RBPA 2018

Solutions

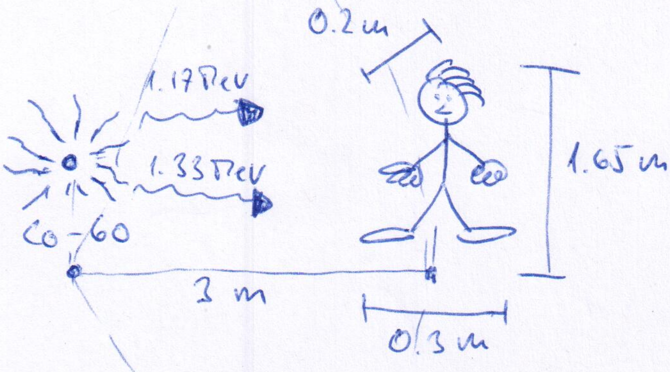


**Problem 1:**

- we have two gamma photons emitted from Co-60 (1173 keV & 1333 keV)

- we start by calculating the solid angle fraction $f_{s.a.}$ of the nurse:

$$f_{s.a.} = \frac{1.65 \times 0.3}{4\pi \times 3^2} = \underline{\underline{0.0043768}}$$




$\rho(\text{tissue}) \simeq 1\ g/cm^3$

$\mu_m(\text{tissue}) = 0.06265\ \frac{cm^2}{g}$ for $\bar{E}_\gamma = 1.25\,MeV$

$m(\text{nurse}) \simeq 60\,kg$

- assuming 20 cm of thickness $x_n$ for the nurse, the fraction $f_{abs}$ that gets absorbed in the nurse (we can use $\mu_m$ for tissue @ 1.25 MeV $= 6.265 \times 10^{-2}\ cm^2/g$) is now:

$$f_{abs.} = 1 - e^{-\mu_m \rho x_n} = 1 - e^{-0.06265 \times 1 \times 20} = \underline{\underline{0.71435}}$$

- the absorbed dose D over the time t of 8 hours is now:

$$D = \frac{A \times E_\gamma \times f_{s.a} \times f_{abs.} \times t}{m}$$

$$D \simeq \frac{5 \times 3.7 \times 10^{10} \times (1173 + 1333) \times 10^{3} \times 1.602 \times 10^{-19} \times 0.0043768 \times 0.71435 \times 8 \times 3600}{60}$$

$$D \simeq 0.1145 \text{ J/kg} = 0.1145 \text{ Gy}$$

Considering $w_R = 1$ & $w_T = 1$

$\Rightarrow$ $\underline{\underline{E \simeq 0.11145 \text{ Sv}}}$ ∴, where we have assumed a mass for the nurse of 60 kg.

We get an effective dose of about $\underline{\underline{111.45 \text{ mSv}}}$.

To reduce the dose to 1 μSv, we need to reduce the dose to a fraction F of the original dose (intensity).

$$F = \frac{1 \times 10^{-6} \text{ Sv}}{0.1145 \text{ Sv}} \simeq 8.97 \times 10^{-6}$$

Choosing lead ($\rho_{lead} \simeq 11.34 \frac{g}{cm^3}$) as the shielding material.

We see that $\mu_m$ for lead @ 1.25 MeV is $5.876 \times 10^{-2}$ cm²/g.

Thus, we get:

$$\frac{I}{I_0} \simeq \frac{E}{E_0} = 8.97 \times 10^{-6} = e^{-\mu_m \rho x}$$

solving for **x**, we get:

$$x = \frac{\ln(8.97 \times 10^{-6})}{\mu_m \cdot \rho} = \underline{\underline{17.4 \text{ cm}}}$$ ∴ (consideration of a good geometry)

We need at least 17.4 cm of lead to shield the source!

A possible design would be a cylinder made of lead, with a diameter of 40 cm and a height of 40 cm, and with the source in a small hole in the center. The weight of such shielding would then be around 570 kg.

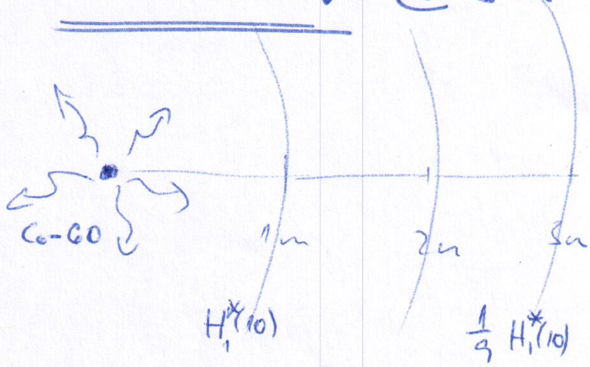
$\boxed{\text{Another approach using calculation of } H^*(10)}$

