

SHAPE FROM X

A horizontal yellow brushstroke with a textured, painterly appearance, spanning the width of the slide below the title.

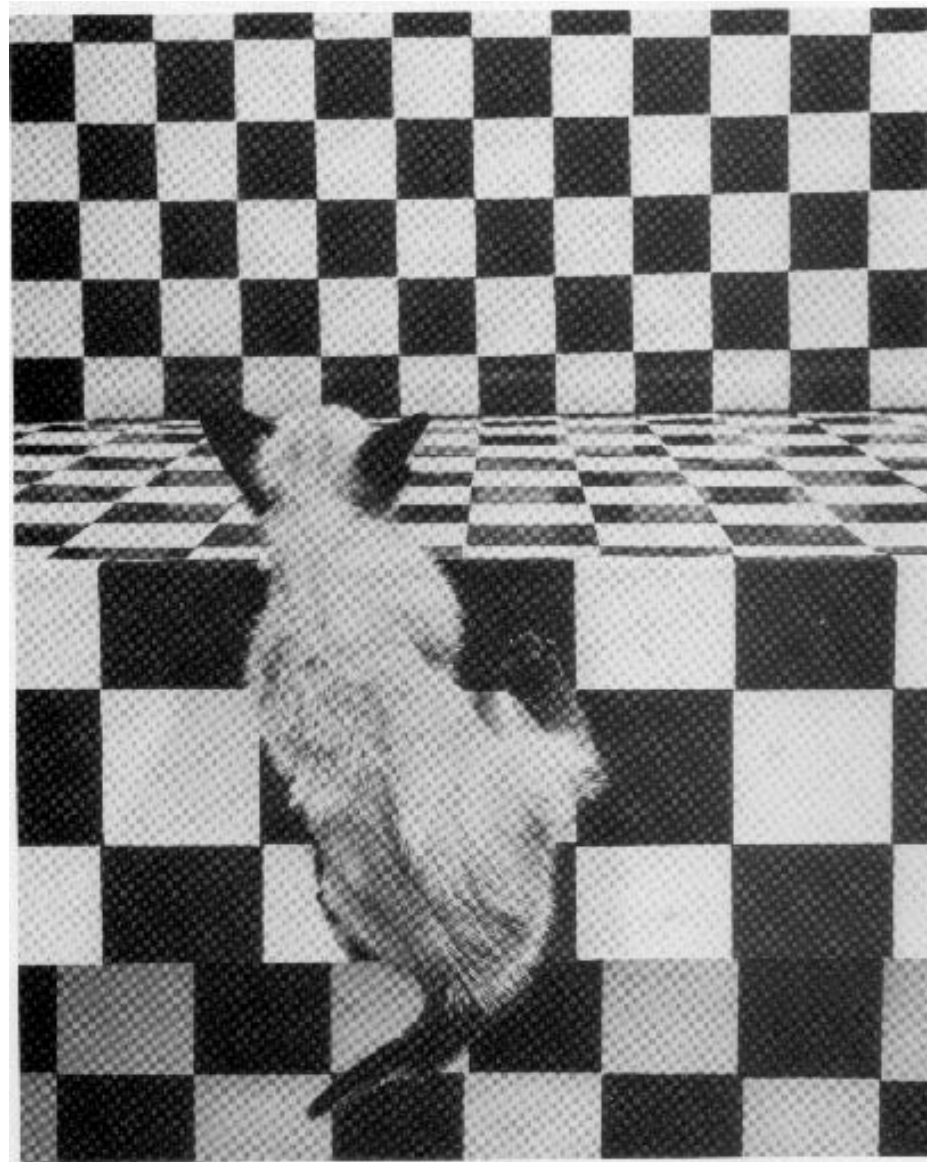
One image:

- Shading
- **Texture**

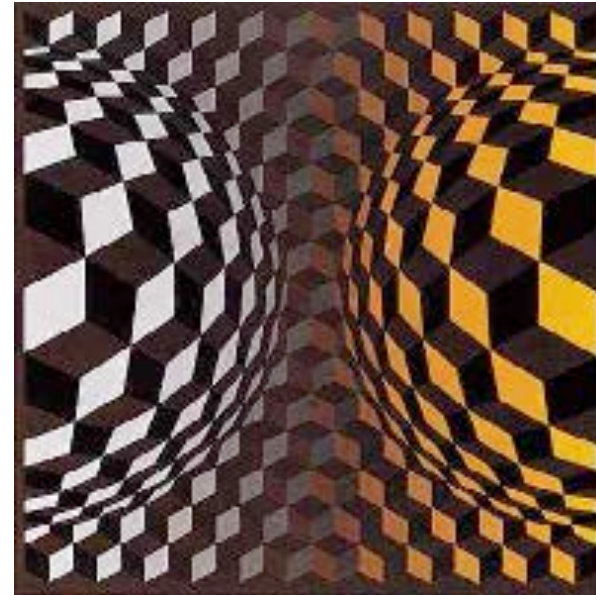
Two images or more:

- Stereo
- Contours
- Motion

SHAPE FROM TEXTURE



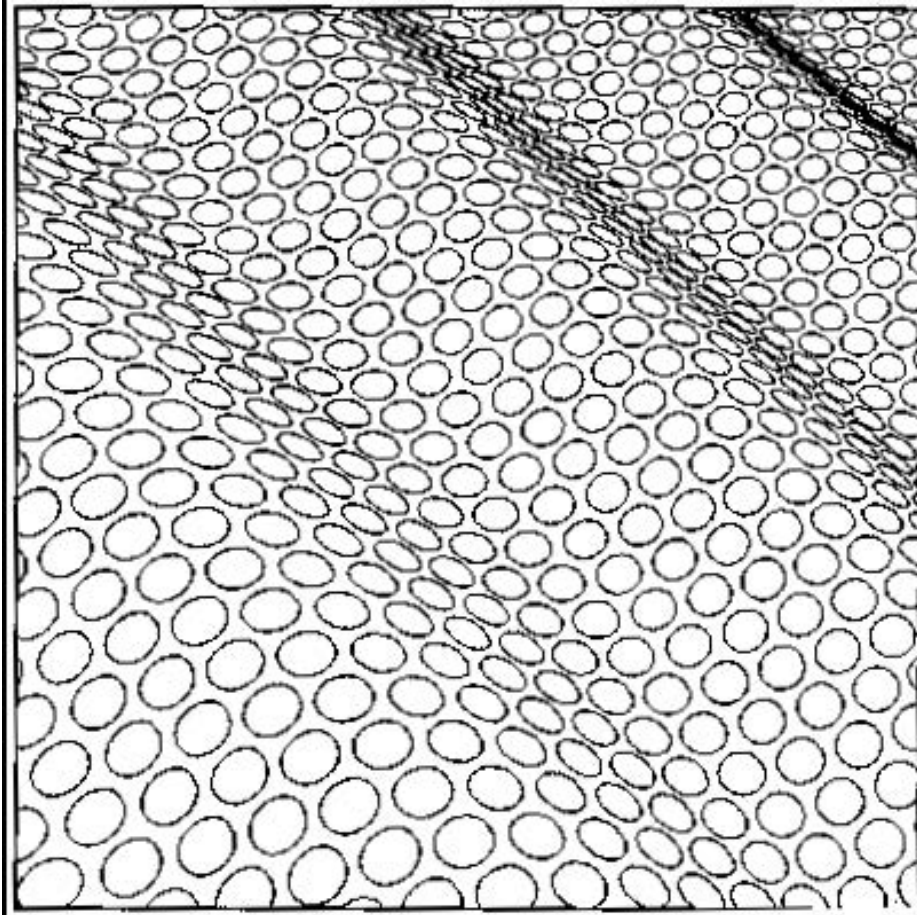
SHAPE FROM TEXTURE



Recover surface orientation or surface shape from image texture.

- Assume texture 'looks the same' at different points on the surface
- This means that the deformation of the texture is due to the surface curvature

STRUCTURAL SHAPE RECOVERY

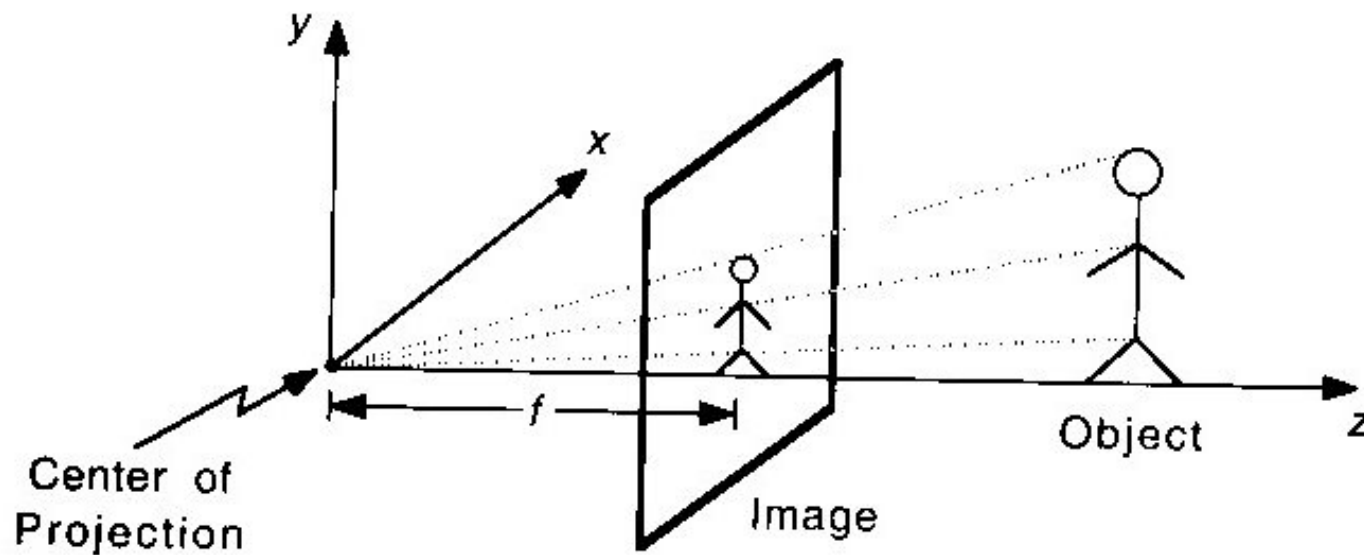


Basic hypothesis: Texture resides on the surface and has no thickness.

--> Computation under:

- Perspective projection
- Paraperspective projection
- Orthographic projection

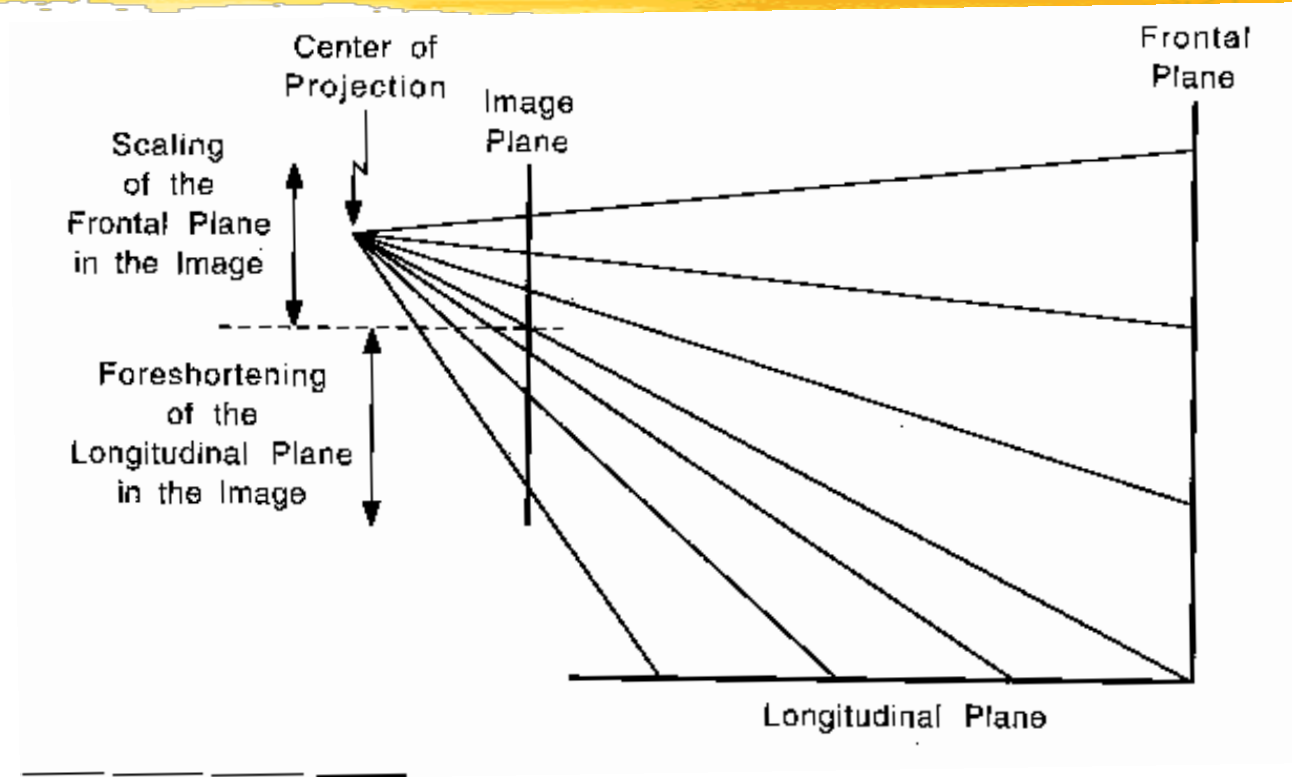
PERSPECTIVE PROJECTION



$$u = f \frac{x}{z}$$
$$v = f \frac{y}{z}$$

Pinhole geometry without image reversal

PERSPECTIVE DISTORTION



Perspective projection distortion of the texture

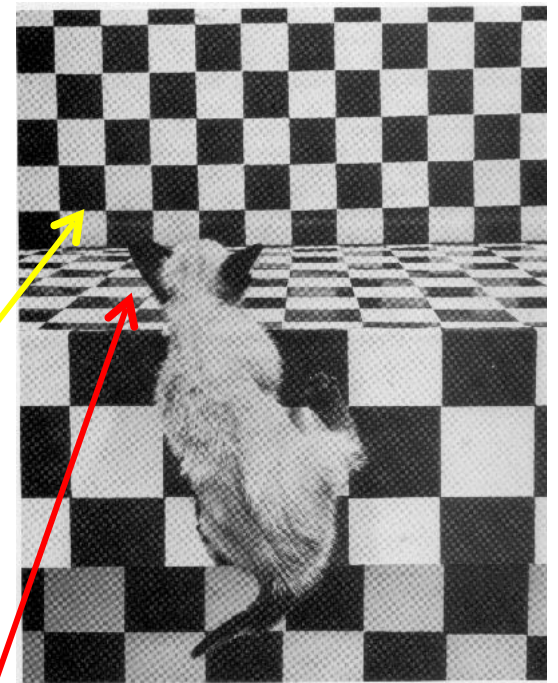
- depends on both depth and surface orientation,
- is anisotropic.

FORESHORTENING

Depth vs Orientation:

Infinitesimal vector $[\Delta x, \Delta y, \Delta z]$ at location $[x, y, z]$. The image of this vector is

$$\frac{f}{z} \left[\Delta x - \frac{x}{z} \Delta z, \Delta y - \frac{y}{z} \Delta z \right]$$

