ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Models and Methods for Random Networks

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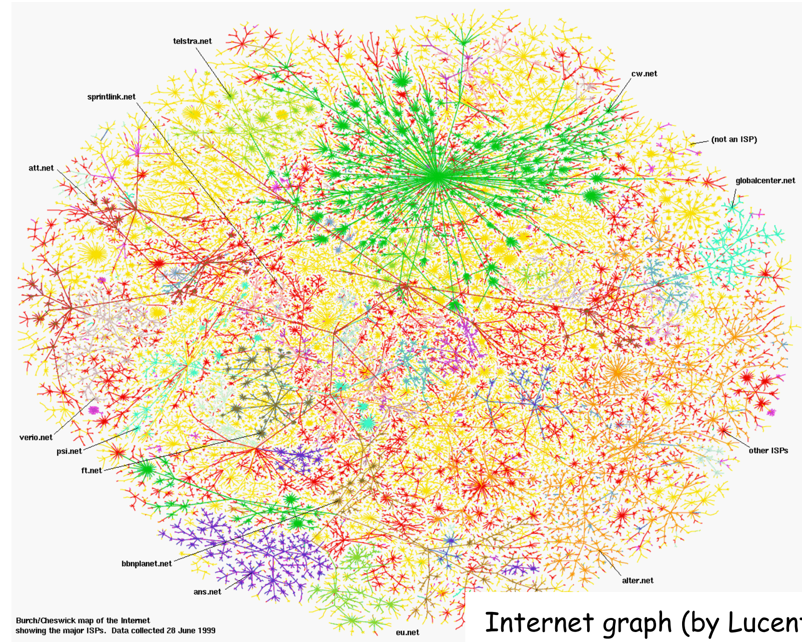
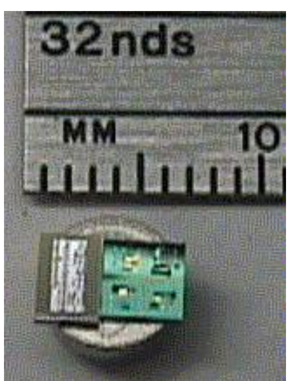
(Matthias Grossglauser)

Communication and Social Networks

- Size explosion
- Small, simple, low-cost components
- Lack of control: self-organize!
- Adversarial conditions, prone to failures, dynamically changing.
- Data representation
- Information processing and propagation

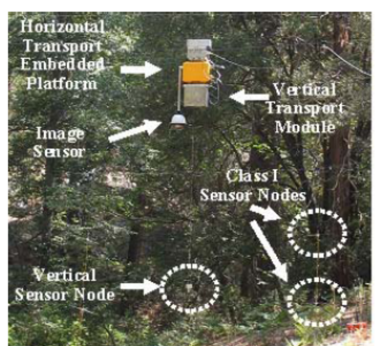


Internet graph (by Lucent)



SmartDust (Berkeley)

Network Info-mechanical System (Cens : UCLA)




ExScal (Ohio State Univ)

New Models and Tools Needed

- Percolation: random graphs embedded in geometric space
  - Bond percolation (square lattice embedding)
  - Site percolation (square lattice embedding)
  - Boolean Poisson model (continuum percolation)
- Random graphs: not constrained by an underlying geometric space
  - Erdös-Rényi model (i.i.d edges)
  - Random regular graph (constant node degree)
  - Small world graph (small diameter, high clustering)
  - Scale free model (power law of node degree)
- Game-Theoretic graphs: considers node preferences and decisions
  - Homophily and Affiliation networks (similar nodes form connections)
  - Network formation games (nodes build edges to satisfy global or local connectivity goals)

New Models and Tools Needed

- How does Information Propagates on Networks?
  - Gossip and voting algorithms.
  - Epidemics: SI, SIR, SIS models.
  - Information Cascades.
- How to find Information on a Network?
  - Network Navigation
  - Decentralized Search.

