Last Name .... First Name..................

Biological Modelling of Neural Networks Exam
23 June 2015

- Confirm that your exam copy has 8 pages total.
- Write your name in legible letters on top of the first page.
- The exam lasts 160 min.
- Write **all** your answers in a legible way on the exam (no extra sheets).
- No documentation is allowed apart from 1 sheet A5 of **handwritten** notes.
- No calculator is allowed.
- Have your student card displayed before you on your desk.

Evaluation:

1. .... / 9 pts
2. .... / 22 pts
3. .... / 11 pts

Total: .... / 42 pts

1 Synaptic Plasticity (9pts)

Throughout this section, we consider a synaptic plasticity rule of the form

$$\frac{dw_{ij}}{dt} = -a_0\nu_j^{\mathrm{pre}} + a_2\nu_j^{\mathrm{pre}}\nu_i^{\mathrm{post}} - a_4(\nu_i^{\mathrm{post}})^4$$

where i denotes the postsynaptic neuron and w_{ij} the synaptic weight from neuron j to neuron i, and a_0, a_2, a_4 are **positive constants**.

(a) Is this a Hebbian rule? Justify your answer

yes, *no* ... because ....

....

number of points: /1

(b) Is this a local rule? Justify your answer

yes, *no* ... because ....

....

number of points: /1

(c) Does this learning rule allow for both potentiation and depression? [Hint: assume $0 < a_4 \ll a_2$ and consider presynaptic neuron ON/OFF; postsynaptic neuron ON/OFF where ON is suitably defined]

yes, *no* ... because ....

....

number of points: /1

(d) Suppose we have a network of 4 neurons. Each neuron is described by a firing rate $\nu = g(h) > 0$. The network is in a homogeneous state, so that all neurons always have the same firing rate: $\nu_1 = \nu_2 = \nu_3 = \nu_4 = \nu$. The initial state of all weights is $w = 0.0001$.

Write down the differential equation for the evolution of the weight from neuron 2 to neuron 4.

....

....

number of points: /1

Assume $a_0 = a_2$ and $0 < a_4 \ll a_2$. Will the weight grow, or decay? Always? Aproach a fixed point or not? [Hint: Consider different initial states of the weight and of the rates!]

Please justify your answers, by mathematical calculation or suitable graphics.

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....

number of points: /3

(e) Would your answers change if $a_0 \ll a_2$?

Yes/No because ....

....

Would your answers change if $a_0 = 0$?

Yes/No because ....

....

number of points: /2

(space for calculations)

2 Synaptic Plasticity in Networks (22pts)

We now combine synaptic plasticity with neuronal dynamics. We consider four rate neurons with gain function

$$\nu = g(h) \quad = \quad \left[1 - (1-h)^2\right] \nu_0 \quad \text{for} \quad 0.1 < h < 1$$

and $g(h) = 0.19\nu_0$ for $h \leq 0.1$ and $g(h) = \nu_0$ for $h \geq 1$;

and dynamics (for neuron i)

$$\tau \frac{dh_i}{dt} \quad = \quad -h_i + \sum_{j \neq i} w_{ij} g(h_j)$$

Each neuron receives input from the three other neurons in the network via weights w_{ij} that follow a dynamics

$$\tau_w \frac{dw_{ij}}{dt} \quad = \quad -\tilde{a}_0 \nu_j^{\text{pre}} + \nu_j^{\text{pre}} \nu_i^{\text{post}} - \tilde{a}_4 (\nu_i^{\text{post}})^4$$

(a) How many **differential equations** are necessary to describe the dynamics in our plastic network of 4 neurons?

....

number of points: /1

(b) Assume that all weights are identical (but time dependent), and that all neuronal firing rates are identical (but time dependent). Assume unit-free variables and **set** $\tilde{a}_0 = 0.00$ **and** $\tilde{a}_4 = 1/4$ **and** $\nu_0 = 8/3$. Write down the two resulting equations (you may keep $g(h)$ without inserting the expression).

$$\tau \frac{dh}{dt} \quad =$$

$$\tau_w \frac{dw}{dt} \quad =$$

number of points: /2

(c) Calculate the nullclines [Hint: you may keep $g(h)$ without inserting the expression and note that $g(h) > 0$ for all h]

....

....

number of points: /3

Space for calculations

(d1) Evaluate the h-nullcline for

$h = 0.0 \longrightarrow w =$..

$h = 0.1 \longrightarrow w =$..

