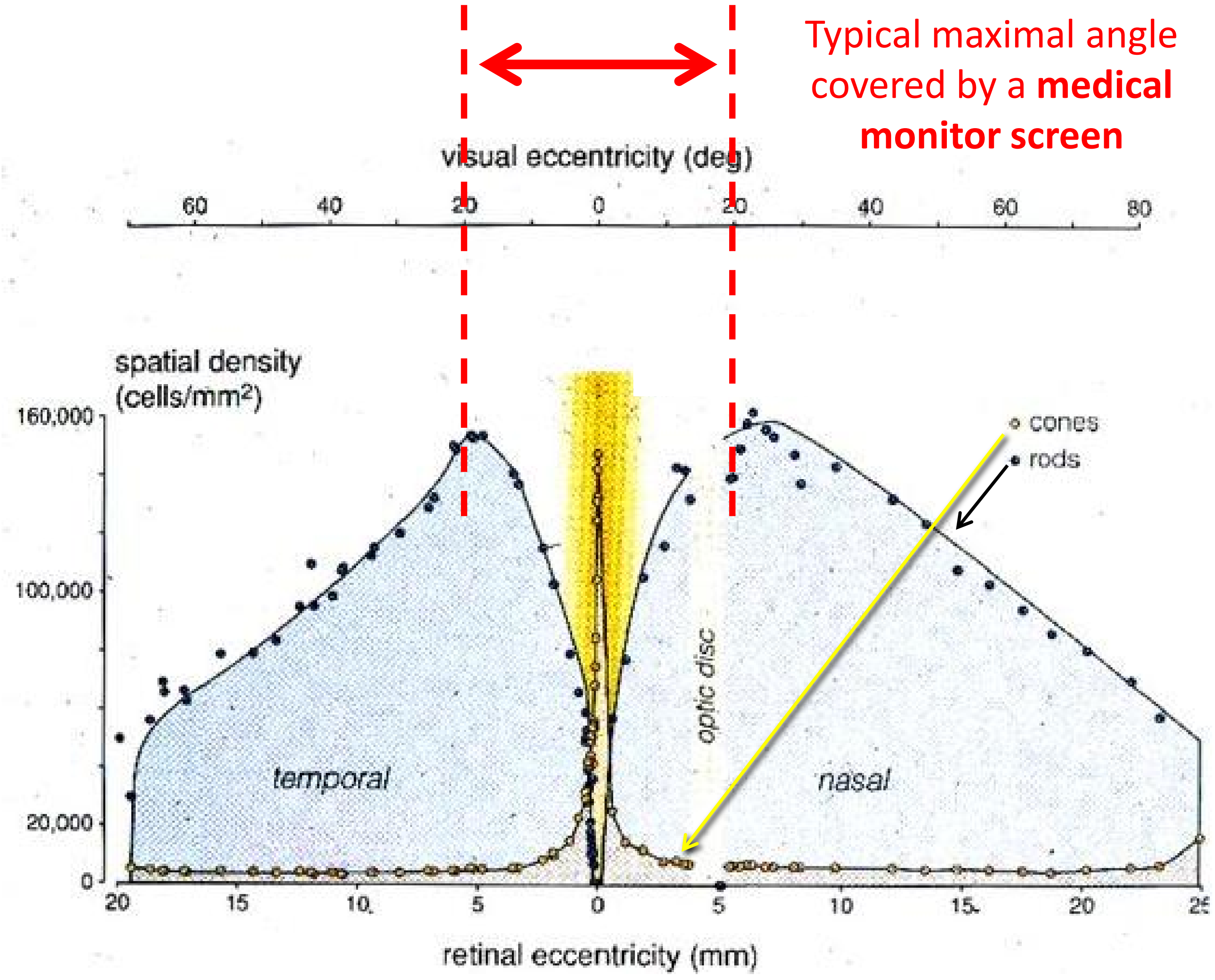
Competing goals

- Maximize **spatial resolution**
- Maximize **field of view**
- Minimize **neural resources**

Solutions

- High resolution **foveal** vision
- Low resolution **peripheral** vision
- Rapid **eye movements**

Typical maximal angle covered by a **medical monitor screen**



http://www.cns.nyu.edu/~david/courses/perception/lecturenotes/retina/retina.html

Viewing with the **fovea** leads to a **much better spatial resolution**

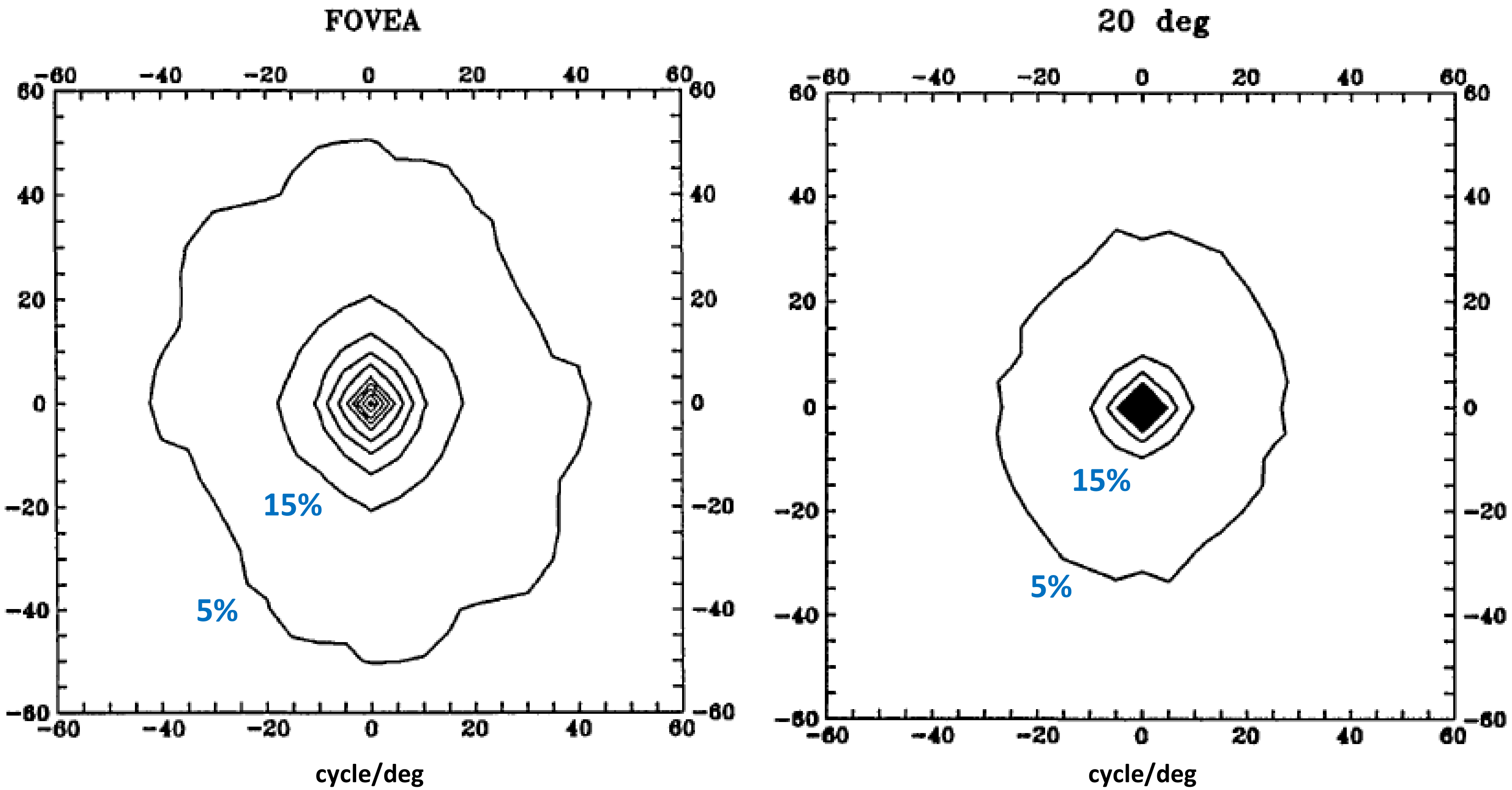
the head is on focus

the whole image is on focus

the **feet** are on focus

http://homepage.psy.utexas.edu/homepage/class/Psy380E/VS_8_retina.pdf

The eye MTF varies with eccentricity



pure **physical measurements** on the retina

(average of four observers)

Navarro *et al*, JOSA-A 10:201-212 (1993)

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https://www.youtube.com/watch?v=Rk2izv-c_ts

MINUTE PHYSICS

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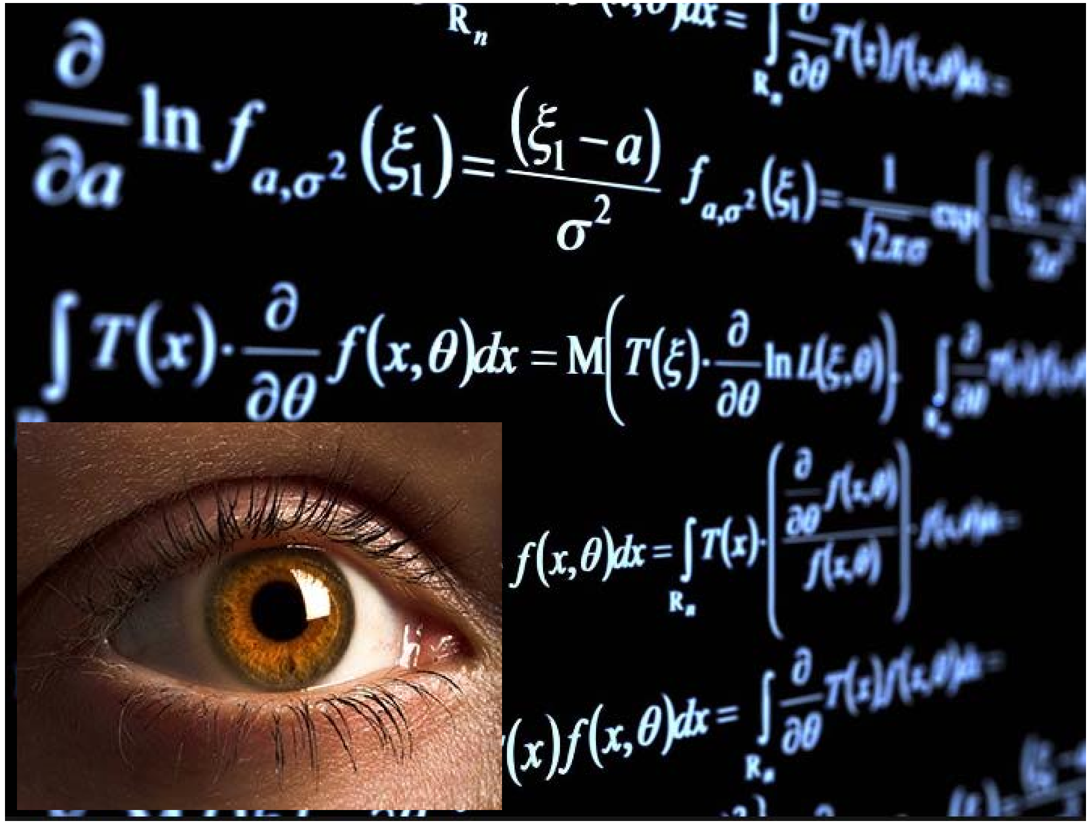


Mathematical model observers

5.