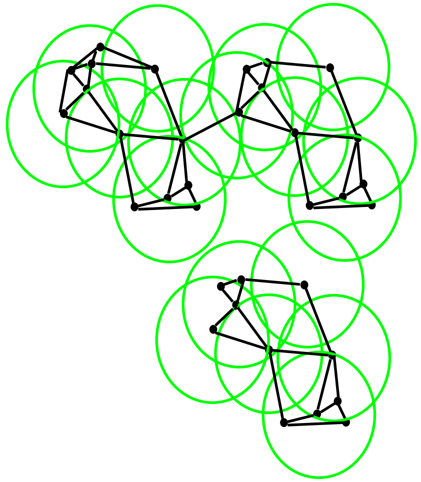
Program

- Models and methods: Random graphs (8h)
  - Erdös-Rényi model (i.i.d edges)
  - Random regular graph
  - Small World Networks, Scale Free Networks
- Models and methods: percolation and game theory (8h)
  - Bond percolation on trees and lattices.
  - Continuum percolation.
  - Full connectivity vs percolation.
  - Network formation games
  - Affiliation Networks and homophily.
- Network and Dynamics Applications to wireless, social (and biological) networks (10h)
  - Connectivity and capacity of wireless multi-hop networks,
  - Navigation, network search discovery
  - Information cascades and epidemics on graphs

About Scaling Laws

- Performance metrics for large random networks:
  - Connectivity
  - Delay
  - Robustness
  - Routing
  - Coverage (wireless multi-hop networks)
  - Troughput
  - Delay
  - Lifetime...
- How do these metrics scale when number of nodes becomes large ?
- In this course, we will see some tools and methods to answer this question.
- An example: the existence of a phase transition

Example 1: Wireless Networks

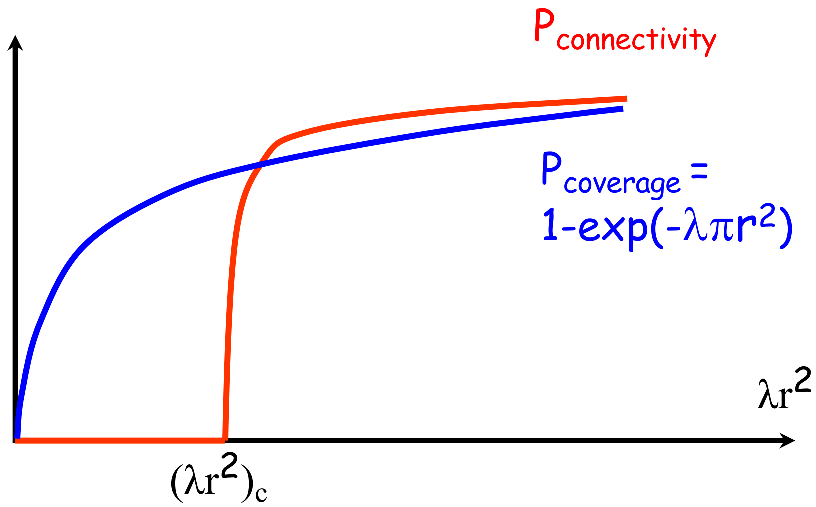


- A simple model: Poisson Boolean model: nodes follow a random spatial distribution with density
- Boolean model: fixed radio range r. Nodes i and j, at positions x_i and x_j, are directly connected iff

