RBPA 2018 WEEK 10

Solution of the Problem 1:

$$^{62}_{29}Cu \rightarrow {}^{62}_{28}Ni + {}^{0}_{1}\beta^{+} + {}^{0}_{0}\nu_e$$

$$\lambda = \frac{\ln 2}{T_{1/2}} \approx \frac{0.693}{9.76 \times 60} = \underline{\underline{1.2 \times 10^{-3}\ s^{-1}}} \quad \therefore$$

Solution of the Problem 2:

$^{238}_{92}U$: 8 × α-decays and 6 × β^--decays

$$Z = 92 - 8 \times 2 + 6 = 82$$

$$A = 238 - 8 \times 4 = 206$$

The final product of this decay series is $^{206}_{82}Pb$ $\therefore$

Solution of the Problem 3:

Momentum Conservation :

$\vec{p} = m\vec{v}$ $\boxed{\vec{p_i} = \vec{p_f}}$ gives

$$\Rightarrow (4\,amu) \times \vec{v_\alpha} = ((210-4)\,amu) \times \vec{v_D}$$ — Daughter

because the mother nucleus is @ rest ($\vec{p_i} = 0$).

We can replace $\vec{v_D} = \frac{4\vec{v_\alpha}}{(210-4)}$ in the equation representing

Energy Conservation:

$$\Rightarrow \boxed{Q = (\Sigma M_i - \Sigma M_f)c^2 = \Sigma E_f - \Sigma E_i}$$

with $\Sigma E_i = 0$

and $\vec{E_{kin}} = \frac{1}{2} m \vec{v}^2$

$$Q = \frac{1}{2}(4\,amu)\vec{v_\alpha}^2 + \frac{1}{2}((210-4)\,amu)\,\vec{v_D}^2$$

$$Q = \frac{1}{2}(4\,amu)\vec{v_\alpha}^2 + \frac{1}{2}((210-4)\,amu)\left(\frac{4\vec{v_\alpha}}{(210-4)}\right)^2$$

$$Q = \frac{1}{2}(4\,amu)\vec{v_\alpha}^2 \cdot \left(1 + \frac{4}{210-4}\right)$$

$$Q = \underbrace{\frac{1}{2}(4\,amu)\vec{v_\alpha}^2}_{\vec{E_\alpha}} \cdot \underbrace{\left(\frac{210}{210-4}\right)}_{\frac{A}{A-4}}$$

$$\boxed{Q = \vec{E_\alpha} \times \frac{A}{A-4}} \Rightarrow \boxed{\vec{E_\alpha} = \frac{A-4}{A}\,Q}$$

Inserting $A = 210$, $Q = 5.50\ MeV$ $\Big\} \Rightarrow \boxed{E_\alpha \simeq 5.4\ MeV}$ ∴

Solution of the Problem #4:

(a) $^{12}_{6}C + ^{2}_{1}H \rightarrow ^{13}_{7}N + ^{1}_{0}n$

(b) $^{13}_{7}N \rightarrow ^{13}_{6}C + ^{0}_{1}\beta^{+} + ^{0}_{0}\nu_e$

Solution of the Problem #5:

No. According to the decay law, the number of the radioactive nuclei in the sample is decreasing exponentially. Thus, there are $\frac{N_0}{2}$ nuclei in the sample in time $T_{1/2}$ and $\frac{N_0}{4}$ nuclei in time $2 \times T_{1/2}$. The number of nuclei, which decay in time $2 \times T_{1/2}$ is $\boxed{\frac{3N_0}{4}}$.:

Solution of the Problem #6:

$\boxed{^{238}_{92}U}$: 5 × α-decays and 2 × β⁻-decays

$$Z = 92 - 5 \times 2 + 2 = 84$$

$$A = 238 - 5 \times 4 = 218$$

RaA element is $\boxed{^{218}_{84}Po}$.:

Solution of the Problem #7

@ $t=0$ there are N_0 radioactive nuclei

@ $t>0$ there are $N(t) = N_0 e^{-\lambda t}$ rad. nuclei

The number of nuclei, which decay:

$$\Delta N = N_0 - N(t) = N_0 (1 - e^{-\lambda t})$$

$\lambda = \frac{\ln 2}{T_{1/2}}$ $\quad t = 1s,\ 1 min,\ 3 min,\ 6 min$

$\Rightarrow \quad \therefore \quad \Delta N = 3.84 \times 10^9,\ 2.1 \times 10^{11},\ 5.0 \times 10^{11},\ 0.75 \times 10^{12}$ nuclei

Solution of the Problem #8:

as in the previous problem:

$$\Delta N (3 T_{1/2}) = N_0 \left(1 - e^{-\frac{\ln 2}{T_{1/2}} 3 T_{1/2}}\right) = \frac{7}{8} N_0 \therefore$$

The sample emitted $7/8\, N_0$ particles in time $3 \times T_{1/2}$.