Anthropomorphic observer:

Mathematical formalism

image $\mathbf{g}$

observer response:
scalar λ

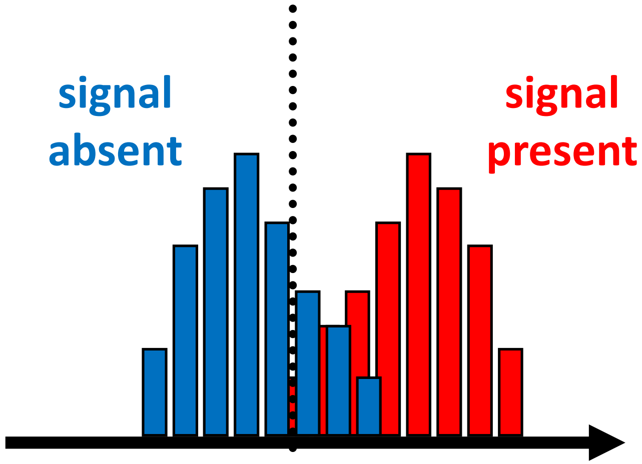
decision D_0

decision D_1

negative t_c **positive**

signal absent

signal present

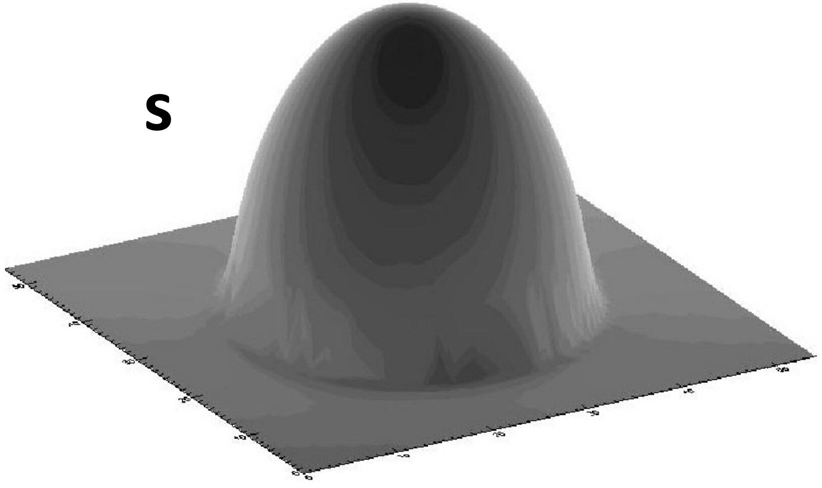


observer response λ
(level of certainty)

Matched-filter observer

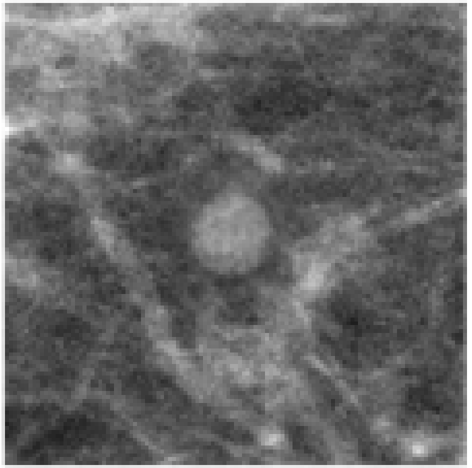
$$\lambda(\mathbf{g}) = \mathbf{s}^{\mathsf{T}}\mathbf{g}$$

s



signal

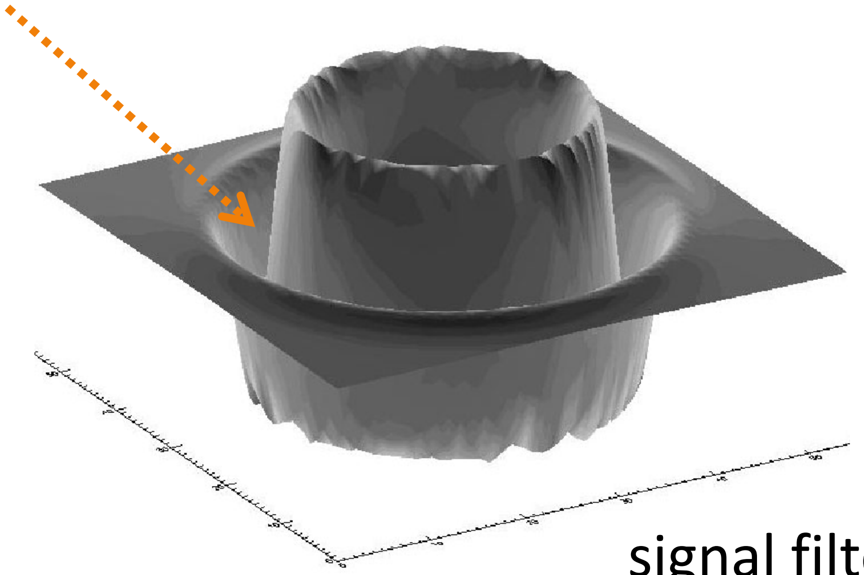
g



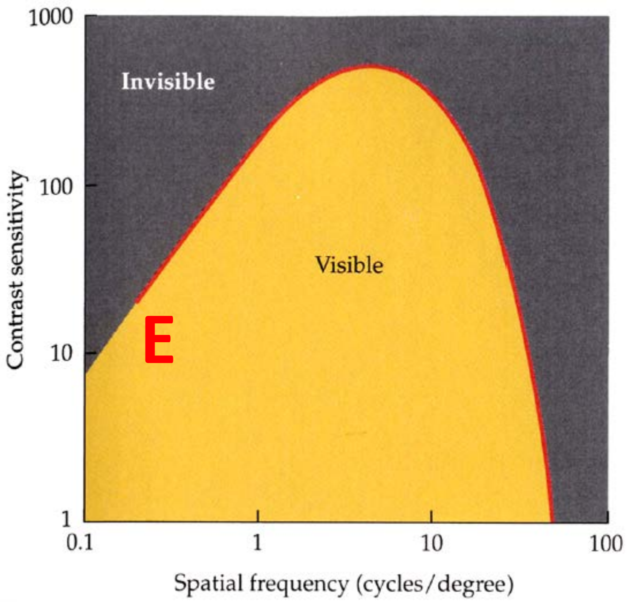
Matched-filter with eye-response observer

$$\lambda_{NPWE}(\mathbf{g}) = \mathbf{s}^T\mathbf{E}^T\mathbf{E}\mathbf{g} = (\mathbf{E}\mathbf{s})^T(\mathbf{E}\mathbf{g})$$

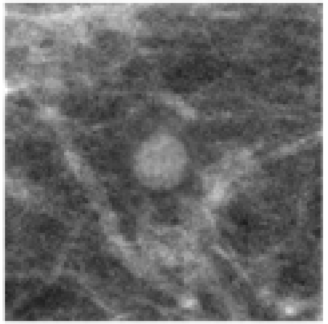
the **inhibition** observed at object's border through **center-surround** process in **human vision** is visible here



signal filtered by E^2



Channelized models



g
(image)

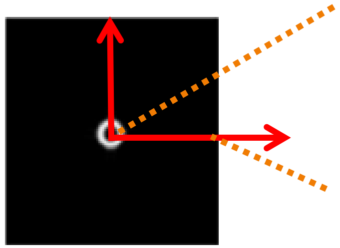
Response to channel **c**

$v = \mathbf{c}^T \cdot FT(\mathbf{g})$

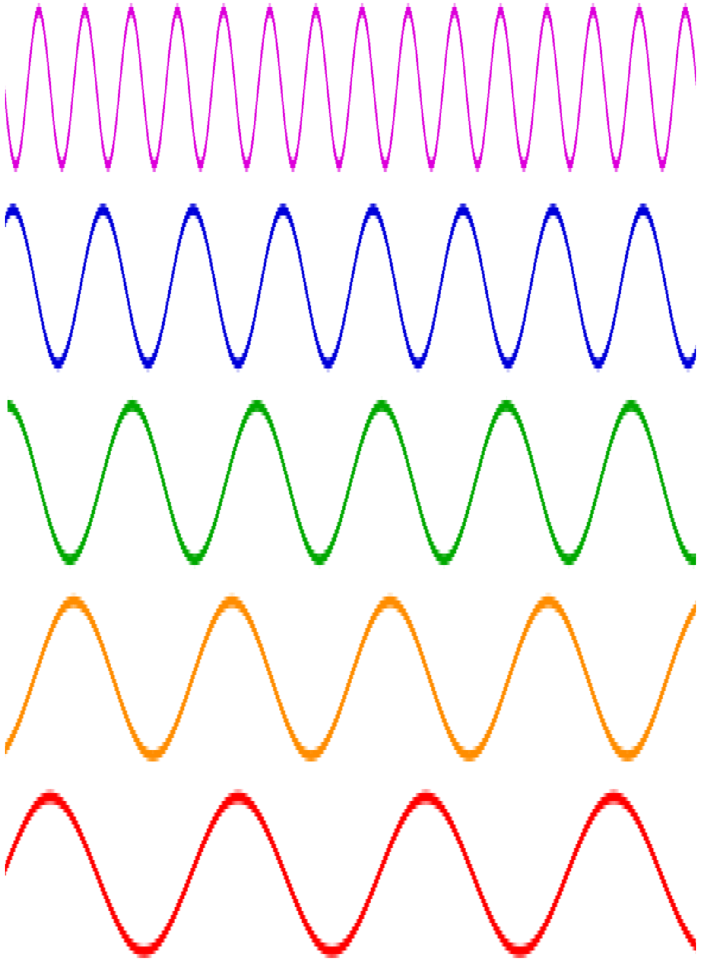
(FT: Fourier transform)
(v is a scalar)

channel
(frequency template)

c
(channel)