$$\left\| x_i - x_j \right\| < r$$

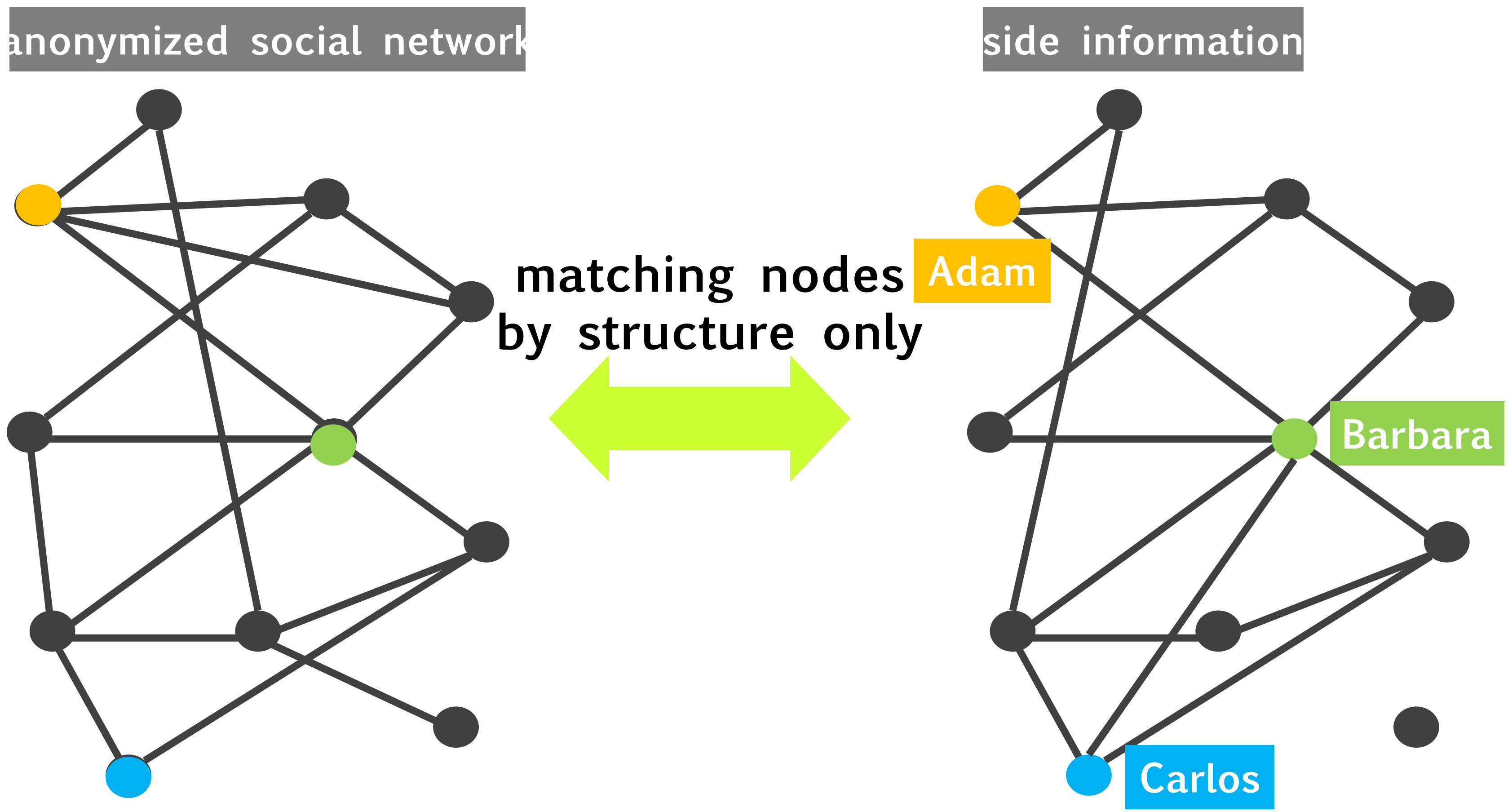
Coverage vs Connectivity

- Ad hoc network : connectivity
- Sensor network : connectivity (probability that an arbitrary node is connected to the base station) and coverage (probability that an arbitrary point is covered by a node).
- Phase transition for connectivity, not for coverage.



