RBPA 2018

Solutions WEEK 11

Problem 1:

(a) From Fig. 7-9 the maximum range of 1.71 MeV β-particles is $810 \frac{mg}{cm^2}$ or $0.81 \frac{g}{cm^2}$. Hence, the thickness of polyethylene required to absorb the 1.71 MeV β-particles is:

$$\frac{0.81 \frac{\cancel{g}}{\cancel{cm^2}}}{0.93 \frac{\cancel{g}}{\cancel{cm^3}}} = \underline{\underline{0.87\ cm}} \therefore$$

(b) Essentially all of the Bremsstrahlung can be assumed to be produced in the water solution since the thin polyethylene walls are of similar density. From Table 8-1 about 0.6 % of 1.7 MeV β-energy is converted to photons due to absorption in water.
As the average β-energy for ^{32}P was given as $\overline{E}_{\beta^-} = 0.695\ MeV$, the photon energy emission rate is:

$$E_{rad} = 3.7 \times 10^{10}\ s^{-1} \times 0.695\ MeV \times 6 \times 10^{-3} =$$

$$= \underline{\underline{1.55 \times 10^{8}\ MeV/s}}$$

(Energy emission rate due to Bremsstrahlung production)

This energy rate is assumed to produce photons of energy equal to the maximum β- energy 1.71 MeV (conservative estimate).

The photon emission rate is then:

$$\frac{1.55 \times 10^8 \ \mathrm{MeV/s}}{1.71 \ \mathrm{MeV/photon}} \simeq \underline{\underline{9.1 \times 10^7 \ \frac{\text{photons}}{s}}}$$

Assuming isotropy, the photon flux @ 1m is:

$$\frac{9.1 \times 10^7 \ \frac{\text{photons}}{s}}{4 \times \pi \times 100^2 \ cm^2} = \underline{\underline{7.24 \times 10^2 \ \frac{\text{photons}}{cm^2 \cdot s}}}$$

<u>N.B.</u> What should be the thickness of lead required to ensure that the dose equivalent rate due to Bremsstrahlung photons is less than 1 mrem/h @ 1m?

- Dose equivalent rate: $\dot{H} = w_R \times \dot{D}$ dose rate

  radiation factor

- Bremsstrahlung photons = γ-rays (continuous) $\Rightarrow w_R = 1$

<u>1 rem unit</u>

- 1 Roentgen Equivalent in Man (or Mammal)
- unit ~~to~~ measure the biological effect of ionizing radiation
- the dosage in rads that will cause the same amount of biological injury as 1 rad γ-rays (or X-rays)

$1 \ \text{rad} = 0.01 \ Gy = 0.01 \ J/kg = 100 \ \frac{\text{ergs}}{g}$

For γ-rays (X-rays) $w_R = 1 \Rightarrow \boxed{100 \ \frac{\text{ergs}}{g} = 1 \ \text{rad} = 1 \ \text{rem}}$

Thus, from Table 8-2 the mass energy absorption coefficient for 1.71 MeV photons for tissue is (by interpolation) 0.027 $\frac{cm^2}{g}$.
Therefore, the photon flux produces an energy absorption rate in tissue of:

$$E_{ab} = \left(7.24 \times 10^2 \frac{1}{\cancel{cm^2 \cdot s}}\right) \times \left(1.71\ \cancel{MeV}\right) \times \left(0.027 \frac{\cancel{cm^2}}{g}\right) \times \left(1.6022 \times 10^{-6} \frac{ergs}{\cancel{MeV}}\right) \times$$

$$\times \left(3600 \frac{\cancel{s}}{h}\right) \simeq 0.1928 \frac{ergs}{g} \frac{1}{h} \simeq \underline{\underline{0.193 \frac{ergs}{g} \frac{1}{h}}}$$

This corresponds to an absorbed dose rate of 1.93 mrad/h or a dose equivalent rate of $\underline{\underline{1.93\ mrem/h}}$.

In order to reduce the dose eq. rate below 1 mrem/h, it is necessary to add a Pb shield. From Table 8-2 the attenuation coefficient μ for lead and 1.71 MeV photons is $\mu = 0.565\ cm^{-1}$. Assuming that there are no scattered photons, one obtains from:

$$1.0\ mrem/h = 1.93\ mrem/h \times e^{-0.565 \cdot x}$$

the required thickness $\boxed{x = 1.16\ cm}$.:

Please note: 1 mrem/h = 10^{-5} Sv/h

Problem 2: (a) From Table 8-3 we get $HVL = 1.47$ cm for 1 MeV photons.

As $200 \text{ mR/h} = \dfrac{800 \text{ mR/h}}{2^n}$ with $n = 2$

$\Rightarrow$ two HVLs or $2 \times (1.47 \text{ cm}) = \underline{2.94 \text{ cm}}$ of iron shield are needed.

(b) Applying the exponential attenuation law:

$I = I_0 e^{-\mu x}$

we find with $\mu = \dfrac{\ln 2}{HVL} = \dfrac{\ln 2}{1.47 \text{ cm}} \simeq \underline{0.47 \text{ cm}^{-1}}$

$\ln \dfrac{I}{I_0} = -\mu x$

$\ln \dfrac{150}{800} = -\mu x \Rightarrow x = \dfrac{1.67398}{0.47} \simeq \underline{3.56 \text{ cm}}$

Problem 3: From Table 8-2 the linear attenuation coefficient μ for 1.5 MeV photons in lead is 0.5927 cm^{-1}, and for good geometry:

$I(x) = 10^5 \dfrac{\gamma}{\text{cm}^2} \times e^{-(0.5927 \times 2)} \simeq \underline{3.06 \times 10^4 \text{ } \gamma/\text{cm}^2}$

From Table 8-4 the buildup factor for 1.5 MeV photons in lead for ($\mu x = 0.5927 \times 2 = 1.1854$) can be extracted (by interpolation) to $\underline{B = 1.45}$, and the fluence including buildup is:

$$I_6(x) = B \times I_0 \times e^{-\mu x} \simeq \underline{\underline{4.43 \times 10^4 \ \gamma/cm^2}}$$

which contains primary unscattered photons and scattered ones with lower energy.