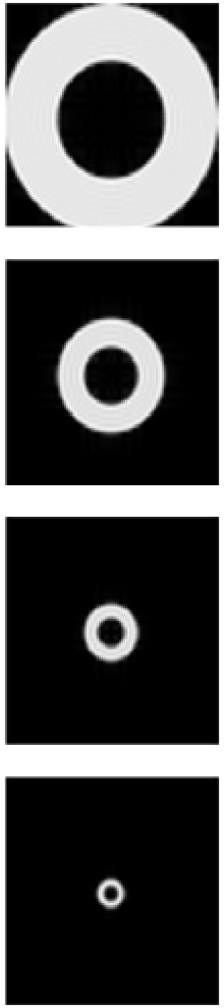


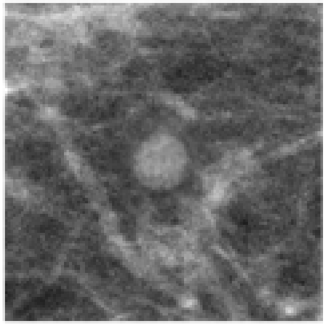
Channelized models



High
frequency band

Low
frequency band





g
(image)

Channelized models

linear channelized model observer

$$\lambda = \mathbf{w}^T \mathbf{v}$$

Why?
Similar to **human brain**
Large **reduction of data**

typical medical image:
512x512=**262'144 pixels**

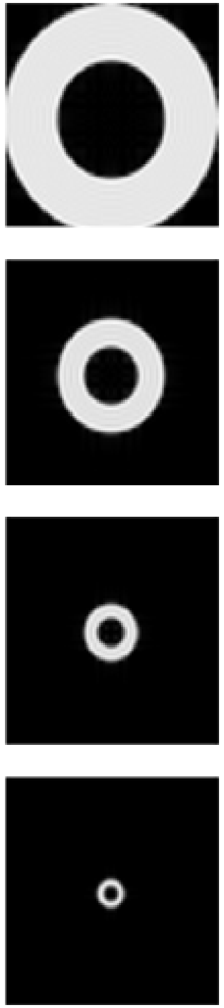
typical number of channels: **6-10 channels**

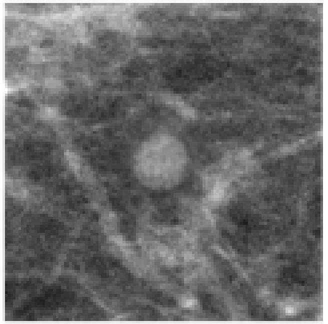
v_1 *(scalar)* ← response to this frequency band

v_2 *(scalar)* ← response to this frequency band

v_3 *(scalar)* ← response to this frequency band

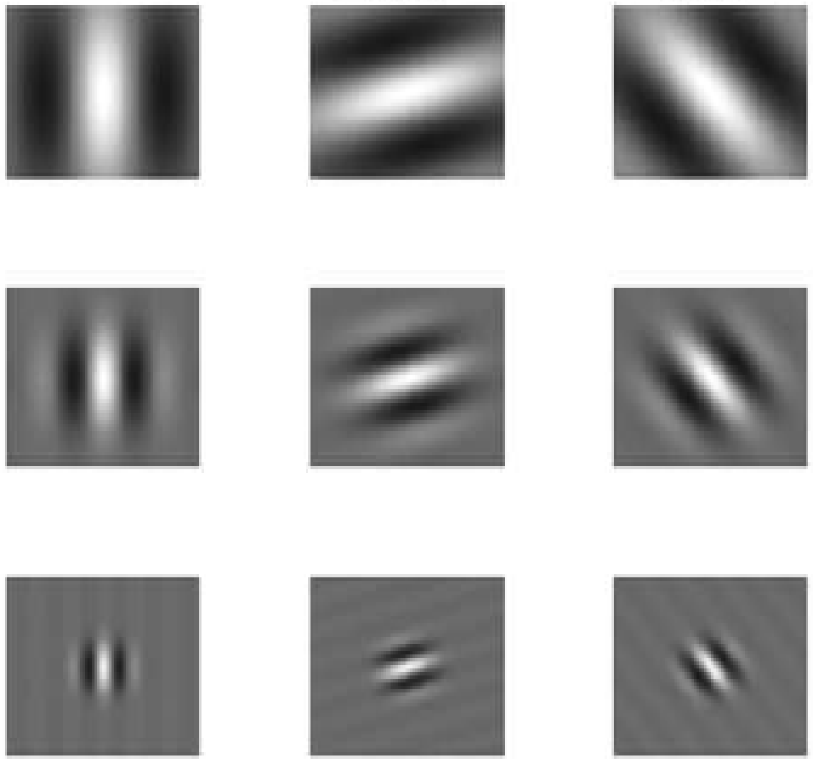
v_4 *(scalar)* ← response to this frequency band





g *(image)*

Example of channels



Gabor channels

Design of anthropomorphic selective channels

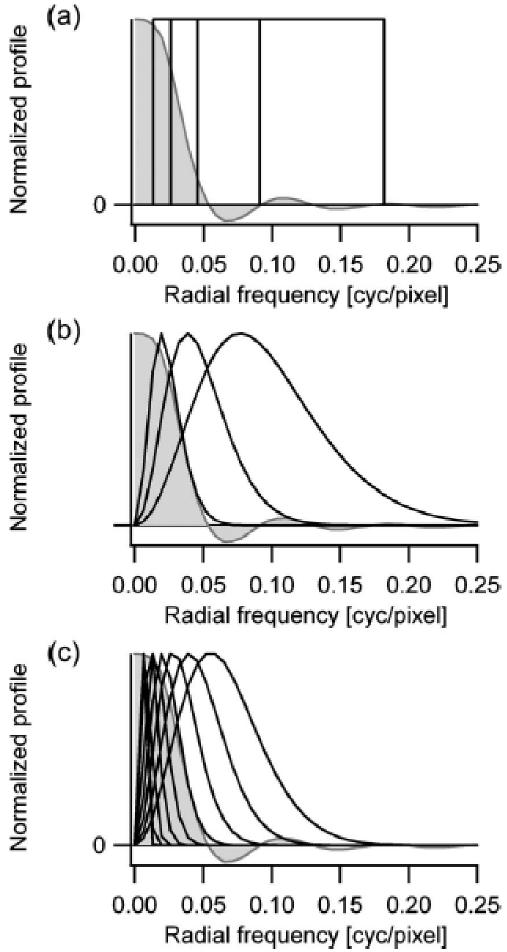


Fig. 1. Channels used for (a) the SQR model, (b) the S-DOG model, and (c) the D-DOG model. The gray area represents the signal frequency profile.

Usually three conditions:

(**1**)
zero response at **zero frequency** in the frequency domain
(no response to a constant)

(**2**)
sparse channels
(only a handful of channels)

(**3**)
overlapping bandwidths that are octavely spaced
(i.e., the width of the N^{th} channel are twice those of the $(N\text{-}1)^{th}$ channel)

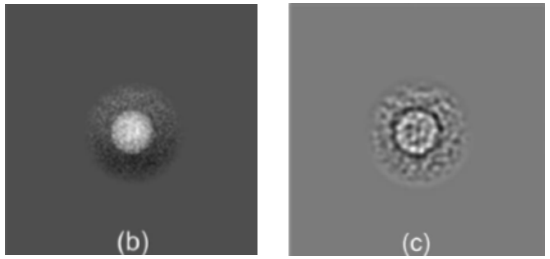
Castella, Abbey Eckstein, Verdun, Kinkel, Bochud ; 'Human linear template with mammographic backgrounds estimated with a genetic algorithm'; JOSA-A 24; pp. B1-B12 (2007)
Xin He and Subok Park, Model Observers in Medical Imaging Research, Theranostics 2013; 3(10):774-786. doi: 10.7150/thno.5138

Examples of anthropomorphic observers that can be presented as linear templates **w**

$$\lambda = \mathbf{w}^{T}\mathbf{g}$$

Images

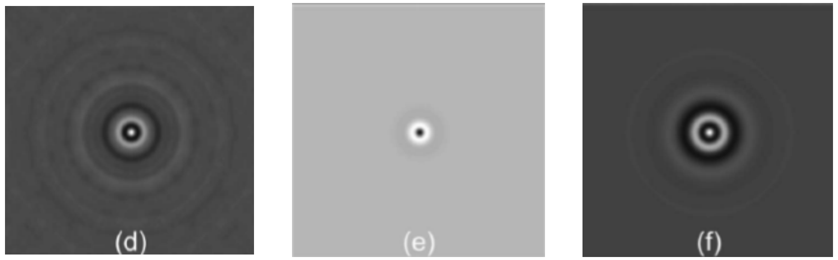
NPW
(= signal)



NPWE
(eye filter)

without signal

CHO



(SQR channels) *(S-DOG channels)* *(D-DOG channels)*

with signal

Castella, Abbey Eckstein, Verdun, Kinkel, Bochud ; 'Human linear template with mammographic backgrounds estimated with a genetic algorithm'; JOSA-A 24; pp. B1-B12 (2007)

Internal noise

Good **anthropomorphic linear model** observers **outperform human** observers

Model observers $\lambda = \mathbf{w}^T\mathbf{g}$ are consistent
*(they always provide the same answer to a given image **g**)*