Example 2: Privacy of Networks

- Adversary has:
  - Anonymized network: unlabeled graph
  - Side information: labeled graph – similar but not identical

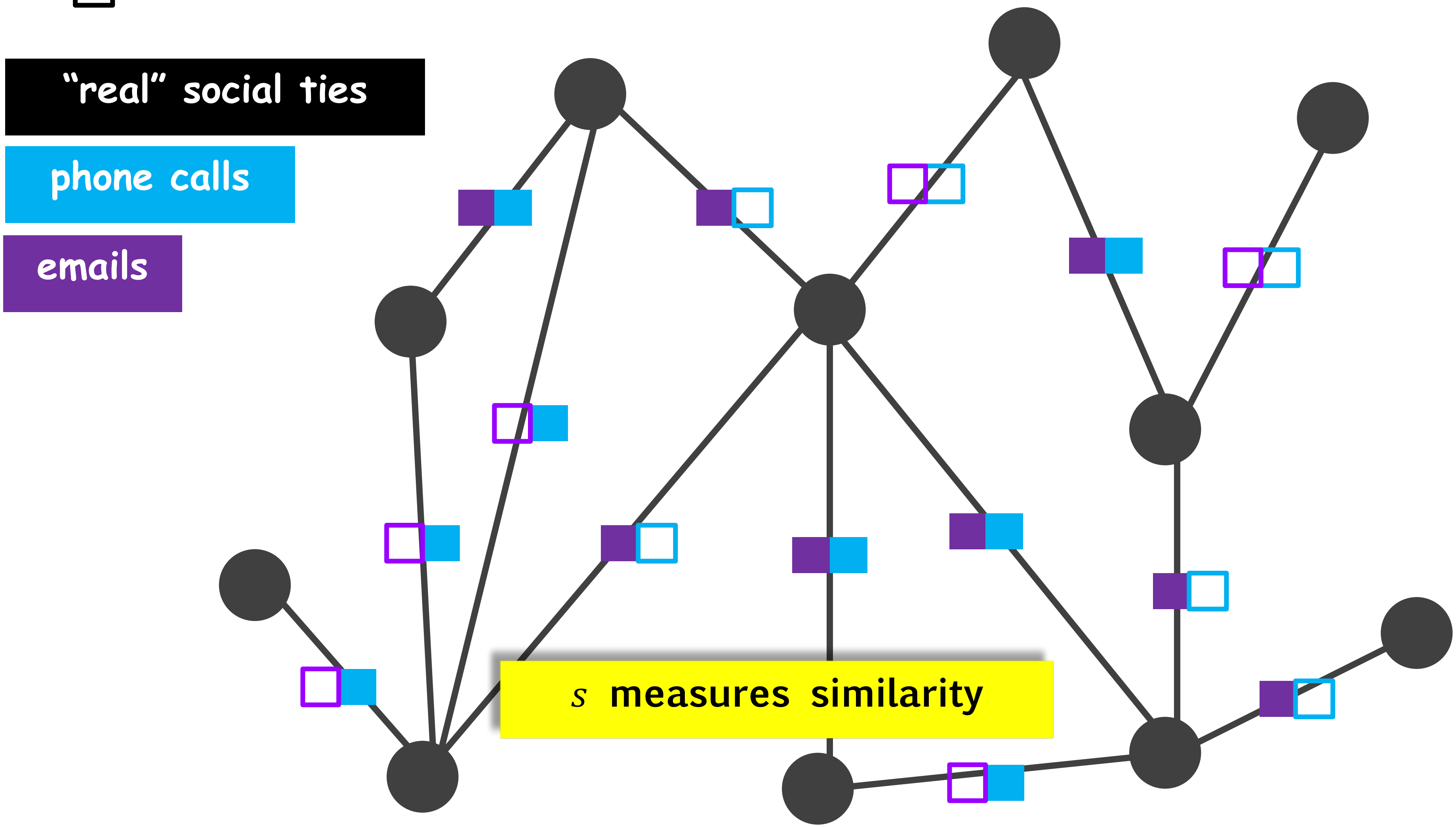


$G(n, p; s)$ Sampling Model

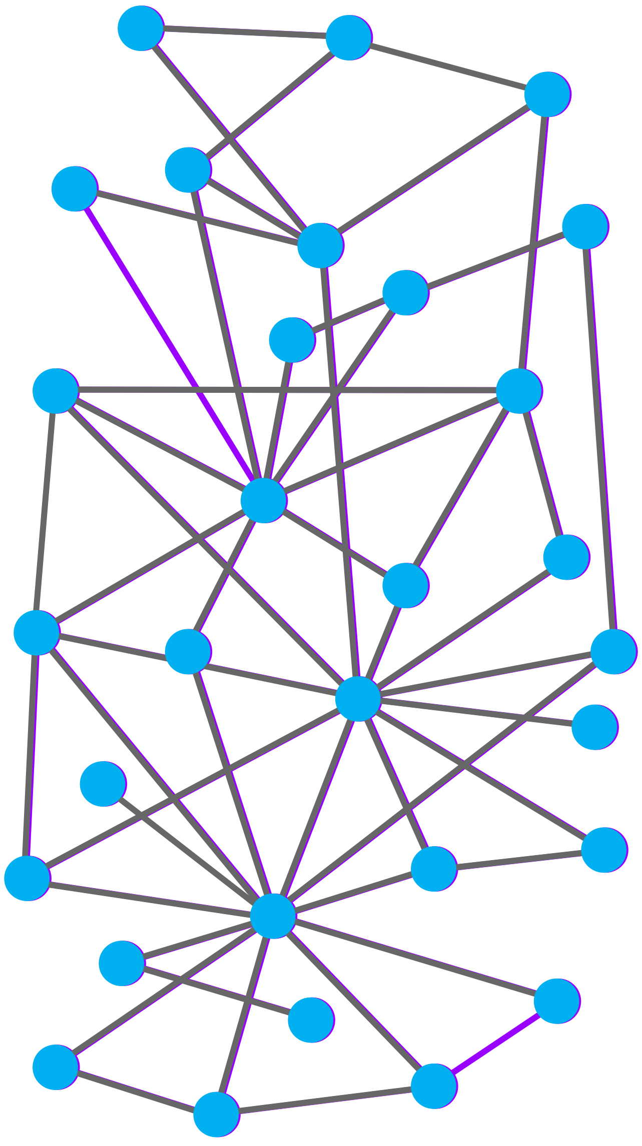
Generator $G = G(n, p)$

■ sampled (s)