$$A = 5 \times 3.7 \times 10^{10}\ Bq = \underline{\underline{185\ GBq}}$$

$\boxed{Co-60}$: $h(10) \simeq 0.366 \ \frac{mSv/h}{GBq}$ @ 1 m

$$H^*(10) = h(10) \times \frac{A \times t}{\underbrace{r^2}_{\text{in } [m]}}$$

$$H^*(10) = 0.366 \ \frac{mSv/\cancel{h}}{\cancel{GBq}} \times \frac{185\ \cancel{GBq} \times 8\cancel{h}}{3^2} \simeq \underline{\underline{60.187\ mSv}} \ @ \ 3\,m$$




$$E = E_{externe} + \cancel{E_{interne}}^{\,0}$$

$$E_{externe} = \underline{H_p(10)} \text{ or } \underline{H^*(10)}$$

$\boxed{E \simeq 60.2\ mSv}$ ∴

$$F = \frac{1 \times 10^{-6}\ Sv}{0.0602\ Sv} \simeq \underline{1.66 \times 10^{-5}}$$

$\rho_{lead} = 11.34\ g/cm^3$

$\mu_m(lead) = 5.876 \times 10^{-2}\ cm^2/g$ @ 1.25 MeV

$$\frac{I}{I_0} \simeq \frac{E}{E_0} = 1.66 \times 10^{-5} = e^{-\mu_m \rho x}$$

$$x = \frac{\ln(1.66 \times 10^{-5})}{\mu_m \rho} \simeq \underline{\underline{16.5\ cm}} \ ∴$$

We need at least 16.5 cm of lead to shield the source!

(consideration of a good geometry)

Problem 2: there are @ least three ways to identify Po-210 with detectors of ionising radiation:

1) one possibility is to measure the energy of the alpha radiation with a well calibrated alpha detector (e.g. a surface barrier silicon detector). The alpha energy of Po-210 should be 5.305 MeV.

2) another possibility is to measure the decay half-life (using the alpha decay) carefully over a number of days, to see that the half-life is in fact 138.38 days.

3) a third possibility is to measure the very weak gamma ray of 803 keV emitted from an excited state of the daughter nucleus Pb-206. A Germanium detector should be used for this, and it is only possible if the source is strong enough (as in this case).

- 10 micrograms of Po-210 correspond to a number of atoms N:

$$N = \frac{m \cdot N_A}{m_u} = \frac{10\times10^{-6} \times 6.022\times10^{23}}{209.98} = \underline{\underline{2.8679\times10^{16}}}$$

- the activity A can now be calculated

$$A = \lambda \times N = \frac{\ln 2}{T_{1/2}} \times N = \frac{0.69315}{138.376\times24\times3600} \times 2.8679\times10^{16}$$

$$A \simeq 1.6627\times10^{9}\ Bq \simeq \underline{\underline{1.66\ GBq}}$$

We can assume that the absorbed dose D is now calculated with the knowledge of $E_\alpha$:

$$D = \frac{A \times E_\alpha \times t}{m_{body}} = \frac{1.6627 \times 10^9 \times 5.305 \times 10^6 \times 1.602 \times 10^{-19} \times 24 \times 3600}{75}$$

$$D \simeq 1.6591 \ J/kg = \underline{\underline{1.6591 \ Gy}}$$

$\boxed{m_{body} = 75 \, kg \text{ (assumption)}}$

To get the effective dose, we should multiply by $W_R = 20$ since we deal with alpha radiation and by $W_T = 1$. The effective dose is then around 33 Sv.

<u>This is a lethal dose!</u>

N.B. We have calculated the effective dose during 24 hours!

Please recall

$$E_{50} = \int_{t_0}^{t_0 + 50\,years} E(t)\,dt$$

$$E_{50} = e_{ing.} \times A_{ing.}$$

Thus, if you use directly $e_{ing.}$ from „ORaP", you will get $E_{50}$!

Problem 3: To calculate the time since the death of the mammoth, we use the decay formula:

$$N = N_0 \exp(-\lambda t) \qquad \lambda = \frac{\ln 2}{T_{1/2}} \qquad T_{1/2} = 5730 \text{ years}$$

$1\,g$ of $^{12}C$ ... $\frac{N_A}{12}$ atoms

- simple assumption:

one $^{14}C$ atom for $10^{12} \times {}^{12}C$ when mammoth died as it is today!

$\Rightarrow$ therefore, 1 gram of natural carbon contains approximately:

$10^{-12} \frac{N_A}{12}$ atoms of the $^{14}C$ isotope

(here, we can neglect the 1.1 % C-13 abundance)

Solving the above formula for the time $t$ (in years), we get:

$$t = -T_{1/2} \times \frac{\ln (N/N_0)}{\ln 2} = -5730 \times \frac{\ln \left(3.945 \times 10^{8} / \left(\frac{1}{12} 6.023 \times 10^{23} \times 10^{-12}\right)\right)}{0.69315}$$

$$\boxed{t \simeq 40\,060 \text{ years}}$$

We see that this particular mammoth died about 40 000 years ago.