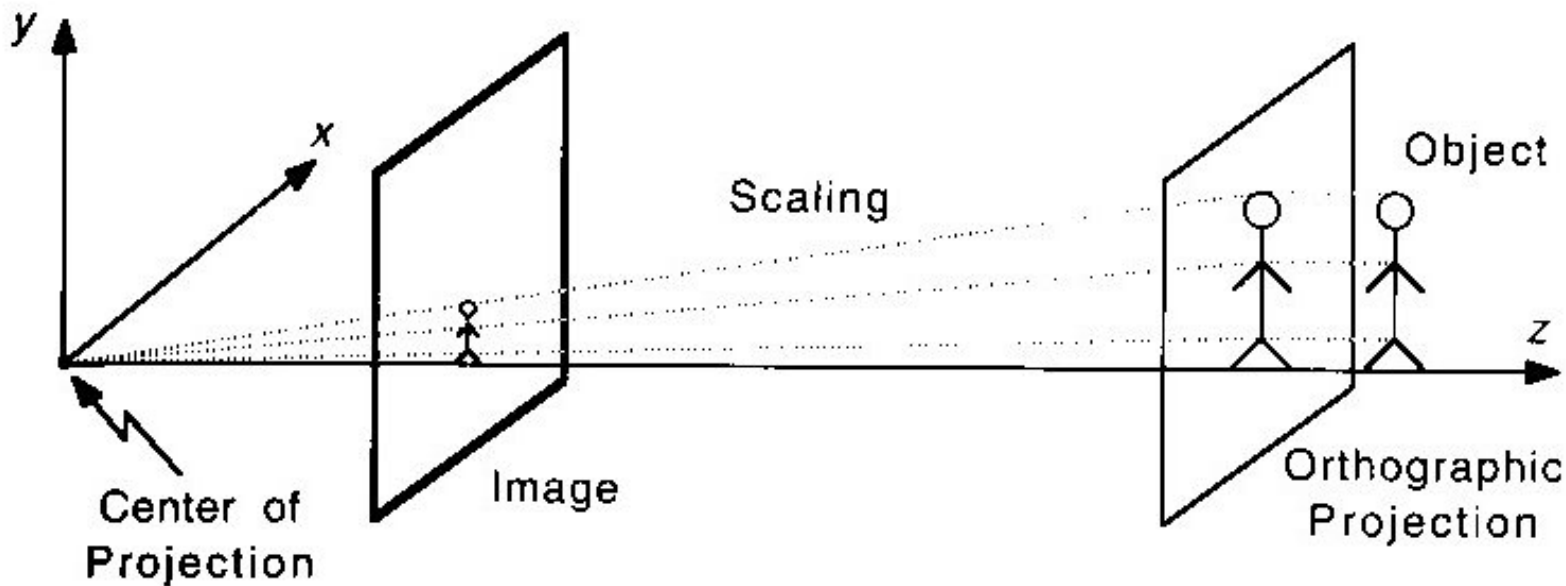


Two special cases:

$\Delta z = 0$: The object is scaled

$\Delta x = \Delta y = 0$: The object is foreshortened

ORTHOGRAPHIC PROJECTION



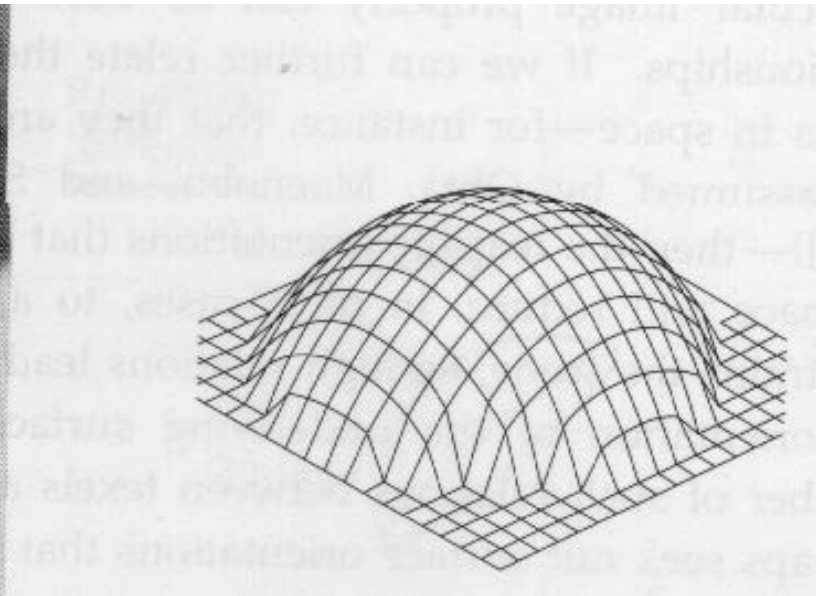
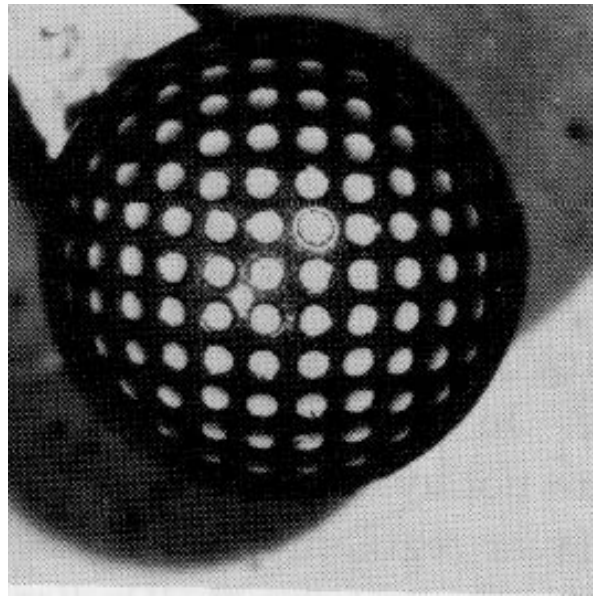
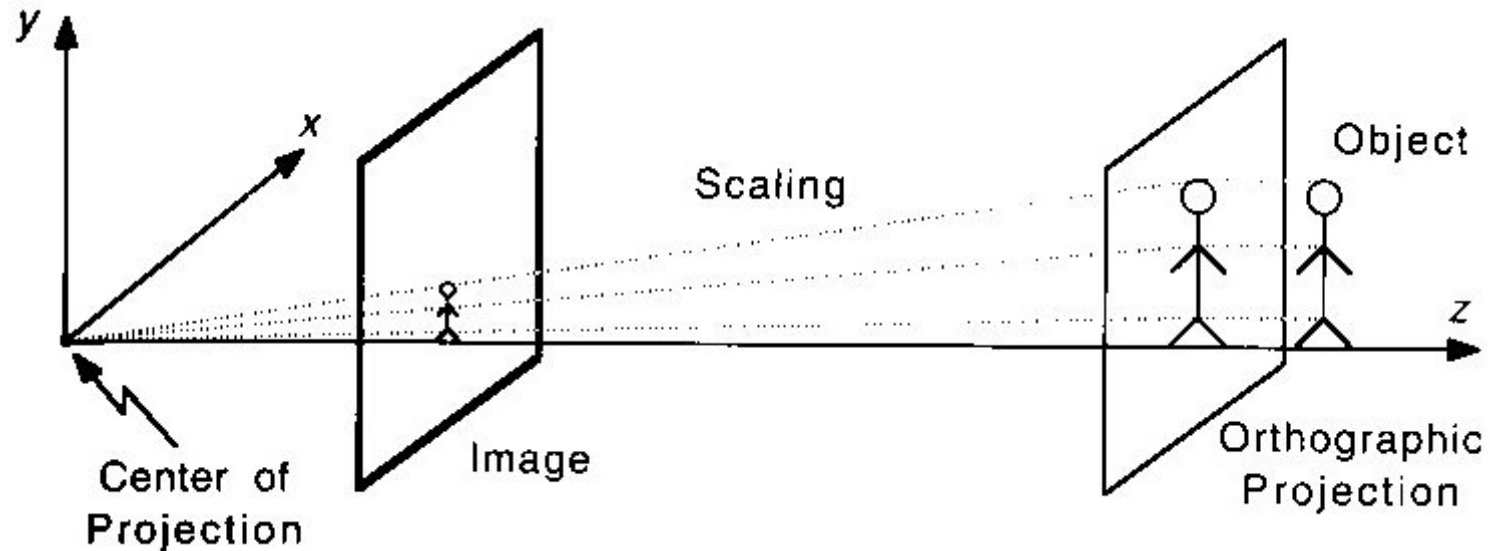
$$u = sx$$