Human are not always consistent
*(if they are shown a given image **g** they may provide another response)*

Model observers can be degraded by adding a random variable ε to their responses
(ε is called internal noise)

$$\lambda = \mathbf{w}^{T}\mathbf{g} + \varepsilon$$

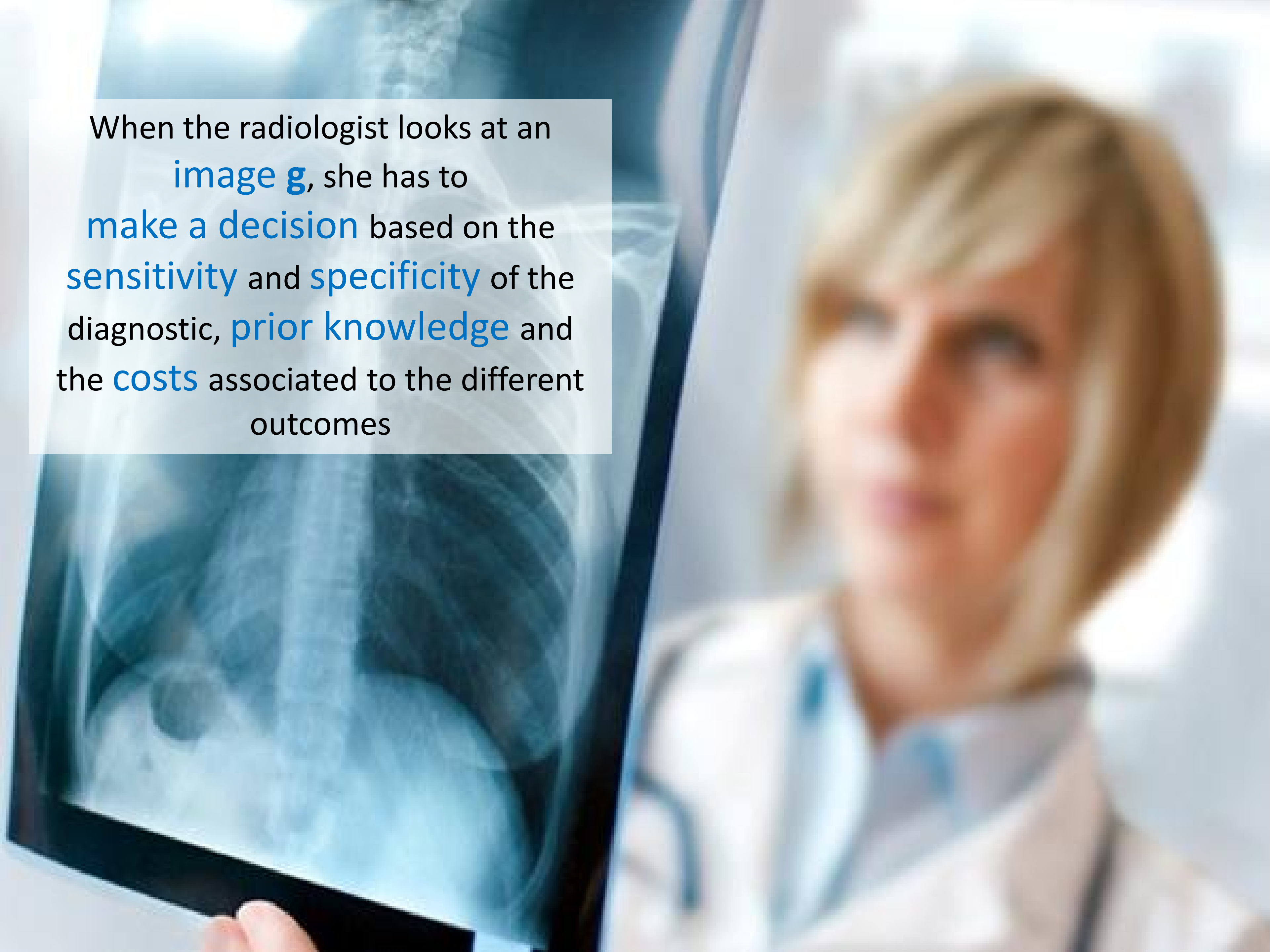
ε is typically a Gaussian random variable that has the effect of spreading the observer's responses
(ε is fitted on the human performances)

Mathematical model observers

6.

Ideal model observer

https://www.preachingtoday.com/illustrations/2018/february/millennials-strive-for-perfection-to-their-harm.html



When the radiologist looks at an **image g**, she has to make a decision based on the sensitivity and specificity of the diagnostic, prior knowledge and the costs associated to the different outcomes

image **g**

(in R^M)

(if there are M voxels)

response λ(**g**)

(in R)

decision D_0

decision D_1

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The **ideal observer (IO)** makes its decision according to its prior knowledge and by **minimizing** the **mean cost**

decision (negative D_0; positive D_1)

hypothesis (reality: signal absent H_0; signal present H_1)

$$\bar{C} = \sum_{\text{alternatives}} P(D,H) \times \text{cost}(D,H)$$

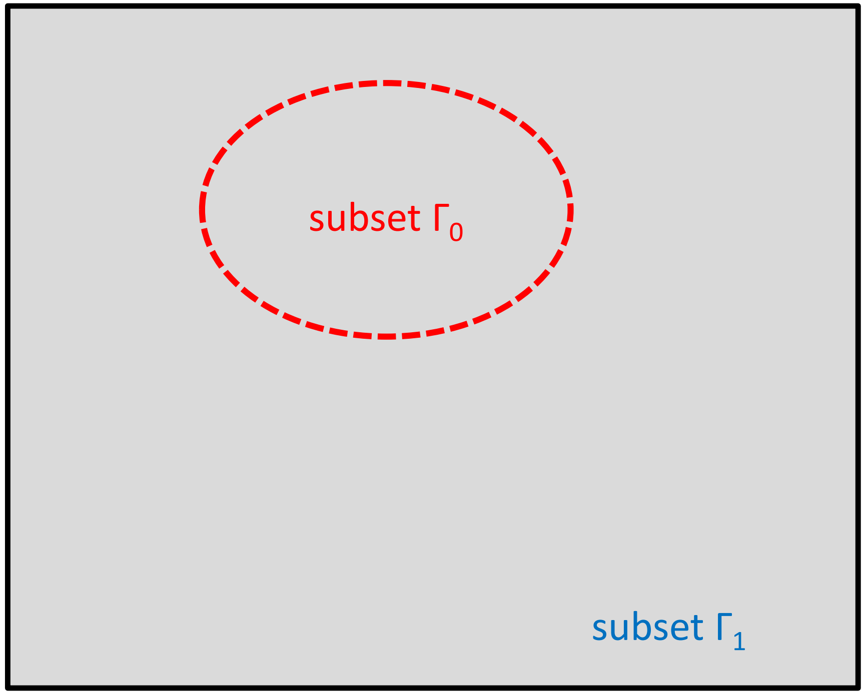
$$= C_{00}P(D_0|H_0)P(H_0) + C_{01}P(D_0|H_1)P(H_1) + C_{10}P(D_1|H_0)P(H_0) + C_{11}P(D_1|H_1)P(H_1)$$

minimizing **mean cost**

$$\bar{C} = C_{00}P(D_0|H_0)P(H_0) + C_{01}P(D_0|H_1)P(H_1) + C_{10}P(D_1|H_0)P(H_0) + C_{11}P(D_1|H_1)P(H_1)$$

+

1. **Decisions** are **deterministic**
2. The observer is **forced to make a decision**



ensemble of all possible images

minimizing **mean cost**

$$\bar{C} = C_{00}P(D_0|H_0)P(H_0) + C_{01}P(D_0|H_1)P(H_1) + C_{10}P(D_1|H_0)P(H_0) + C_{11}P(D_1|H_1)P(H_1)$$

➕

1. **Decisions** are **deterministic**
2. The observer is **forced to make a decision**

⬇ (…)

The decision of the IO is based on a **likelihood ratio**

likelihood to observe this image **g** if H_1 is true

likelihood ratio ← $\Lambda(\mathbf{g}) = \frac{p(\mathbf{g}|H_1)}{p(\mathbf{g}|H_0)} > \text{threshold}$

likelihood to observe this image **g** if H_0 is true

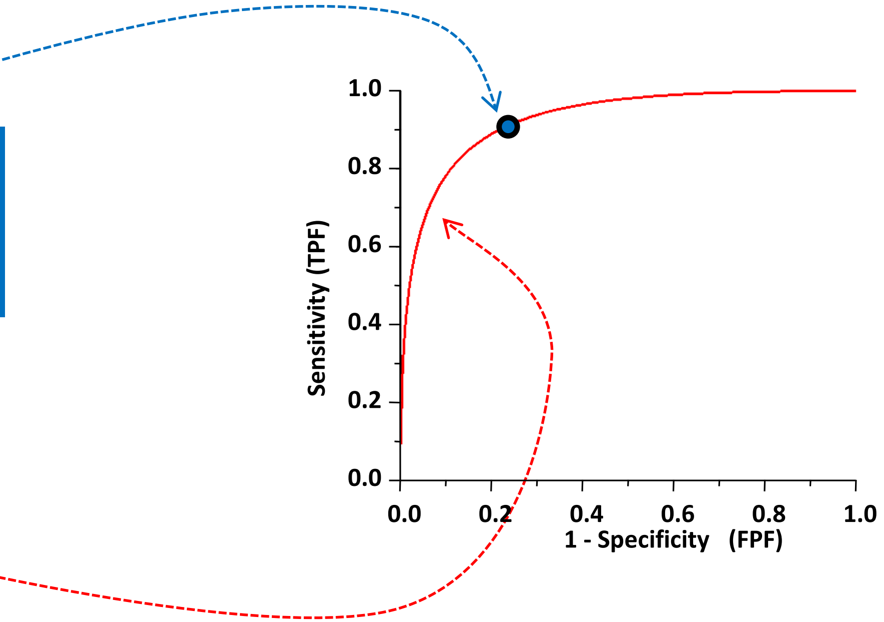
The IO requires the full probability density function on image $\mathbf{g}$ under each hypotheses

$$\Lambda(\mathbf{g}) = \frac{p(\mathbf{g}|H_1)}{p(\mathbf{g}|H_0)} \underset{D_0}{\overset{D_1}{\gtrless}} \frac{C_{10} - C_{00}}{C_{01} - C_{11}} \frac{p(H_0)}{p(H_1)}$$

