Last Name .... First Name..................

Neural Networks and Biological Modelling Exam
22 June 2016

- Write your name in legible letters on top of this page.
- The exam lasts 160 min.
- Write **all** your answers in a legible way on the exam (no extra sheets).
- No documentation is allowed apart from 1 sheet A5 of **handwritten** notes.
- No calculator is allowed.
- Have your student card displayed before you on your desk.
- **Check that your exam has 12 pages**

Evaluation:

1. ..../6 pts
2. .... / 18 pts
3. .... / 8 pts
4. .... / 14 pts (10 plus 4 bonus points)

Total: .... / 46 pts (inclues N=4 bonus points)

1 Biophysics of ion channels (6 points)

We consider the following model of a ion channel

$$I_{ion} = g_0 \, x^p \, (u - E)$$

where u is the membrane potential. The parameters g_0, p and $E = 0$ are constants.

(a) What is the name of the variable E? ....

Why does it have this name, what does it signify (give answer in one short sentence)

....
....
number of points: 1

(b) The variable x follows the dynamics

$$\frac{dx}{dt} = -\frac{x - x_0(u)}{\tau}$$

where $x_0(u)$ is monotonically increasing and bounded between zero and one. Suppose that, at $t = 0$, we make a voltage step from a fixed value E to a new constant value u_0. Give the mathematical solution $x(t)$ for $t > 0$.

$x(t) =$ ....
number of points: 2

Experimental colleagues tell you that they are able to apply voltage steps as in (b) and that by measuring the current they want to determine the parameters g_0 and p of the ion channel in (a) and (b)

(c) How should they proceed to measure the parameter p? What would be different between the case p=1 and $p = 2$? You can sketch a little figure to illustrate your answer.

....
....
....

number of points: 2

(d) Under the assumption that $x_0(u)$ is bounded between zero and 1, how can they measure g_0?

....
....
....

number of points: 1

2 Phase Plane Analysis and separation of time scales (18 points)

Synaptic plasticity happens at different time scales. The weight w of a synapse evolves on a time scale $\tau_w = 1/\gamma$ but it is coupled to a slower variable z (time scale $t\tau_z = 1/\epsilon$). You may want to think of z as the physical size of the synapse and of w as the number of AMPA receptors.

Here are the two equations that we will study:

$$\frac{dw}{dt} = \gamma\left[f(w) - w + z + H\right] \tag{1}$$

$$\frac{dz}{dt} = \epsilon\left[f(z) - z + w\right] \tag{2}$$

Normally the input H is zero. But during a pairing protocoll the joint activity of pre- and postsynaptic neuron give rise to a positive 'Hebbian' drive $H > 0$. The constants γ and ϵ have appropriate units (You may assume $\gamma = 0.25$ and $\epsilon = 0.05$).

The function f is a third-order polynomial

$$f(x) = -2x(1-x)(2-x) \tag{3}$$

where $x = w$ or $x = z$.

(a) Assume $H = 0$ and plot the two nullclines. To do so evaluate the w-nullcline at

$w = 0.0 \longrightarrow$ $z =$ ....

$w = 0.5 \longrightarrow$ $z =$ ....

$w = 1.0 \longrightarrow$ $z =$ ....

$w = 1.5 \longrightarrow$ $z =$ ....

$w = 2.0 \longrightarrow$ $z =$ ....

and the z-nullcline at

$z = 0.0 \longrightarrow$ $w =$ ....

$z = 0.5 \longrightarrow$ $w =$ ....

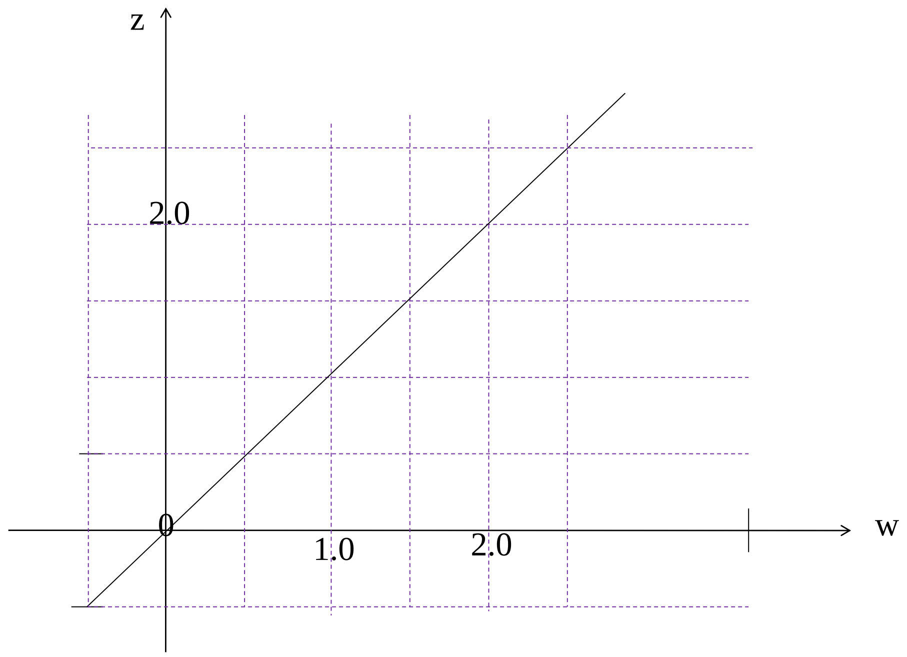
$z = 1.0 \longrightarrow$ $w =$ ....