$$v = sy$$

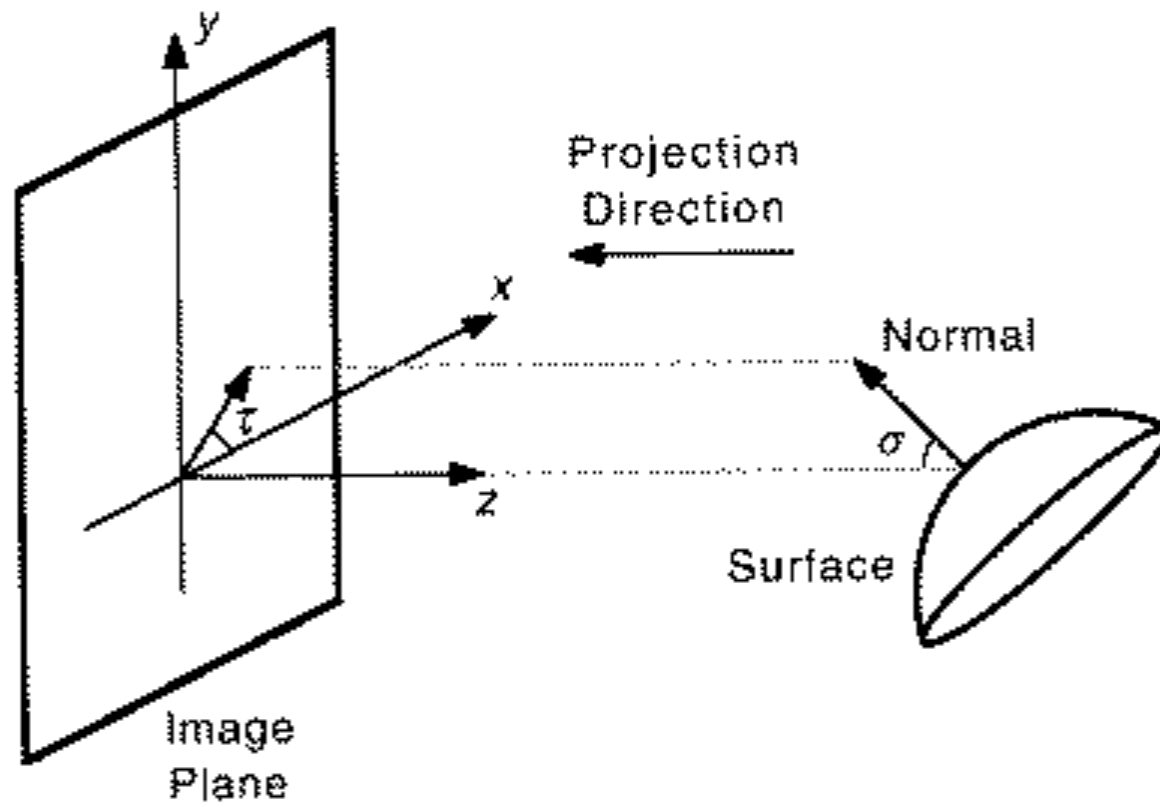
Special case of perspective projection:

- Large f
 - Objects close to the optical axis
- Parallel lines mapped into parallel lines.

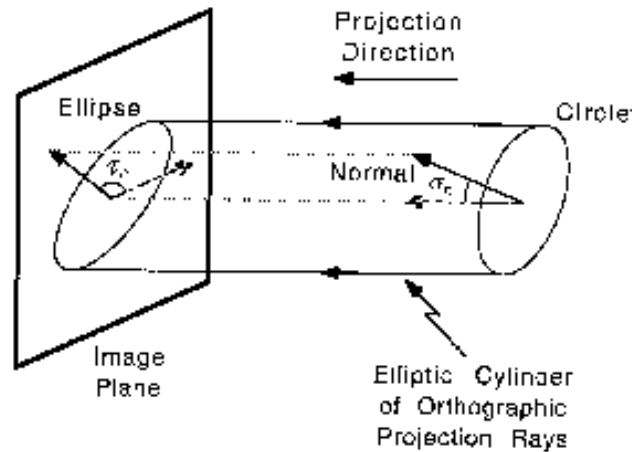
ORTHOGRAPHIC PROJECTION



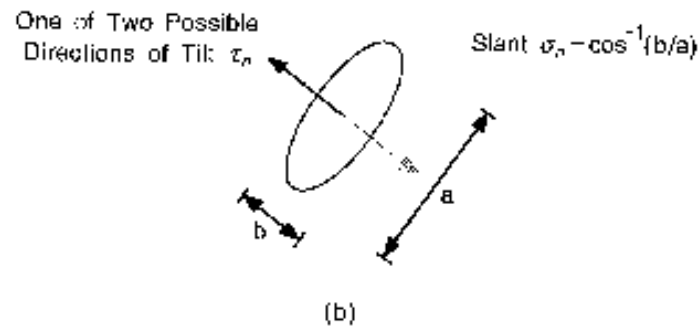
TILT AND SLANT



ORTHOGRAPHIC PROJECTION



(a)