The ROC curve is defined by the decision variable $\Lambda(\mathbf{g})$

The operating point is defined by the costs and the prevalence *(a priori probabilities)*

$$\Lambda(\mathbf{g}) = \frac{p(\mathbf{g}|H_1)}{p(\mathbf{g}|H_0)} \underset{D_0}{\overset{D_1}{\gtrless}} \frac{C_{10} - C_{00}}{C_{01} - C_{11}} \frac{p(H_0)}{p(H_1)}$$



Ideal observer (IO)

$$\Lambda(\mathbf{g}) = \frac{p(\mathbf{g}|H_1)}{p(\mathbf{g}|H_0)} \underset{D_0}{\overset{D_1}{\gtrless}} \frac{C_{10} - C_{00}}{C_{01} - C_{11}} \frac{p(H_0)}{p(H_1)}$$

$\Lambda(\mathbf{g})$ is an **ideal observer** (IO) because

"it utilizes **all statistical information** available regarding the task to **maximize task performance** as measured by Bayes risk or some other related measures of performance"

It is not the observer that performs always correctly

Ideal observer (IO)

$$\Lambda(\mathbf{g}) = \frac{p(\mathbf{g}|H_1)}{p(\mathbf{g}|H_0)} \underset{D_0}{\overset{D_1}{\gtrless}} \frac{C_{10} - C_{00}}{C_{01} - C_{11}} \frac{p(H_0)}{p(H_1)}$$

If the probability density function of the images is Gaussian (**multinormal**)

$$p(\mathbf{g}|H_i) = \frac{1}{(2\pi)^{M/2}\sqrt{\det(\mathbf{K})}} e^{-\frac{1}{2}(\mathbf{g}-\overline{\mathbf{g}}_i)^T \mathbf{K}^{-1}(\mathbf{g}-\overline{\mathbf{g}}_i)}$$

M: number of pixels
***K**: covariance matrix*

$$\Lambda(\mathbf{g}) = \left(\mathbf{K}^{-1}\mathbf{s}\right)^T \mathbf{g}$$

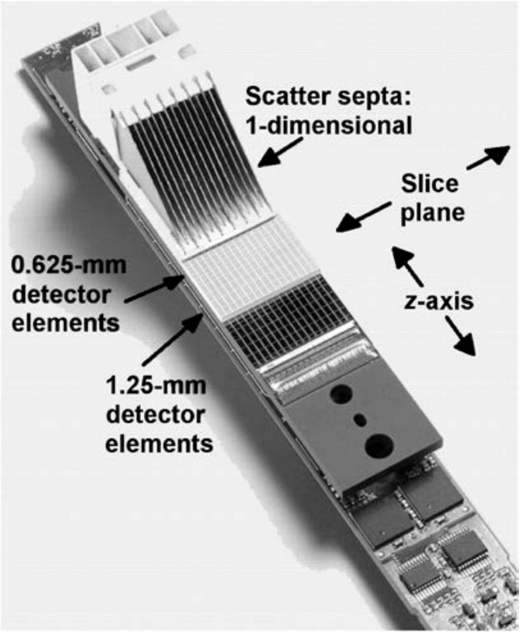
$$= \left(\mathbf{K}^{-\frac{1}{2}}\mathbf{s}\right)^T \mathbf{K}^{-\frac{1}{2}}\mathbf{g}$$

In **medical imaging**, **model observers** are developed for two general purposes

Hardware system **optimization**

(such as scatter rejection, detector sensitivity, MTF, …)

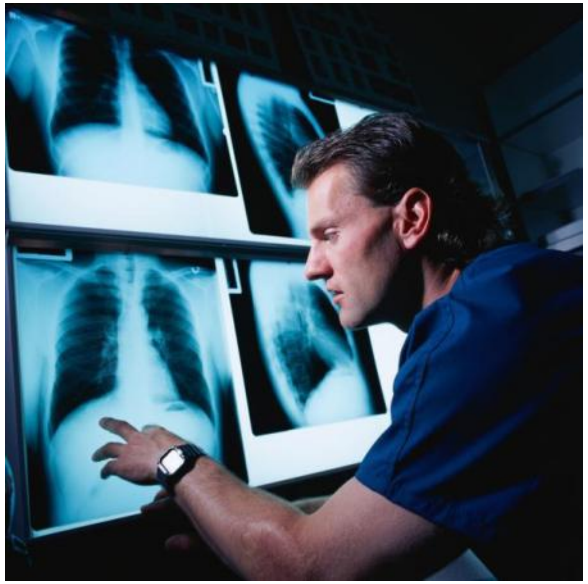
We need to **acquire** useful information



Optimization of **software** systems

(such as image reconstruction or processing methods)

We want to **display** accessible information **to the radiologist**



Xin He and Subok Park, Model Observers in Medical Imaging Research, Theranostics 2013; 3(10):774-786. doi: 10.7150/thno.5138

In **medical imaging**, **model observers** are developed for two general purposes

Hardware system **optimization**
(such as scatter rejection, detector sensitivity, MTF, …)

Ideal observer

We need to **acquire** useful information

A model observer that **extracts as much** statistical **information as possible** from the images for a given task

For signal detection,
Bayesian **ideal observer** (IO)
or ideal linear observer is often used

Optimization of **software** systems
(such as image reconstruction or processing methods)

Anthropomorphic observer

We want to **display** accessible information **to the radiologist**

A model that **mimics human-observer** performance.
Human studies are resource demanding
(e.g. multiple parameters for reconstruction algorithms or acquisition protocols)

For signal detection,
linear IO or approximations of IO
with **some human features**
(limited frequency efficiency, internal noise)

Xin He and Subok Park, Model Observers in Medical Imaging Research, Theranostics 2013; 3(10):774-786. doi: 10.7150/thno.5138

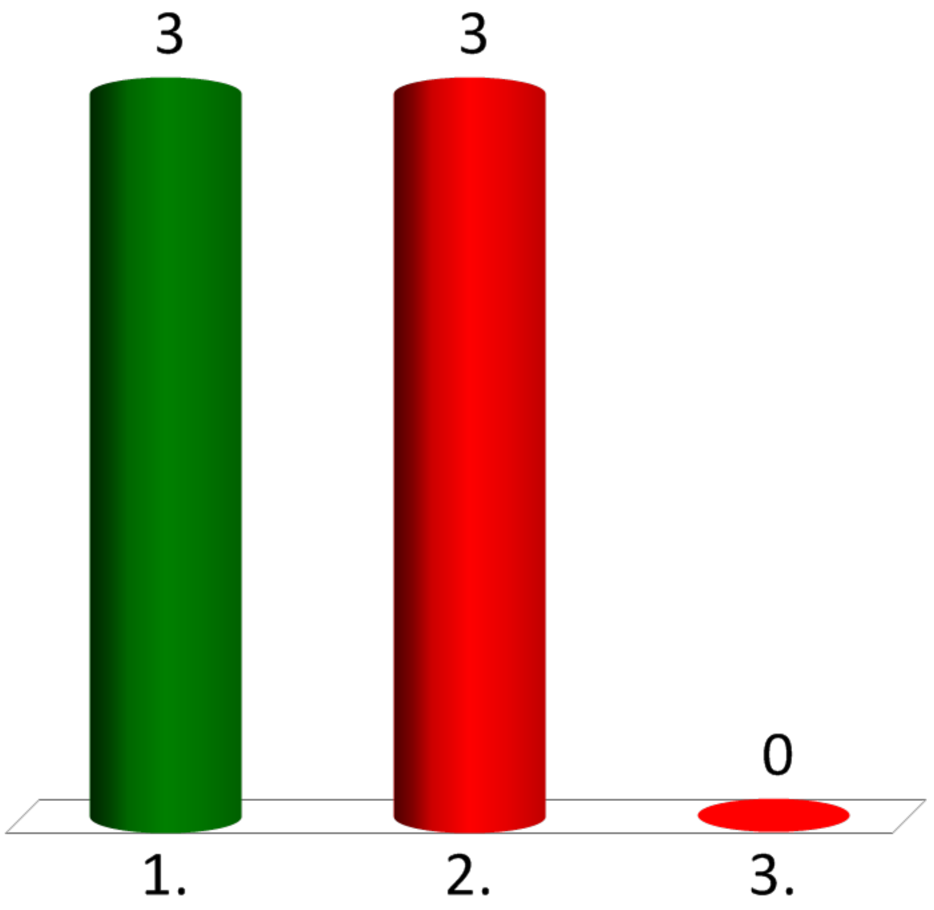
Mathematical model observers

7.

Summary questions

Which set of parameters define **objective image quality** in medical imaging?