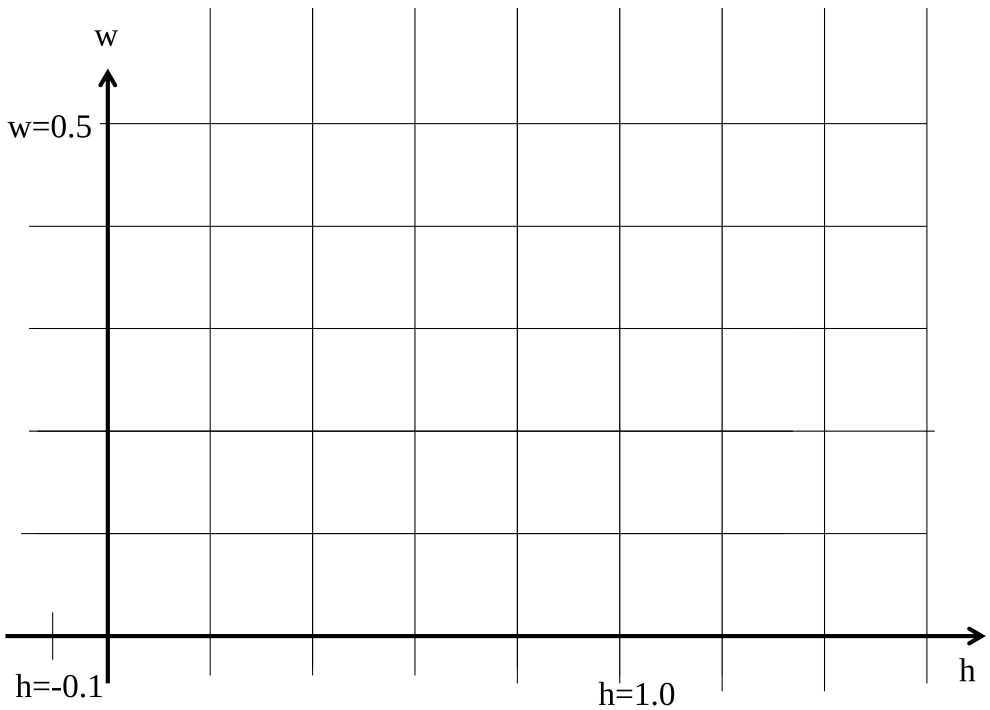
$h = 0.5 \longrightarrow w =$..

$h = 1.0 \longrightarrow w =$..

number of points: /2

[Hints: insert the definitions of g in the regimes $h \leq 0.1$, $0.1 < h < 1$ and $h \geq 1$; note that for $h = 0.5$ we have $g(h) = 2$, and for $h = 0.1$ we have $g(h) = 0.19 \cdot 8/3$ as you can easily verify by inserting the value for ν_0].

(d2) sketch the nullclines in the range $0 < w < 0.4$ and $-0.1 < h < 1$ in the space here:
number of points: /4



(e) in the graph above, draw arrows on all relevant parts of the nullclines
number of points: /2

(f) in the above graph indicate **qualitatively** the direction of arrows in all relevant regions of the phase plan. Consider in particular the points (h=1, w=0.2); (h=0, w=0.2); (h=1, w=0); (h=0.2, w=0)

(You may assume $\tau_w = 3\tau$.)
number of points: /2

(g) in the above graph sketch the evolution of a network where the four neurons start with an input potential of $h = 1.0$ and weights $w_{ij} = 0.2$. You may assume that $\tau_w \gg \tau$. number of points: /2

(h) in the above graph sketch the evolution of a network where the four neurons start with an input potential of $h = 0.2$ and weights w_{ij} =0.01. You may assume that $\tau_w \gg \tau$. number of points: /2

(i) Is the network useful as a memory unit? If yes, why? Would your answer change if we set $a_0 > 0$? Would your answer change if we set $a_0 < 0$? **Justify your answers**.

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number of points: /2

3 Stochastic Neuron Model (11pts)

Throughout this exercise, we consider stochastically spiking neurons with a stochastic intensity (or 'hazard')

$$\rho(t) \;=\; g(h) = [1-(1-h)^2]\,\nu_0 \quad \text{for} \quad 0 < h < 1$$

and $g(h) = 0$ for $h \leq 0$ and $g(h) = \nu_0$ for $h \geq 1$. After each spike, the neuron has an **absolute refractory time** of Δ during which it does not fire (i.e., the harzard vanishes).

(a) Calculate the mean firing rate ν for $h = 0.5$ and $h = 2$.

$h = 0.5 \longrightarrow \nu =$ ....
....

$h = 2.0 \longrightarrow \nu =$ ....
number of points: /2

(b)What is the interval distribution $P(s)$ for the case $h = 2.0$.

$h = 2.0 \longrightarrow P(s)$ ....
....
number of points: /2

(c) we now have $N = 200$ neurons. The input to all neurons switches every 500ms between a value of h_1 and a value of h_2, starting at $t = 0$ with $h = h_1$.

Suppose you have a measurement device that allows you to record the spikes of each of the 200 neurons.

What is the expected number of spikes $\bar{n}_{\text{spikes}}$ after 5 seconds of measurements ? (keep the values h_1 and h_2 as well as Δ arbitrary)

$\bar{n}_{\text{spikes}} =$..
....
number of points: /2

(d) Suppose that, during the above experiment with $N = 200$ neurons, the stimulus is switched on at $t = 0$ and is constant thereafter with a value $h_1 = h_2 = 0.5$. Suppose $\nu_0 = 200$Hz and $\Delta = 10$ms. What is the expected population activity $A(t)$ at $t = 4$s?

$A(t)$ =..

....

number of points: /2

(e) Suppose that neurons have been silent for $t < 0$ (e.g., $h = -1$ for $t < 0$) and the above input $h = 0.5$ is switched on at $t = 0$.

e1 What is the expected population activity A at $t = 1$ms? (justify your answer).

$A(t) \approx$ ... .
because ....

....

e2 Is it different from the value calculated in (d)? (justify your answer)

Different/not different ....
because ....

....

e3 How many neurons do you expect to fire between $t = 0$ and $t = 10$ms? A rough estimate is sufficient: nearly all neurons, about half but rather more, about half but rather less, significantly less than half etc. [Hint: how many neurons survive without firing?]

Out of the $N = 200$ neurons, I expect that .... fire because

....

....

number of points: /3