Conservatively, the best estimate for the energy fluence is:

$$I_6(x)_E = 4.43 \times 10^4 \ \gamma/cm^2 \times 1.5 \ MeV = \underline{\underline{6.65 \times 10^4 \ MeV/cm^2}} \quad \therefore$$

<u>Problem 4:</u> This kind of design problem has to be solved iteratively. First, we need to compute the thickness for the unscattered beam ($B = 1$).

As given, the exposure rate in air is related to the photon flux by:

$$\text{Exposure}\ (mR/h) = 0.0658 \times \phi\ (cm^{-2} \cdot s^{-1}) \times E\ (MeV) \times \left(\frac{\mu_{en}}{\rho}\right)_{air} \left(\frac{cm^2}{g}\right)$$

For 1 MeV photons, <u>Table 8-2</u> gives $\left(\frac{\mu_{en}}{\rho}\right)_{air} \simeq 0.0279 \ \frac{cm^2}{g}$

Then, the flux that produces 1 mR/h is:

$$\phi = \frac{1 \ mR/h}{0.0658 \times 1 \ MeV \times 0.0279 \ \frac{cm^2}{g}} \simeq \underline{\underline{544.72 \ \frac{\gamma}{cm^2 \cdot s}}}$$

The flux @ 60 cm from a point source that isotropically emits 10^8 photons/s is:

$$\phi_1 = \frac{10^8 \ \gamma/s}{4 \times \pi \times R^2 \ cm^2} = \frac{10^8 \ \gamma/s}{4 \times \pi \times 60^2 \ cm^2} \simeq \underline{\underline{2210 \ \frac{\gamma}{cm^2 \cdot s}}}$$

To reduce this flux ϕ_1 to ϕ we need, as the linear attenuation coefficient for 1 MeV photons in iron is $0.472\ cm^{-1}$ (from Table 8-2), an iron shielding with a thickness x of:

$$\phi = \phi_1 e^{-\mu x} \Rightarrow \ln\left(\frac{\phi}{\phi_1}\right) = -\mu x \Rightarrow \underline{\underline{x \simeq 2.96711\ cm}}$$

This is the thickness without a buildup factor!

In order to take the scattering of photons into account the buildup factor $B(1\,MeV, (\mu x)_{Fe})$ is needed.

From Table 8-4 we get for $\mu x = 0.472\ cm^{-1} \times 2.967\ cm = 1.40$ and 1 MeV photon energy the value (by interpolation) $B = 2.25$.

With this information we obtain a flux:

$$\phi_2 = 2210 \times 2.25 \times e^{-0.472 \times 2.967} \frac{f}{cm^2 \cdot s} = \underline{\underline{1225.7 \frac{f}{cm^2 \cdot s}}}$$

which still yields an exposure $\gg 1\ mR/h$!

Thus, the following process should be established:

$$\phi_i \rightarrow \text{new } x \rightarrow \mu x \rightarrow B \rightarrow \phi_{i+1}$$

MUST BE REPEATED UNTIL $\boxed{\phi_{i+1} = \phi}$ $\therefore$

Problem 5: (a) Point sources that emits S $\left(\frac{\text{neutrons}}{s}\right)$ can be specified @ a distance R (cm) in terms of a neutron flux ϕ:

$$\phi\left(\frac{n}{cm^2 \cdot s}\right) = \frac{S\left(\frac{n}{s}\right)}{4 \times \pi \times R^2 \, (cm^2)}$$

Table 14-4 indicates that the flux corresponding to a dose equivalent rate of 1 mrem/h due to 4.5 MeV neutrons is about 6.4 $\frac{\text{neutrons}}{cm^2 \cdot s}$

OR 0.156 mrem/h per 1 $\frac{\text{neutron}}{cm^2 \cdot s}$

The dose equivalent rate @ 100 cm from the unshielded neutron source is:

$$\dot{H}_0\,(\text{mrem}/h) = \frac{3 \cdot 10^7 \, \cancel{s^{-1}} \times 0.156 \, \frac{\frac{\text{mrem}}{h}}{\frac{1}{\cancel{cm^2} \cdot \cancel{s}}}}{4 \times \pi \times 100^2 \, \cancel{cm^2}} \simeq \underline{\underline{37.2 \text{ mrem}/h}}$$

∴

(b) If the source is shielded by water, scattering of neutrons will take place and a buildup factor B is needed. With B = 5 (see slide 25 / WEEK 11 Supplementary lecture) and Σ_{nr} extracted from Table 14-5 as 0.103 cm^{-1} one obtains:

$$\dot{H}\,(25\,cm\;H_2O) = \dot{H}_0 \times B \times e^{-\Sigma_{nr} x} = 5 \times 37.2 \times e^{-0.103 \times 25} \simeq$$

$$\simeq \underline{\underline{14.2 \text{ mrem}/h}} \quad ∴$$