1. task
   type of signal + bkg
   observer
   figure of merit
2. resolution
   contrast
   noise
3. fidelity of representation



6

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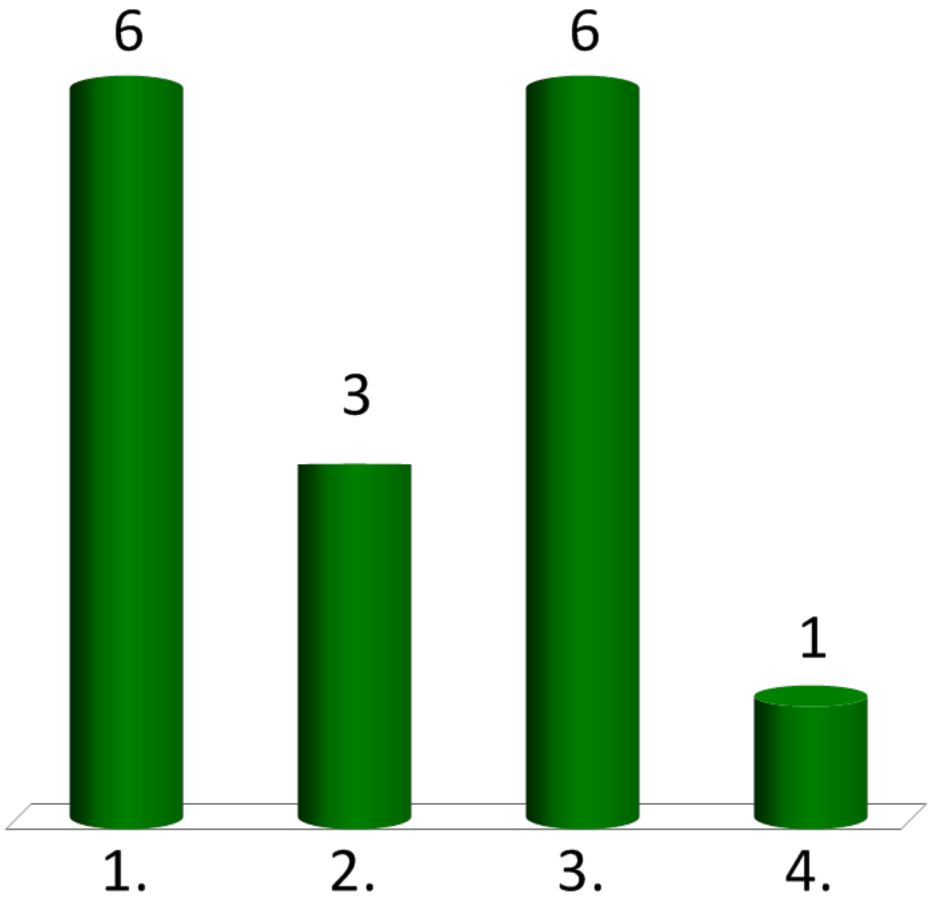
Why are model observers useful in medical imaging?

(multiple responses possible)

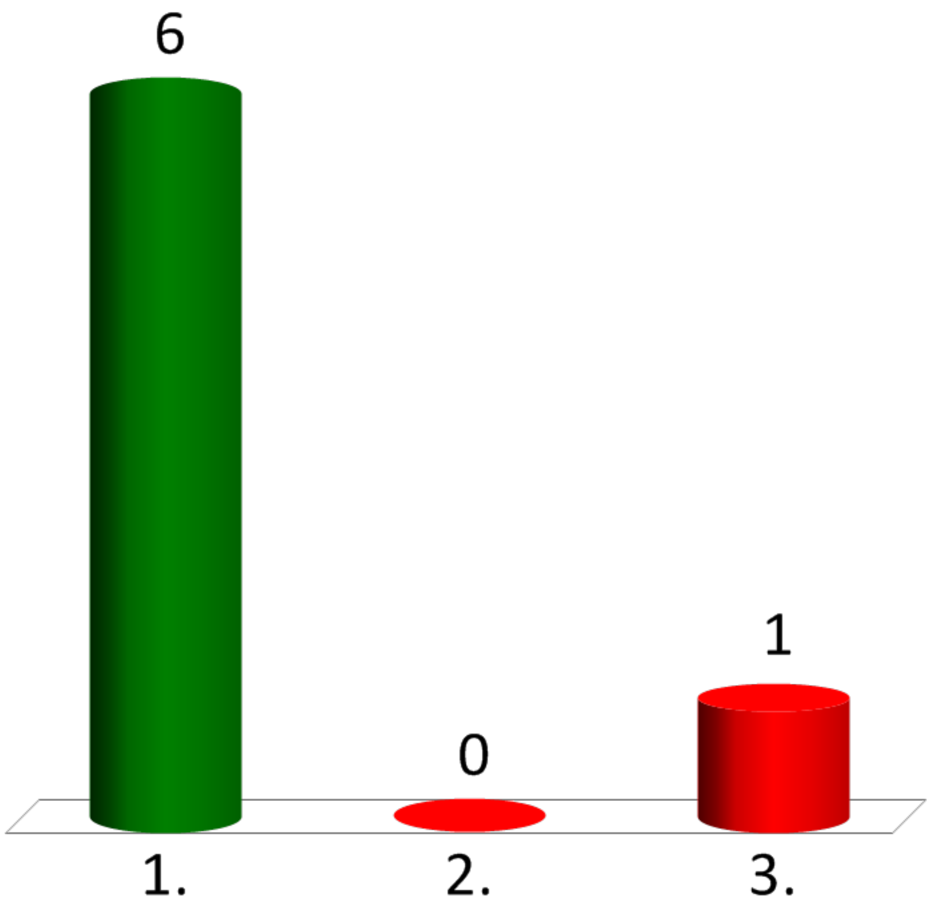
1. they objectively characterize image quality
2. they are consistent (they do not change their mind one week or another)
3. they allow to test a large number of parameters
4. they can be used at the design state when only simulated images exist



7

What is the **dimension** of the **response** of a **model observer**?

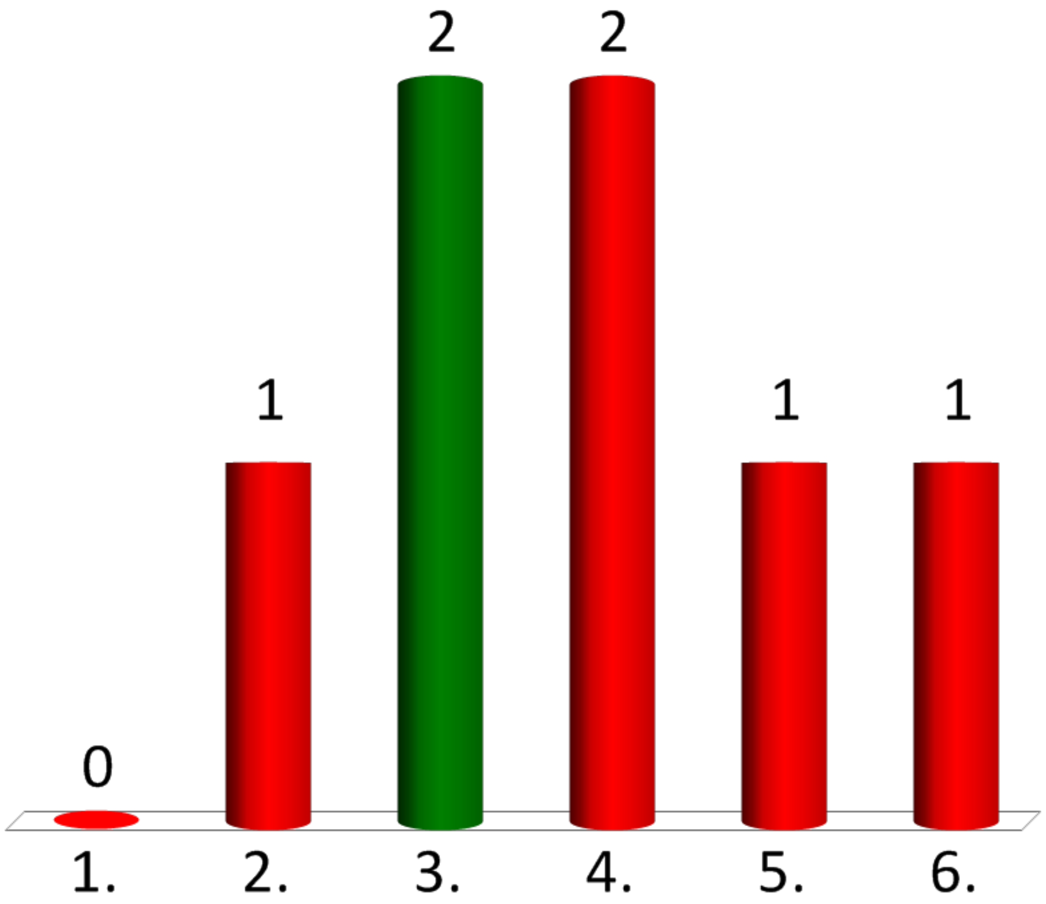
1. scalar
2. vector
3. matrix



7

What is the visible angle of the sun

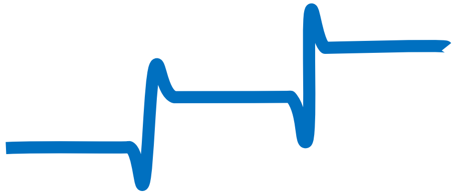
1. 0.05°
2. 0.1°
3. 0.5°
4. 1°
5. 5°
6. 10°