Problem 4: The radioisotope $^{11}C$ is a $\beta^+$-emitter. It decays directly to the ground state of the daughter nuclide $^{11}B$. From the annihilation of the positron, we get (for each decay) two 511 keV photons emitted in opposite direction. Since the source is injected in the patient body, this radiation is emitted from within the patient. The half-life of $^{11}C$ is 20.39 minutes. We can therefore assume that all $^{11}C$ nuclei will decay in the body of the patient (during and after the PET measurement).

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We know for the activity, $A_0$, that $A_0 = \lambda \times N_0 = \frac{N_0 \times \ln 2}{T_{1/2}}$

From this we get the number, $N_0$, of C-11 nuclides (@ the time of injection):

$$N_0 = \frac{A_0 T_{1/2}}{\ln 2} = \frac{1.5 \times 3.7 \times 10^{10} \times 60 \times 20.39}{0.69315} = \underline{9.7957 \times 10^{13}}$$

- the kinetic energy of the $\beta^+$-particle can be converted in different ways (e.g. bremsstrahlung), but here we can assume that the kinetic energy is all converted by the collisions in the body.

- Assuming an average of 0.5 times the Q-value (1982 keV) for the kinetic energy (since the neutrino takes the other half), we get an absorbed energy of 991 keV in the body for each decay.

- In addition, the patient will absorb some fraction of the 511 keV photon intensity. Assuming an average of 10 cm from the point of decay to the surface of the body, we can check the absorbed fraction for 511 keV photons:

$$\varepsilon_1 = 1 - \frac{I}{I_0} = 1 - e^{-\mu x} = 1 - e^{-9.598 \times 10^{-2} \times 1.06 \times 10} = 1 - 0.36154 =$$

$$= \underline{\underline{0.63846}}$$

where the density and abs. coeff. for soft tissue is used.
($\rho = 1.06$ g/cm3 ; $\mu/\rho = 9.598 \times 10^{-2}$ cm$^2$/g @ 511 keV)

- The total effective dose, E, (here it is equal to the absorbed dose: (gamma, beta, full body): $W_R = 1$ $W_T = 1$

$$E = D = \frac{N_0 \times (E_{photons} \times \varepsilon_1 + E_{Beta})}{m} = \frac{9.7957 \times 10^{13} (2 \times 511 \times 0.63846 + 991) \times 10^3 \times 1.602 \times 10^{-19}}{80} =$$

$$= \underline{\underline{0.32233 \text{ Sv}}}$$

where the factor $\underline{2}$ comes from the fact that two 511 keV are emitted, and where the body mass is assumed to be 80 kg.

It should be noted that this calculated dose of c. 322 mSv is quite high. For diagnostics of this type it is unusual to have a much higher full body doses than 10 mSv.

For the nurse:

We assume that she is only affected by the 511 keV photons. Since we have estimated above that about 36% of the radiation escapes the patient, we need only to calculate the absorption, $\mathcal{E}_2$, in the nurse.

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- we assume that the nurse is (sitting down) 1.2 m high, 40 cm wide, and (on average) 25 cm thick

$$\mathcal{E}_2 = 1 - e^{-\mu x} = 1 - e^{-9.598 \times 10^{-2} \times 1.06 \times 25} = \underline{\underline{0.921407}}$$

- assuming that the nurse sits 2 meters away from the patient, we get the solid angle fraction:

$$f_{s.a.} = \frac{1.2 \times 0.4}{4\pi \times 2^2} = \underline{\underline{0.0095493}}$$

- the # of C-11 nuclei @ the end of the PET measurements (30 min.) is now $N(t=30\,\text{min})$. The number of decays during 30 min., $N_{d_{30}}$, is therefore:

$$N_{d_{30}} = N_0 - N(30\,\text{min.}) = 9.7957 \times 10^{13} \left(1 - e^{-\ln 2 \left(\frac{30}{20.39}\right)}\right) = \underline{\underline{6.2628 \times 10^{13}}}$$

We see that the effective dose for nurse is now (again, it is equal to the absorbed dose: (gamma, , full body)):

$$E = D = \frac{N_{d_{30}} \left(E_{photons} \times (1 - \mathcal{E}_1) \times \mathcal{E}_2 \times f_{s.a.}\right)}{m}$$

$$= \frac{6.2628 \times 10^{13} \times (511 \times 0.36154 \times 0.921407 \times 0.0095493) \times 10^3 \times 1.602 \times 10^{-19}}{70}$$

$$\simeq \underline{\underline{233\,\mu Sv}}$$, where we have assumed 70 kg for the nurse.