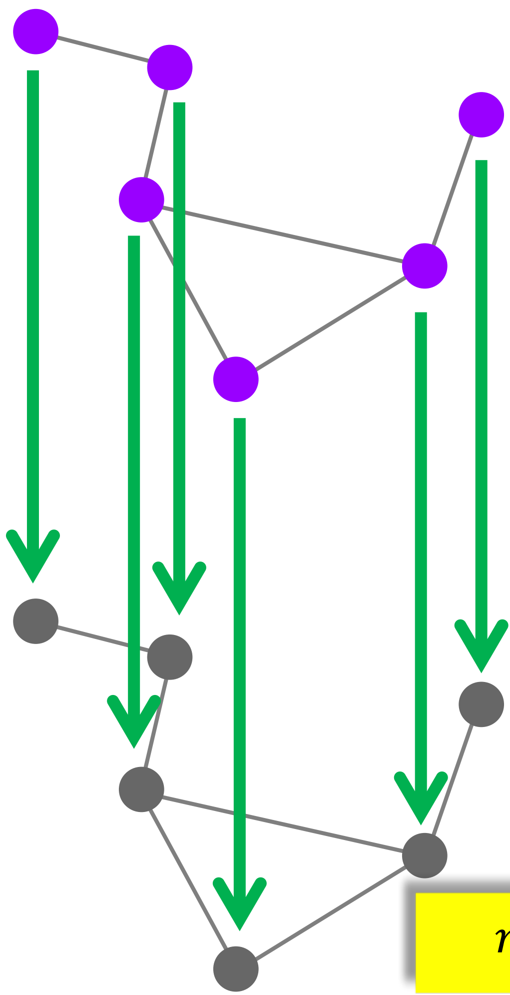
□ not sampled $(1 - s)$

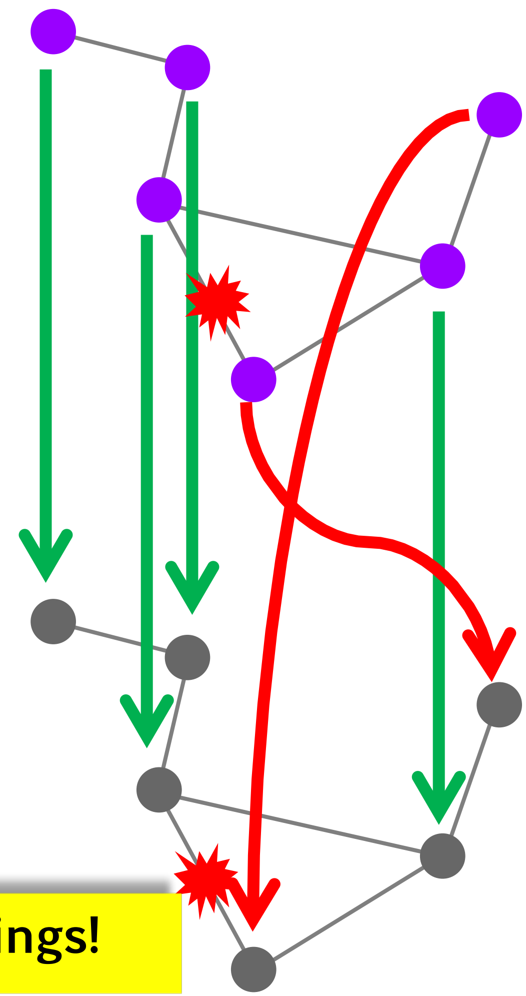


$G(n, p; s)$: Two Correlated $G(n, ps)$'s



Mappings and Edge Mismatch





$n!$ **possible mappings!**

$\Delta(\pi_0) = 0$

$\Delta(\pi) = 2$

Approach

- Assumption:
  - Attacker has infinite computational power
  - Can try all possible mappings π and compute edge mismatch function $\Delta(\pi)$
- Question:
  - Are there conditions on p, s such that