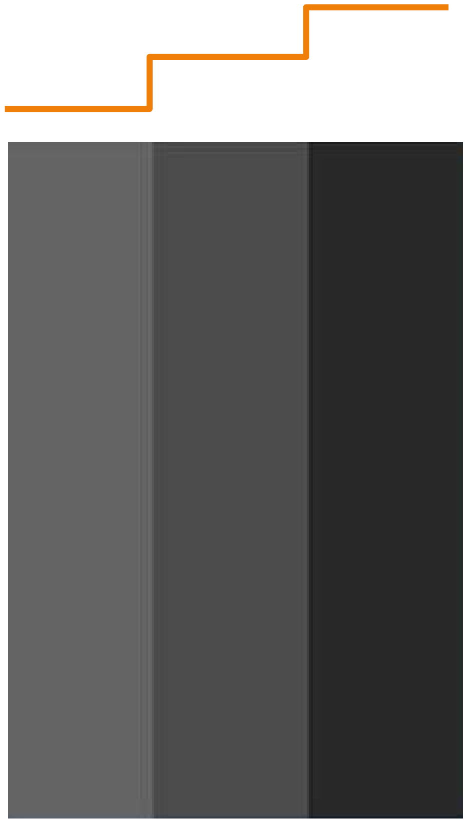


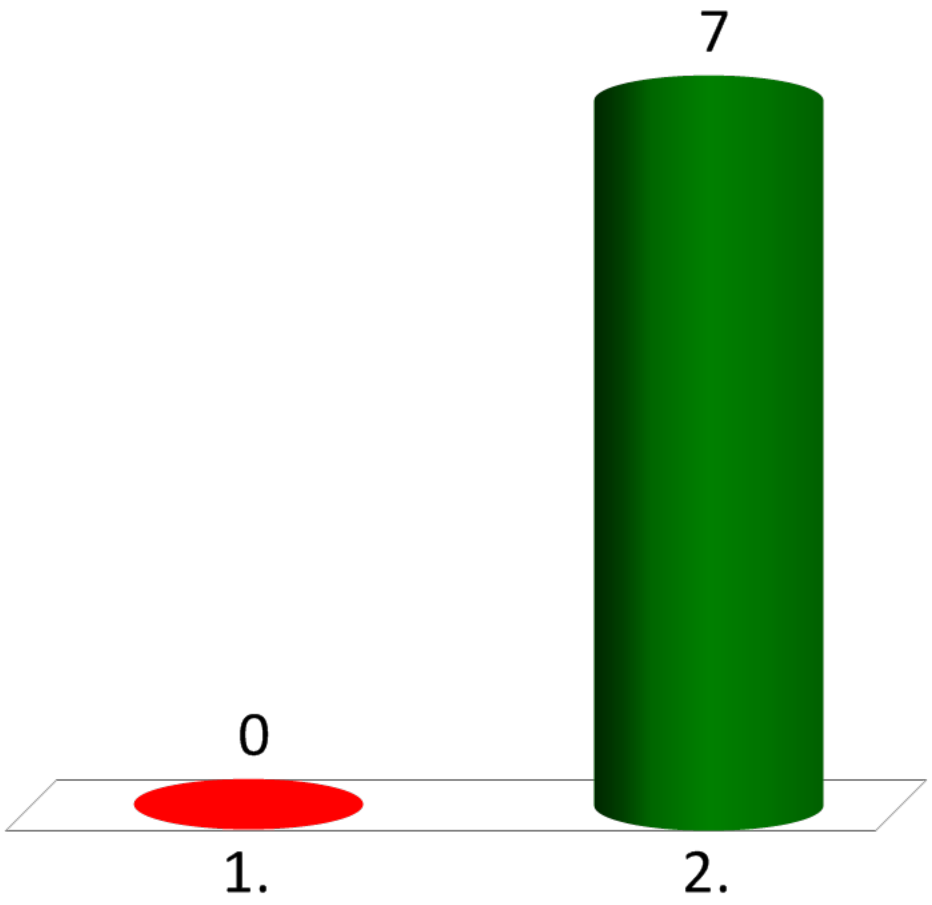
7

What is our typical perception of gray-scale ladder?

1. 

2. 





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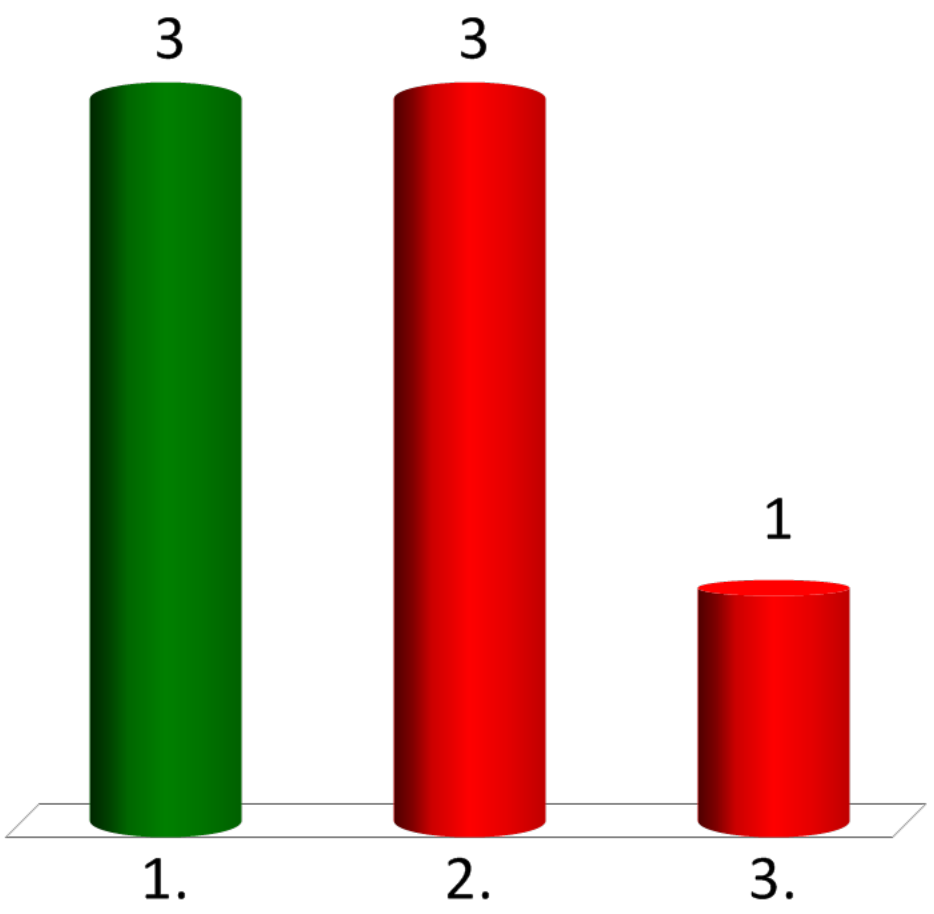
Which mathematical expression is related to the **matched-filter** observer?

1. $\mathbf{s}^{T}\mathbf{g}$
2. $(\mathbf{Es})^{T}\mathbf{Eg}$
3. $(\mathbf{K}^{-1}\mathbf{s})^{T}\mathbf{g}$



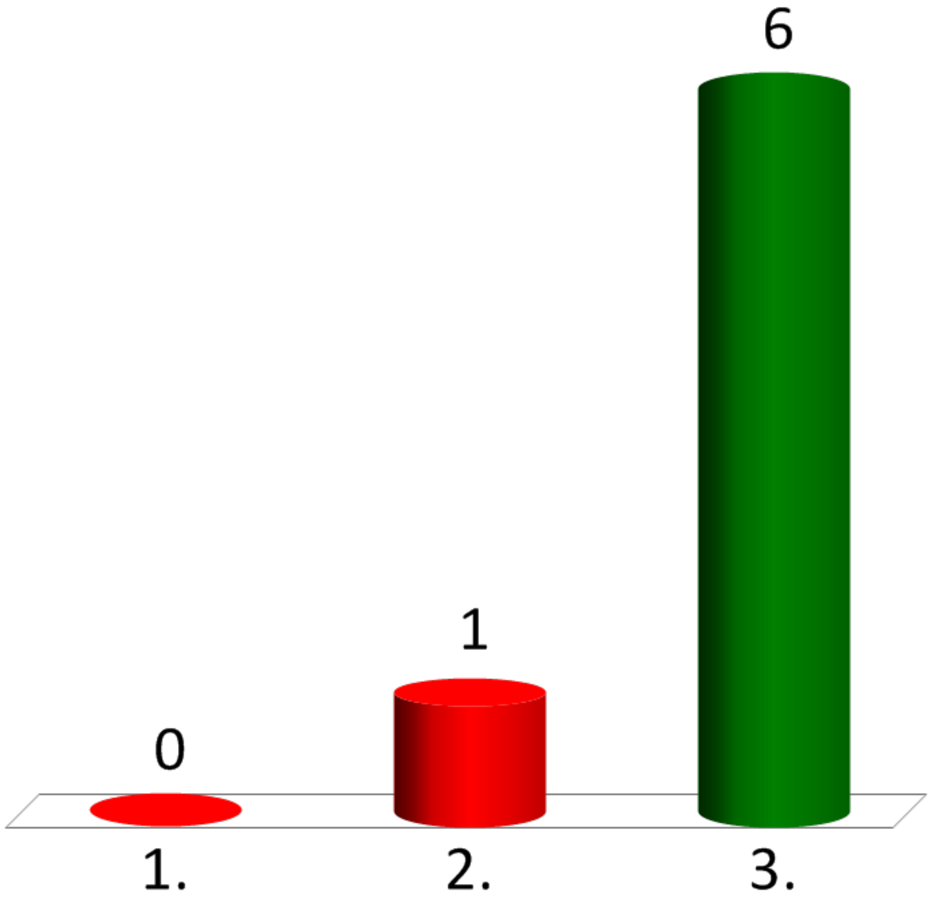
7

Which mathematical expression is related to the **prewhitening matched-filter** observer?

1. $\mathbf{s}^T \mathbf{g}$

2. $(\mathbf{Es})^T \mathbf{Eg}$

3. $\left(\mathbf{K}^{-\frac{1}{2}}\mathbf{s}\right)^T \mathbf{K}^{-\frac{1}{2}}\mathbf{g}$



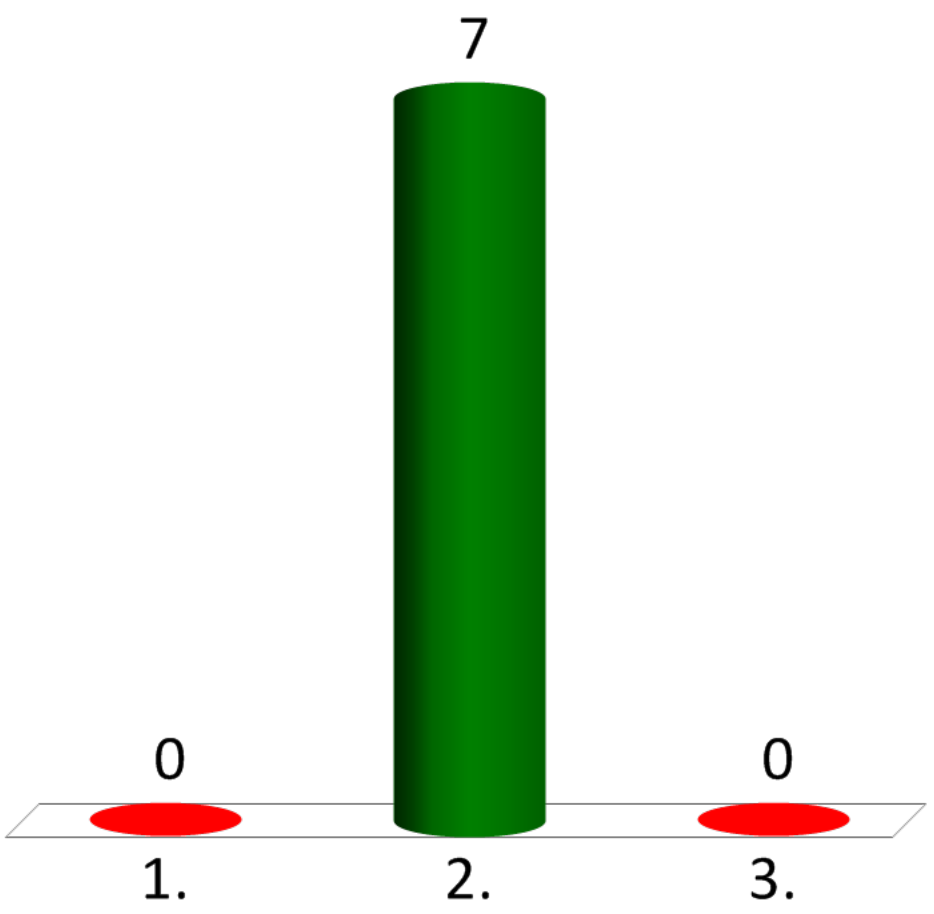
7

Which mathematical expression is related to the **matched-filter with eye-response** observer?

