Networks out of Control:

Homework Set 2

Exercise 1

We identify a weaker condition for $G(n,p)$ to have diameter at most 2, than the one seen in class: Show if $p \geq c\frac{\log n}{\sqrt{n}}$, where $c > 0$ is some constant, then $G(n,p)$ has diameter at most 2 a.a.s.

Exercise 2

Consider two families of random graphs $G_n = G(n,p)$, with $p(n) = n^{-6/5}$, and another independent random graph $H_n = G(\log n, (\log n)^{-8/7})$. Can you show that H_n does not appear in (i.e., is not a subgraph of) G_n for large n?

Hint: Identify a subgraph that is likely to appear in H, then show that this subgraph does not appear in G.

Exercise 3

We have seen in class that $t(n) = 1/n$ is a threshold function for the appearance of cycles of *fixed* order k in $G(n,p)$. Observe that this does not directly imply that $t(n)$ is also a threshold function for the appearance of cycles of *any* order. Prove that the assertion is in fact true.

Exercise 4

Let V denote, as usual, the vertex set of $G(n,p)$, with $0 < p < 1$ constant. A subset W of V is called *independent* if and only if no pair of vertices of W are adjacent. Show that $G(n,p)$ contains a.a.s no independent set of more than $2c\log_q n$ vertices, where $c > 1$ and $q = 1/(1-p)$.

Hint: Let X_k denote the number of independent subsets of V, with k vertices. Compute first $\mathbb{E}[X_k]$, and next an upper bound of $\mathbb{E}[X_k]$ with $k = 2c\log_q n$.