$z = 1.5 \longrightarrow$ $w =$ ....

$z = 2.0 \longrightarrow$ $w =$ ....

Plot the two nullclines in the figure on the next page. Annotate your lines by writing e.g., w-nullcline or z-nullcline.

number of points: 3



(b) In the above graph, add an arrow indicating the direction of flow at the point $(w = 2, z = 0)$.

number of points: 1

(c) Keep in mind that $\gamma = 0.25$ and $\epsilon = 0.05$ and add, in the above graph, representative qualitative arrows indicating the flow in **six different regions** of the phase plane and **on all segments of the nullclines**

number of points: 4

(d) Suppose that, because of a past pairing protocol, the synapse is in a state $(w = 2, z = 0)$. Indicate the trajectory of the synapses in the above graph and label it with A. (As before $\gamma = 5\epsilon$.)

number of points: 1

(e) Suppose that, because of a past pairing protocol, the synapse is in a state $(w = 2, z = 1)$. Indicate the trajectory of the synapses in the above graph and label it with B. (As before $\gamma = 5\epsilon$.)

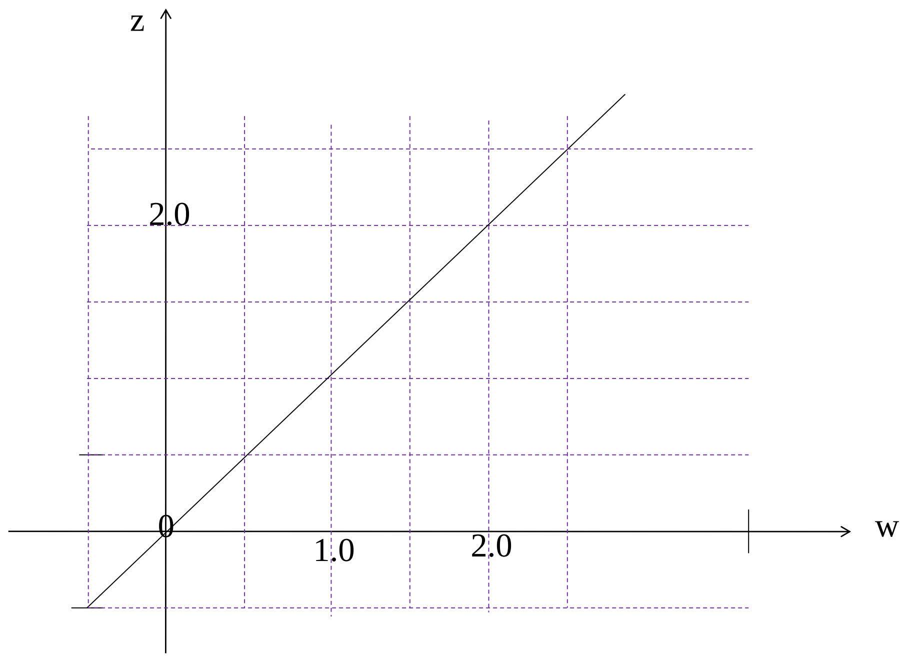
number of points: 1

(f) For the cases discussed in (d) and (e) draw qualitatively the trajectory of the synaptic weight $w(t)$ as a function of time. Use the space here:

number of points: 2

(g) At time $t = 100$, a pairing protocol is applied which corresponds to a 'Hebbian' input $H = 1.5$ which is sustained for a long time.

Redraw qualitatively the two nullclines in the graph here:



number of points: 2

(h) Interpret your results: What happens qualitatively to the nullclines and the fixed points? What does it mean for induction of synaptic plasticity?

....

....

....

....

....

....

number of points: 2

(i) In the above graph, draw qualitatively a trajectory starting at (0,0). Please use $\epsilon \ll \gamma$.

number of points: 2

Free space for your calculations, do not use to write down solutions/answers.