Solution of the Problem #9:

The number of neutrons, which will decay in time t is given as $\Delta N(t) = N_0 - N(t) = N_0(1-e^{-\lambda t})$, where N_0 is the number of neutrons in time $t=0$ and $N(t)$ is the number of neutrons in time $t>0$, which have passed the length $l = v \times t$

The speed of neutrons can be determined from their kinetic energy $\Rightarrow$ $v = \sqrt{\frac{2E_{kin}}{m_n}}$

Relative number of neutrons, which will decay in length l is equal:

$$\eta = \frac{\Delta N}{N_0} = \frac{N_0(1-e^{-\lambda t})}{N_0} = 1 - e^{-\lambda t} = 1 - e^{-\frac{\ln 2}{T_{1/2}} l \sqrt{\frac{m}{2E}}}$$

The exponent in the expression above has a value:

$$\frac{\ln 2}{T_{1/2}} l \sqrt{\frac{m_n}{2E_{kin.}}} = \frac{0.693}{1037 \times 60} \times 1 \times \sqrt{\frac{1.67 \times 10^{-27}}{2 \times 0.025 \times 1.6 \times 10^{-19}}} =$$

$$\doteq 5.1 \times 10^{-7} \ll 1 \Rightarrow \boxed{1 - e^{-x} \approx x}$$

Thus, $\boxed{\eta = 5.1 \times 10^{-7}}$ ∴ In length of one meter there will be $5.1 \times 10^{-7} N_0$ neutrons, which will decay

Solution of the Problem 10:

The initial volume activity of ^{24}Na after the injection into the blood of a man is

$$a_0 = \frac{A_0}{V}$$, where V is the volume of the blood.

Activity is decreasing exponentially with the time. Thus,

$$a_v(t) = \frac{A_0}{V} e^{-\lambda t}$$

$$\therefore \quad V = \frac{A_0}{a_v} e^{-\lambda t} = \frac{2 \times 10^3}{265 \times 10^3} e^{-\frac{\ln 2}{15} \cdot 5} = \underline{\underline{6\ l}}$$

The volume of the blood of a man was 6 l.

Solution of the Problem 11:

The highest X-ray energy is $\boxed{E_{max} = 195\ keV}$

with $E = h \cdot \nu$ and $c = \nu \cdot \lambda$ one obtains:

$$\lambda_{min} = \frac{hc}{E_{max}} \simeq \underline{\underline{6.36 \times 10^{-12}\ m}} \quad \therefore$$

Note: $h = 6.626\,069\,57(29) \times 10^{-34}\ J \cdot s$
$= 4.135\,667\,516(91) \times 10^{-15}\ eV \cdot s$
$= 6.626\,069\,57(29) \times 10^{-27}\ erg \cdot s$

$c = 299\,792\,485\ m \cdot s^{-1}$

Solution of the

Problem 12: The specific activity S is given by:

$$S = \frac{1}{A} \times \frac{1}{T_{1/2}} \times \ln 2 \times N_A \quad / 1g$$

$$S = \frac{1}{A} \times \frac{1}{T_{1/2}} \times 4.175 \times 10^{23} \quad / 1g$$

⇒ with $A \approx 3 \frac{g}{mol}$ and $T_{1/2} = 12.26$ years $\approx 3.87 \times 10^8$ s

the result: $S \approx 3.6 \times 10^{14}$ Bq/g ∴ can be obtained.

Solution of the

Problem 13: As energy = charge × voltage

$$E = Q \times U$$

we get with $Q = +2e$ and $U = 3MV$

the result:

$$E = 2e \times 3MV = 6MeV$$ ∴

NB: α-particle (electric charge 2e)

$\alpha, \alpha^{2+}, He^{2+}, {}^4_2He^{2+}, {}^4_2He$ → as it is possible that the ion gains e's from the environment. Also, e's are not important in nuclear chemistry

The nomenclature is not well defined, and therefore, not all high-velocity helium nuclei are considered by all as α-particles. Some use „doubly ionized helium nuclei" (He^{2+})