This dose 233 μSv is not "high" in itself, but we should remember that the nurse might be exposed several times per week.

For the radioprotection, we can then make a wall (for the nurse to sit behind, e.g. 1 meter high, 1 meter wide) @ the side of the PET equipment.

The thickness, x, of this wall should be enough to reduce the 511 keV photon intensity by a factor of 300:

$$\frac{1}{300} = e^{-\mu x} \Rightarrow x = \frac{\ln(300)}{\mu} = \frac{\ln(300)}{0.1614 \times 11.34} \simeq \underline{\underline{3.078 \text{ cm}}}$$

where we have used lead as material.

We see that it is enough to make the lead wall about 30 mm thick.

Problem 5:

- First, we see (from the Table of isotopes and/or the nuclide chart) that Co-57 decays by electron capture, populating (mainly) the excited state @ E = 136 keV in Fe-57. We see that around 11% (12/112) of the intensity decay by emitting a 136 keV gamma, and the rest (89%) by emitting two gammas; 122 keV and 14 keV. From now on, we consider these three gammas.
- We assume that Dr. Amy has the source close to her body. We can first assume that she has it 20 cm in front of her, and that she is 1.6 m high, 40 cm wide, and (on average) 25 cm thick, and weighs 60 kg.
- We also assume that Dr. Bernadette has the same body size.
- In this case it gives more than 50% solid angle ratio. But we realise that if the source was placed very close to the body, e.g. @ 1 mm distance, then only approximate 50% of the radiation would be directed into the body. So, from that we realise that a rectangular body area divided by a sphere (of 40 cm radius) is not a very good approximation @ this close range. On the other hand, a more

detailed model is a bit difficult to come up with quickly. If we assume that the solid angle fraction is 50%, than we have assumed the "worst-case-scenario" of Dr. Amy holding the source @ very short distance. Thus, this is our assumption from now on.

- Dr. Bernadette on the other hand stands behind Dr. Amy, we assume @ a distance of 1.5 meters from the source. Now, we use the rectangle/sphere method, and get a solid angle fraction of:

$$f_{s.a.} = \frac{1.6 \times 0.2}{4 \times \pi \times 1.5^2} = \underline{\underline{0.023}}$$

- For the abs. coeff. ($\mu/\rho$) of soft tissue, we have, for the 14 keV energy (we use the value @ 15 keV) 1.7 $cm^2/g$. For the 122 keV and 136 keV lines we use the mean value between 100 keV and 150 keV, e.g. 0.16 $cm^2/g$.

- The transmitted intensity fraction ($I/I_0$) of the three energies for a 25 cm thick body is:

$$t_{14} = e^{-\mu x} = e^{-1.7 \times 1.06 \times 25} \simeq \underline{\underline{2.7 \times 10^{-20}}}$$

$$t_{122} = t_{136} = e^{-\mu x} = e^{-0.16 \times 1.06 \times 25} \simeq \underline{\underline{0.0144}}$$

where the density for soft tissue is used.

$\Rightarrow$ we see immediately that the 14 keV photons are completely stopped by Dr. Amy's body (and will not reach Dr. Bernadette)

$\Rightarrow$ For the 122 and 136 keV gammas, only about 1.44% will pass through the body of Dr. Amy; i.e. 98.56% will be absorbed.

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Now, we can calculate the absorbed dose for Dr. Amy:

$$D = \frac{0.5 \times 200 \times 10^{-3} \times 3.7 \times 10^{10} \times (0.11 \times 136 \times 0.9856 + 0.89 \times (14 + 122 \times 0.9856)) \times 10^{3} \times 1.602 \times 10^{-19} \times 25}{60}$$

$$\boxed{D \approx 3.3 \times 10^{-5}\ Gy} \quad \therefore$$

The effective dose (full body, gamma) for Dr. Amy is now c. $\underline{0.33\ \mu Sv}$.

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Now, we can calculate the dose for Dr. Bernadette:

$$D = \frac{0.023 \times 200 \times 10^{-3} \times 3.7 \times 10^{10} \times (0.11 \times 136 \times 0.0144 + 0.89 \times 122 \times 0.0144) \times 10^{3} \times 1.602 \times 10^{-19} \times 25}{60}$$

$$\boxed{D \approx 2.0 \times 10^{-8}\ Gy} \quad \therefore$$

The effective dose (full body, gamma) for Dr. Bernadette is now c. $\underline{20\ nSv}$. Dr. Amy shields the radiation, saving Dr. Bernadette from receiving a much higher dose.

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To conclude, the dose Dr. Amy has received is quite low (comparable to an hour of background radiation), and Dr. Bernadette receives an extremely low dose from the incident, thanks to the shielding of Dr. Amy.