3 Mean-field models (8 points)

Consider a homogeneous network of N neurons. Each neuron has the same parameters and receives $K \leq N$ inputs from other neurons. An input spike causes a postsynaptic potential $w_0 \epsilon(s)$ with two rectangular phases: $\epsilon(s) = +3mV$ for $0 < s < 1$ms and $\epsilon(s) = -4mV$ for $1 < s < 2$ms. At all other times $\epsilon(s) = 0$. Note that $s = 0$ corresponds to the spike arrival time. The parameter w_0 is positive ($w_0 > 0$).

The total input potential of neuron i is

$$u_i(t) = u^{\text{ext}} + \sum_k \sum_f w_0 \epsilon(t - t_k^f) \tag{4}$$

where $u^{\text{ext}} \geq 0$ denotes external input to the network and the sums run over all presynaptic neurons and all firing times, respectively.

Each neuron emits spike trains with a rate of 10Hz if the input potential is below 10mV and spike trains with a rate of 80Hz if the input potential is above 12mV. The firing rate increases linearly in between.

Assume that connections are random in the following sense: in a network of N neurons, **each neuron receives exactly $K = N/2$ inputs. Take** $w_0 = 100\, J_0/K$ and assume that N is larger than 10 000 (formally you can assume $N \to \infty$).

(a) Assume stationary asynchronous firing and determine the population activity graphically. Consider four cases: $u^{ext} = 5mV$ and $u^{ext} = 20mV$ and choose two different values of $J_0 > 0$.

(space for your graphics here)

Complete your graphics on the previous page and write a short comment on your result:

....

....

number of points: 4

(b) Find the solutions for $u^{ext} = 20mV$ analytically, as a function of the parameter $J_0 > 0$. (Hint: there are three different regimes).

(space for your calculations here)

If $J_0 <$ .... then ....

....

If $J_0 >$ .... and $J_0 <$........ then ....

....

If $J_0 >$ .... then ....

....

number of points: 4

4 Stochastic Spike Arrivals and Spiking Probability (10 points + 4 bonus points)

Consider a neuron in a network that receives K inputs from other neurons. An input spike at time $s = 0$ causes a postsynaptic potential $w_0\epsilon(s)$ with two rectangular phases: $\epsilon(s) = +3mV$ for $0 < s < 1$ms and $\epsilon(s) = -4mV$ for $1 < s < 2$ms. At all other times $\epsilon(s) = 0$.

The total input potential of neuron i is

$$u(t) = \sum_k \sum_f w_0\epsilon(t - t_k^f) \tag{5}$$

where the sums run over all presynaptic neurons and all firing times, respectively.

In the following we assume that all presynaptic neurons fire spikes stochastically (i.e., a Poisson process), each neuron with rate ν.

(a) Determine the mean input potential of the postsynaptic neuron. (assuming K presynaptic neurons, each one firing at rate ν)

$< u >=$

....

number of points: 2

(b) Evaluate the mean input potential for $K = 1000; \nu = 1Hz; w_0 = 1$. Pay attention to the units

$< u >=$

....

number of points: 1

(c) Determine the variance of the potential of the postsynaptic neuron (assuming K presynaptic neurons firing at rate ν where K and ν are arbitrary positive parameters)

$< u^2(t) >=$

....

number of points: 2

(d) Using your results of (a) and (c), determine the standard deviation of the membrane potential

$< (\Delta u(t))^2 >^{0.5} = \sqrt{< u^2(t) > - < u >^2} =$

....

number of points: 2

(e) Evaluate the standard deviation for $K = 1000; \nu = 1Hz; w_0 = 1$. Pay attention to the units

$< (\Delta u(t))^2 >^{0.5} =$

....

number of points: 1

(f) The postsynaptic neuron fires a spike as soon as the membrane potential hits the threshold $\vartheta = 4$mV from below. Consider initially $K = 1000, \nu = 1Hz, w_0 = 1$. Now suppose that we **rescale the weights as** $w_0 = 1000/K$ **and we increase the number of presynaptic neurons** K **from 1000 to 2000 or 4000.**

Does the likelihood of the postsynaptic neuron to fire a spike increase or decrease with K? (Justify your answer on the next page)

Increase or Decrease? ....

My justification is:

....

....

....

number of points: 2

(g) **Bonus:** Suppose the postsynaptic neuron fires a spike as soon as the membrane potential hits the threshold $\vartheta = 4$mV from below.

Take $K = 1000, \nu = 1Hz, w_0 = 1$.

Do you expect the neuron to fire a spike in an observation period of 100 milliseconds? Justify your answer.

....

....

My justification is (intuitive, mathematical, OR graphical)

....

....

....

number of points: 2

(h) **BONUS** Consider the autocorrelation $< \Delta u(t) \Delta u(t + \Delta) >$. Is the autocorrelation at $\Delta = 0.9ms$ positive, zero, or negative? (Give an intuitive, or graphical, or mathematical argument).

I predict that $< \Delta u(t) \Delta u(t + \Delta) >$ is ....

because, intuitively, ...

....

....

number of points: 2