Models and Methods for Random Networks: Exercise Set 3

Exercise 1 Let $\mathbb{T}$ be a random tree, where the number of children of vertices are i.i.d. random variables. Namely, each vertex at layer n has a random number D of children at layer $(n+1)$, independently of the number of children of other vertices. Suppose that D can take 2 values $\{0, 2\}$, with $\mathbb{P}(D = 0) = 1 - q$ and $\mathbb{P}(D = 2) = q$ for some $0 < q < 1$. We define an i.i.d. bond percolation process, with open edge probability p on this tree.

1. Compute the percolation probability $\theta(p)$.

2. Compute the percolation threshold p_c.

Exercise 2 Let $\mathbb{T}^2$ be a binary tree. We define the following i.i.d. site percolation process: a vertex is declared open with probability p, and closed with probability $(1-p)$, independently for each vertex. When a vertex at layer n is open, the two edges that connect it to its two children at layer $(n+1)$ are both open. When a vertex at layer n is closed, the two edges that connect it to its two children at layer $(n+1)$ are both closed.

1. Compute the percolation probability $\theta(p) = \mathbb{P}_p(|C| = \infty)$.

2. Compute the percolation threshold p_c.

3. Compute the mean cluster size $\chi(p)$.

4. Compare the three results obtained above with the values of i.i.d. bond percolation on the binary tree seen in class. Which ones are identical ? How can some quantities be identical, whereas others are different ? Explain briefly.