1. $\mathbf{s}^{T}\mathbf{g}$

2. $\left(\mathbf{E}\mathbf{s}\right)^{T}\mathbf{E}\mathbf{g}$

3. $\left(\mathbf{K}^{-\frac{1}{2}}\mathbf{s}\right)^{T}\mathbf{K}^{-\frac{1}{2}}\mathbf{g}$

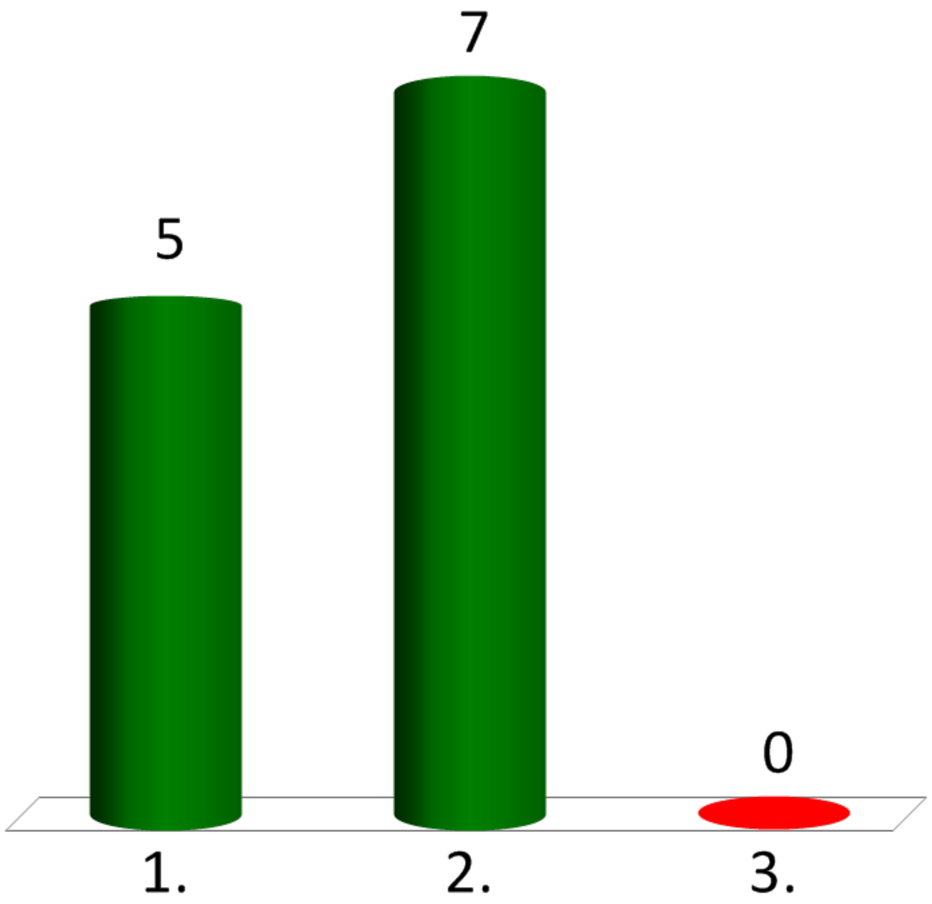


7

What can we say about the **ideal observer**?

(multiple responses possible)

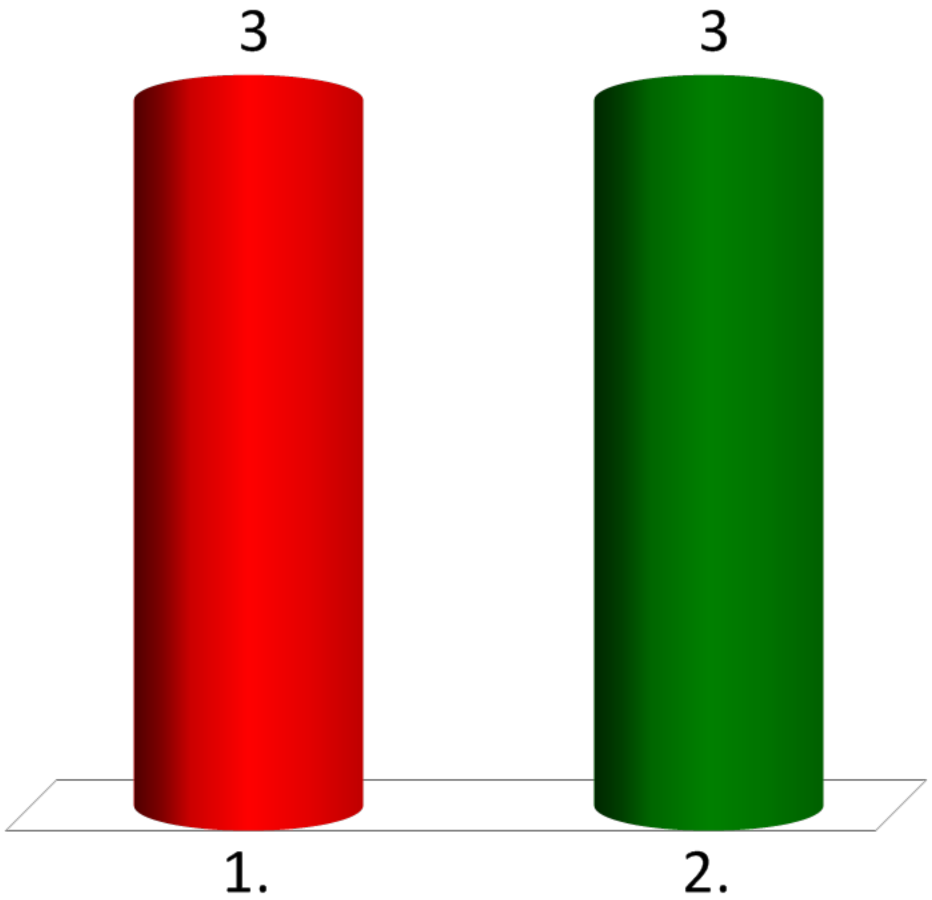
1. it uses all the available statistical information
2. it computes the likelihood ratio of the observed image **g**
3. it performs always correctly



7

Which type of **model observer** is useful to characterize the **image software**?

1. ideal observer
2. anthropomorphic observer



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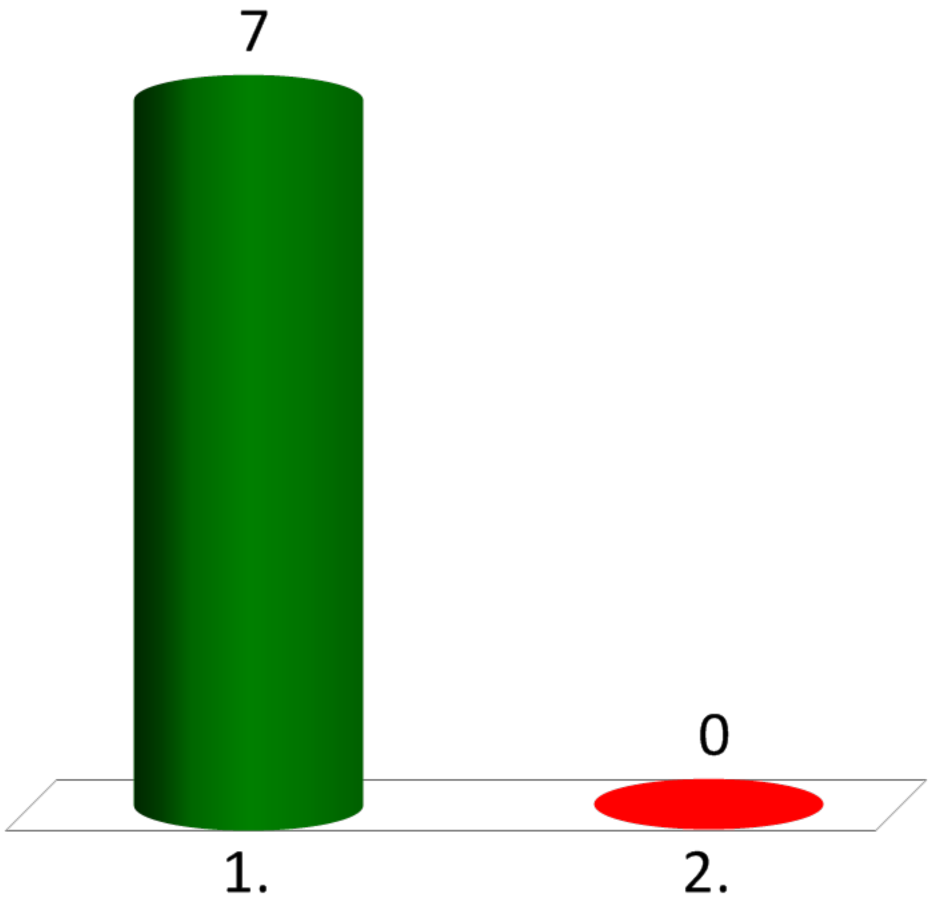
Which type of **model observer** is useful to characterize the information available in the **raw image data**?

1. ideal observer
2. anthropomorphic observer



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