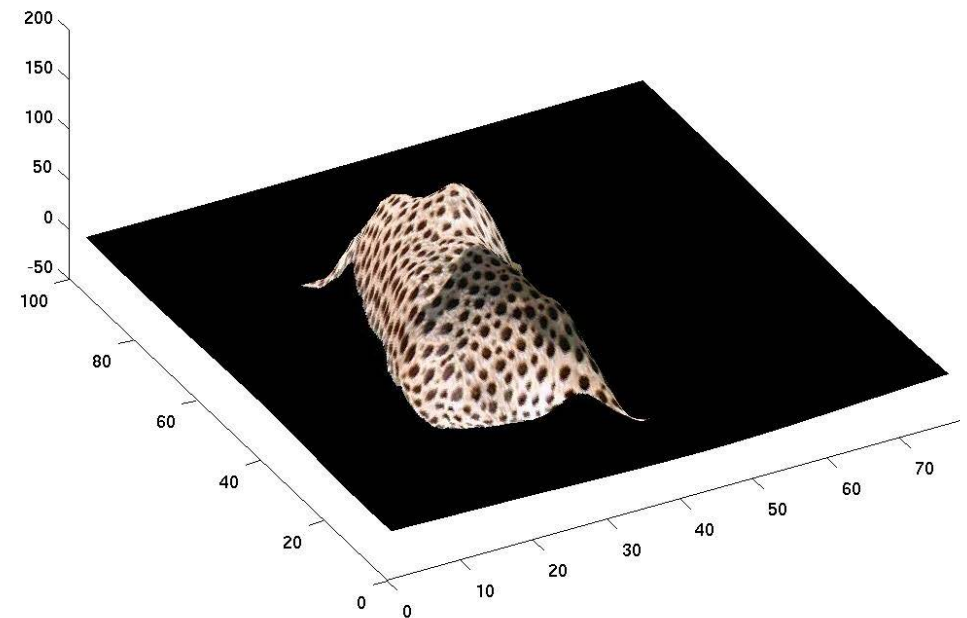
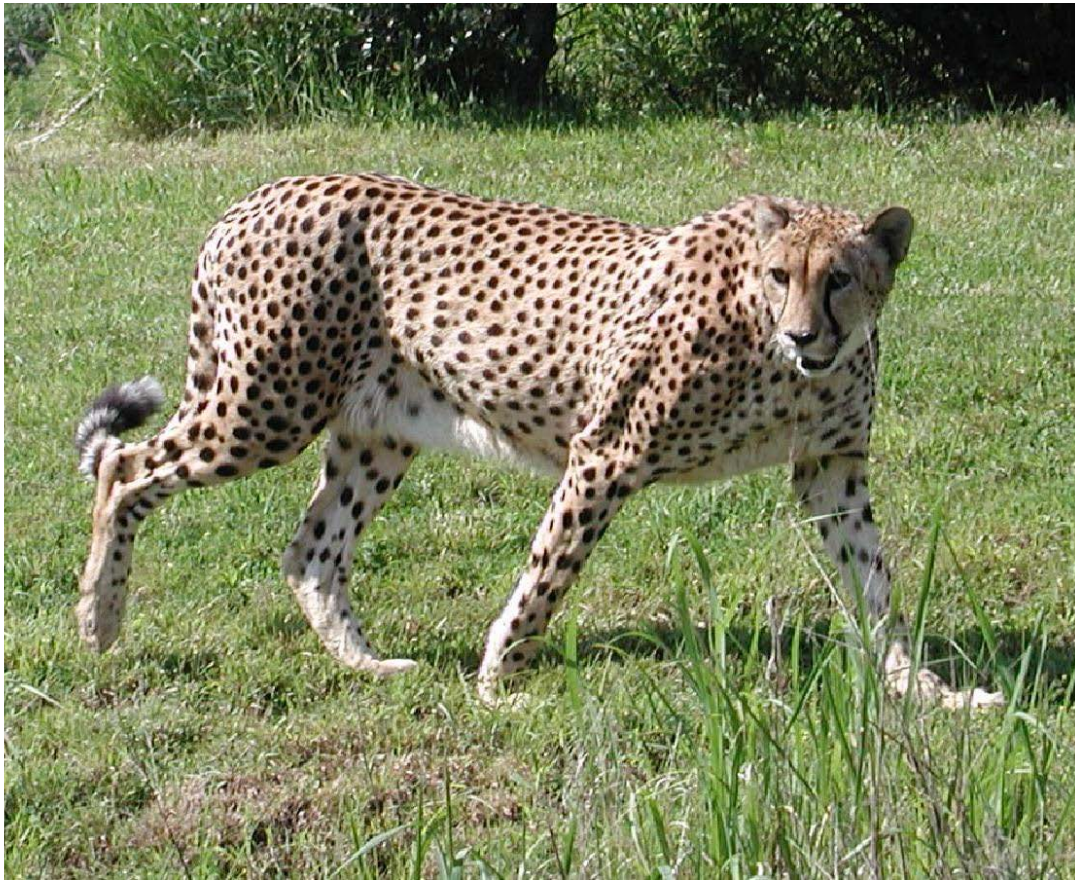


(b)

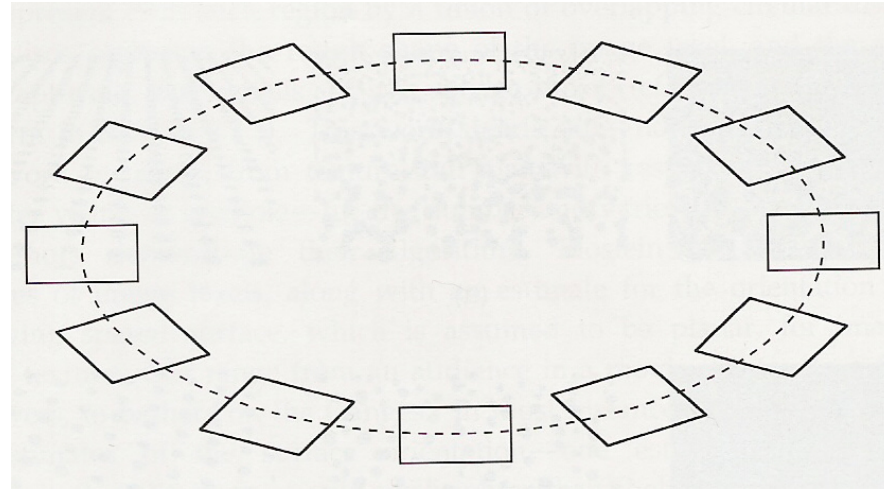
Tilt: Derived from the image direction in which the surface element undergoes maximum compression.