$$P\{\pi_0 \text{ unique min of } \Delta(\pi)\} \to 1$$

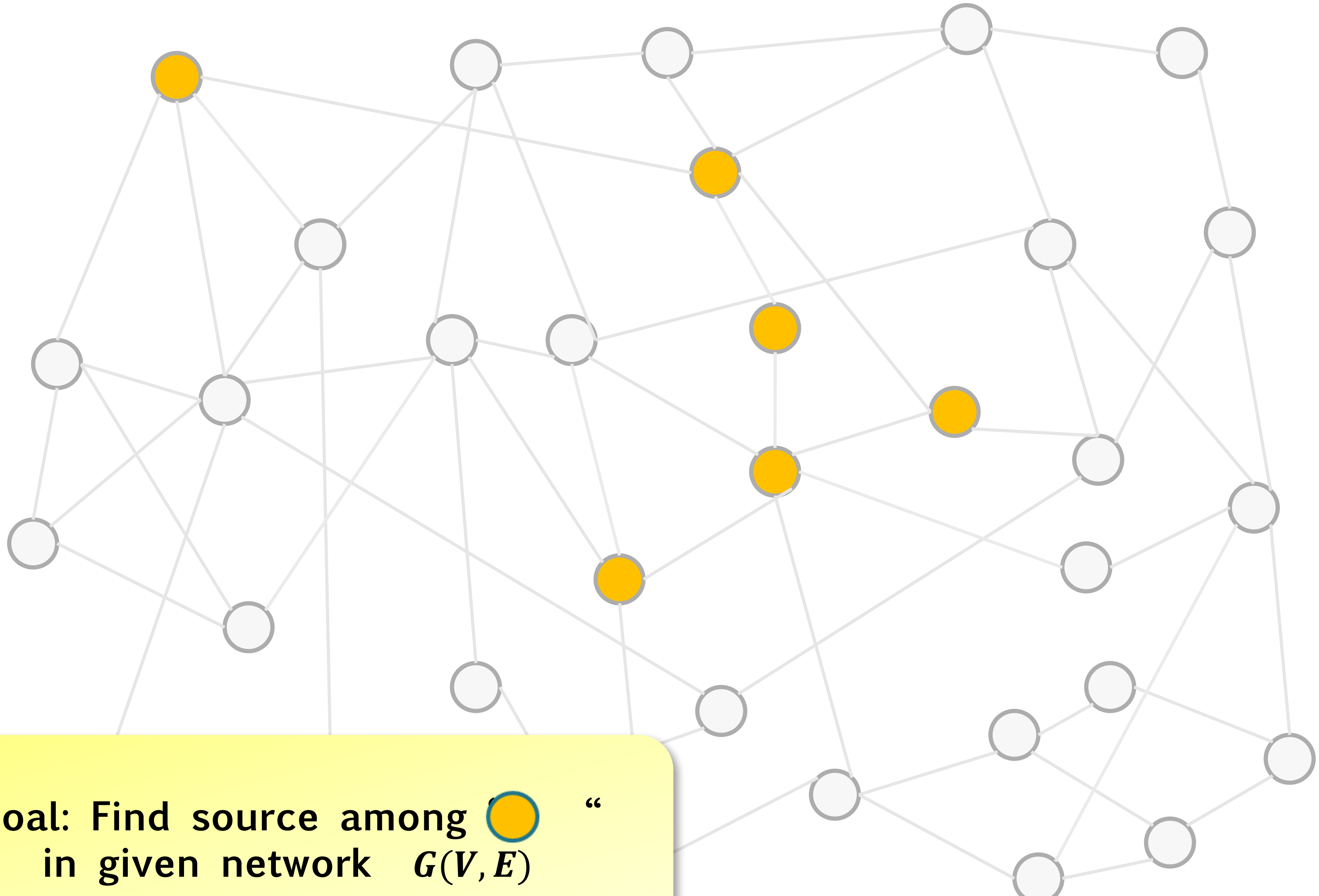
  - If yes: adversary would be able to match vertex sets only through the structure of the two networks!
- Note:
  - $G(n, p; s)$ model: statistically uniform, low clustering, degree distribution not skewed -> conjecture: harder than real networks

Example 3: Rumor Models

- Application: detecting source of an epidemic
  - Who leaked document X into the blogosphere?
  - Which bank started the financial crisis?
  - Who spread a rumor among your circle of friends?
  - Who is patient zero?
- Models of epidemics
  - Infected nodes infect susceptible neighbors = spreading the rumor
  - Network model: contacts, influence, dependence

Susceptible-Infected (SI) Rumor Model

SI Rumor Model



Goal: Find source among “
in given network $G(V,E)$

Summary

- Random networks:
  - Capture uncertainty, decentralized organization, unplanned growth of many social, biological, and technical networks
  - Engineering for such networks: macroscopic, not microscopic
- Asymptotics:
  - Little focus on details (e.g., routing algorithm for small networks), but focus on scale: fundamental relationships for very large networks
- Two large classes ←→ global connectivity constraint:
  - Geometric (must be close to be linked): percolation
    - Wireless; forest fire
  - Unconstrained: random graphs
    - Social net; Internet
- Structural properties; evolution of...; processes on...
- Main tools:
  - Probability
  - Random processes
  - Combinatorics
  - Geometry
  - Game Theory

Organization

- Class notes
  - Moodle
- Instructors:
  - Elisa Celis (BC 249) and Patrick Thiran (BC 201)
- Teaching Assistant:
  - Head TA: Farnood Salehi (BC 250)
  - Amedeo Esposito
- Grading:
  - 4 to 6 homework sets. Max 10 pts.
  - Term paper (presented during the (two) last week(s) of the semester) Max 40 pts.
  - Final exam (session). Max 50 pts.