Slant: Derived from the extent of this compression.

CHEETAH



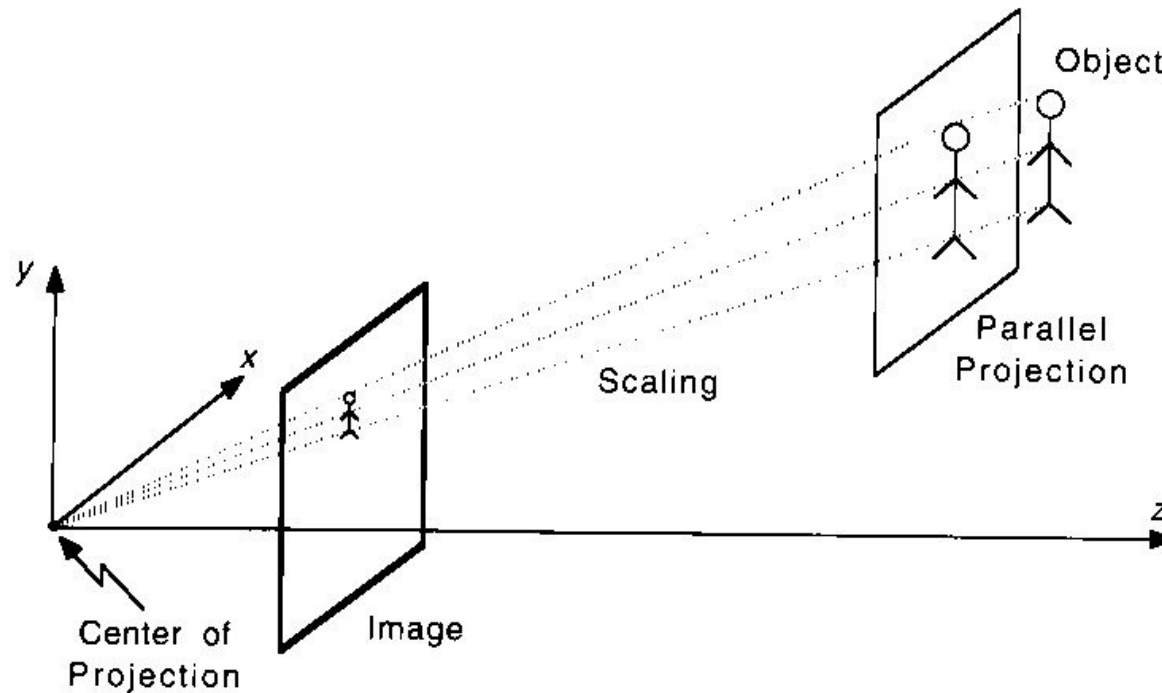
PERPENDICULAR LINES



Orthographic projections of squares that are rotated with respect to each other in a plane inclined at $\omega=60^\circ$ to the image plane.

$$\frac{\|(\mathbf{p}_1 / l_1) \times (\mathbf{p}_2 / l_2)\|}{\|\mathbf{p}_1 / l_1\|^2 + \|\mathbf{p}_2 / l_2\|^2} = \frac{\cos(\omega)}{1 + \cos^2(\omega)}$$

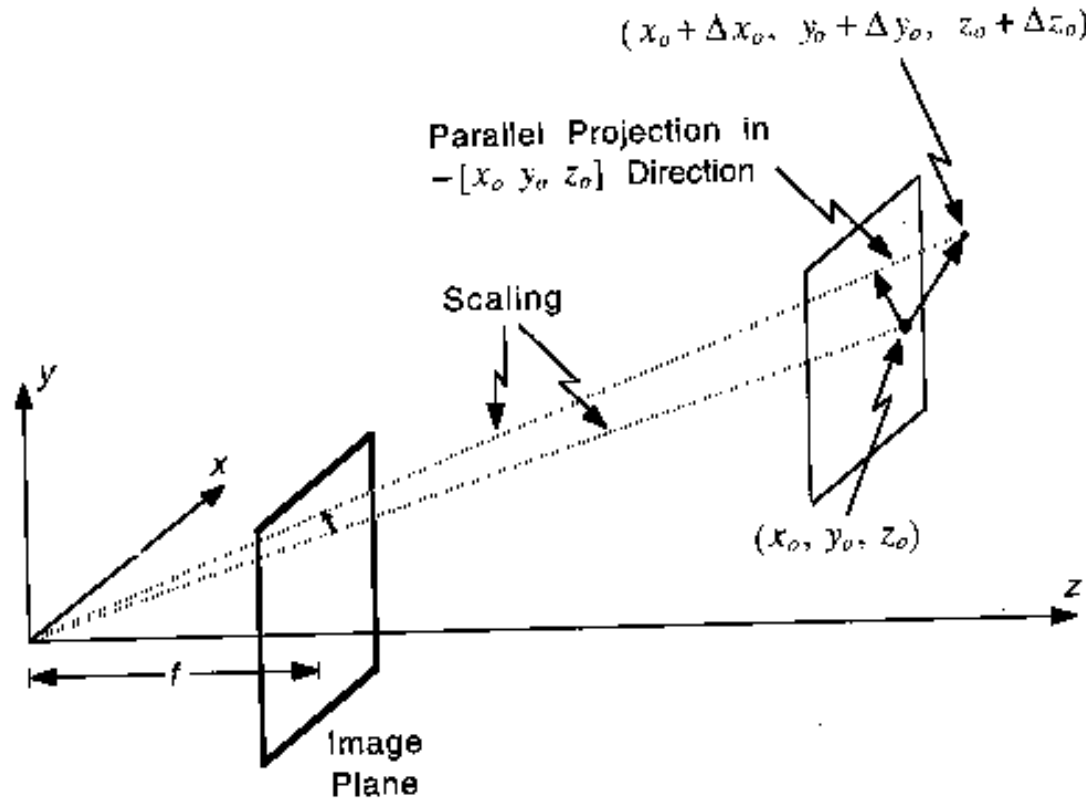
PARAPERSPECTIVE PROJECTION



Generalization of the orthographic projection:

- Object dimensions small wrt distance to the center of projection.
- Parallel projection followed by scaling

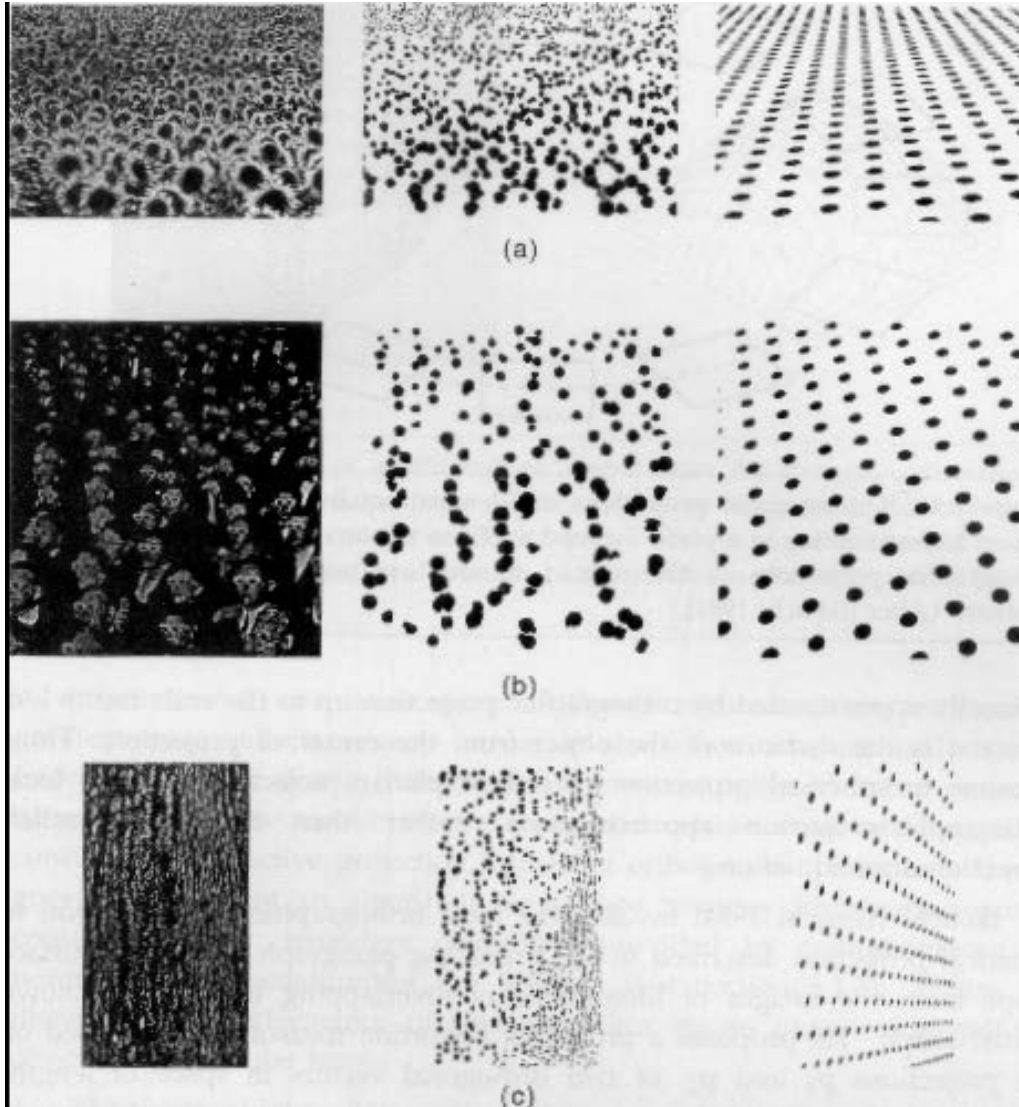
PARAPERSPECTIVE PROJECTION



For planar texels:

$$A' = -\frac{f^2}{z_0^3} \mathbf{n} \cdot [x_0 y_0 z_0] A$$

PARAPERSPECTIVE PROJECTION



Texels:

- Image regions that are brighter or darker than their surroundings.
 - Assumed to have the same area in space.
- Given enough texels, it becomes possible to estimate the normal.

TEXTURE GRADIENT



STATISTICAL SHAPE RECOVERY



Measure texture density as opposed to texel area, that is, the number of textural primitives per unit surface.

Assuming the texture to be homogeneous, we have: $\psi \mathbf{n} \propto \mathbf{b}$

$$\psi = \begin{bmatrix} u_1 & v_1 & 1 \\ \dots & \dots & \dots \\ u_n & v_n & 1 \end{bmatrix}^t$$
$$\mathbf{b} = [b_1, \dots, b_n]^t$$

Image coordinates.

$$\Rightarrow \mathbf{n} = \frac{\psi \mathbf{n}}{\|\psi \mathbf{n}\|}$$

Function of density.

STRENGTHS AND LIMITATIONS



Strengths:

- Emulates an important human ability.

Limitations:

- Requires regular texture.
- Involves very strong assumptions.
- Deep learning might weaken them.