

Biological Modeling of Neural Networks

EPFL

Week 13

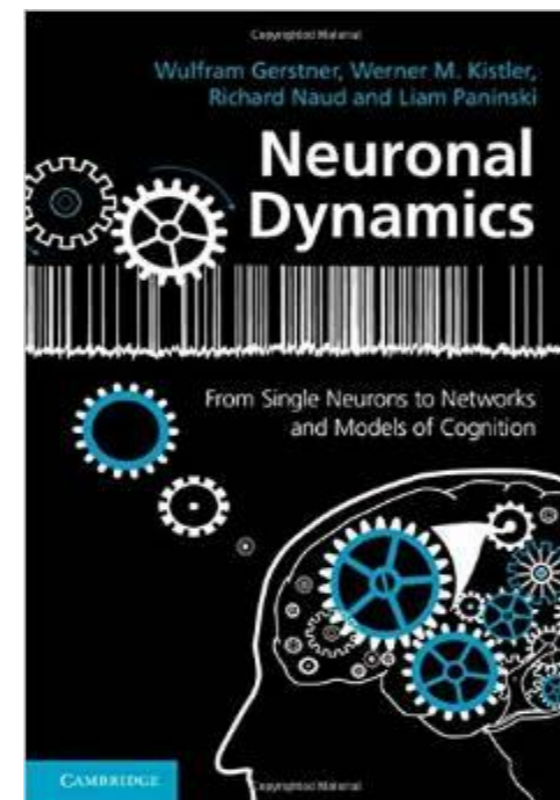
Synaptic plasticity and Learning

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Reading for plasticity:
NEURONAL DYNAMICS
- Ch. 19.1-19.3

Cambridge Univ. Press



1. Synaptic plasticity

motivation and aims

2. Classification of plasticity

short-term vs. long-term

unsupervised vs. reward modulated

3. Model of short-term plasticity

4. Models of long-term plasticity

- Hebbian learning rules

- Bienenstock-Cooper-Munro rule

5. Spiking Models of plasticity

- STDP as Hebbian learning

- Model of STDP: synaptic traces

6. From STDP to rate models

7. Triplet STDP model

8. Online learning of memories

1. Behavioral Learning – and Memory

Learning actions:

→ riding a bicycle

Remembering facts

→ previous president of the US

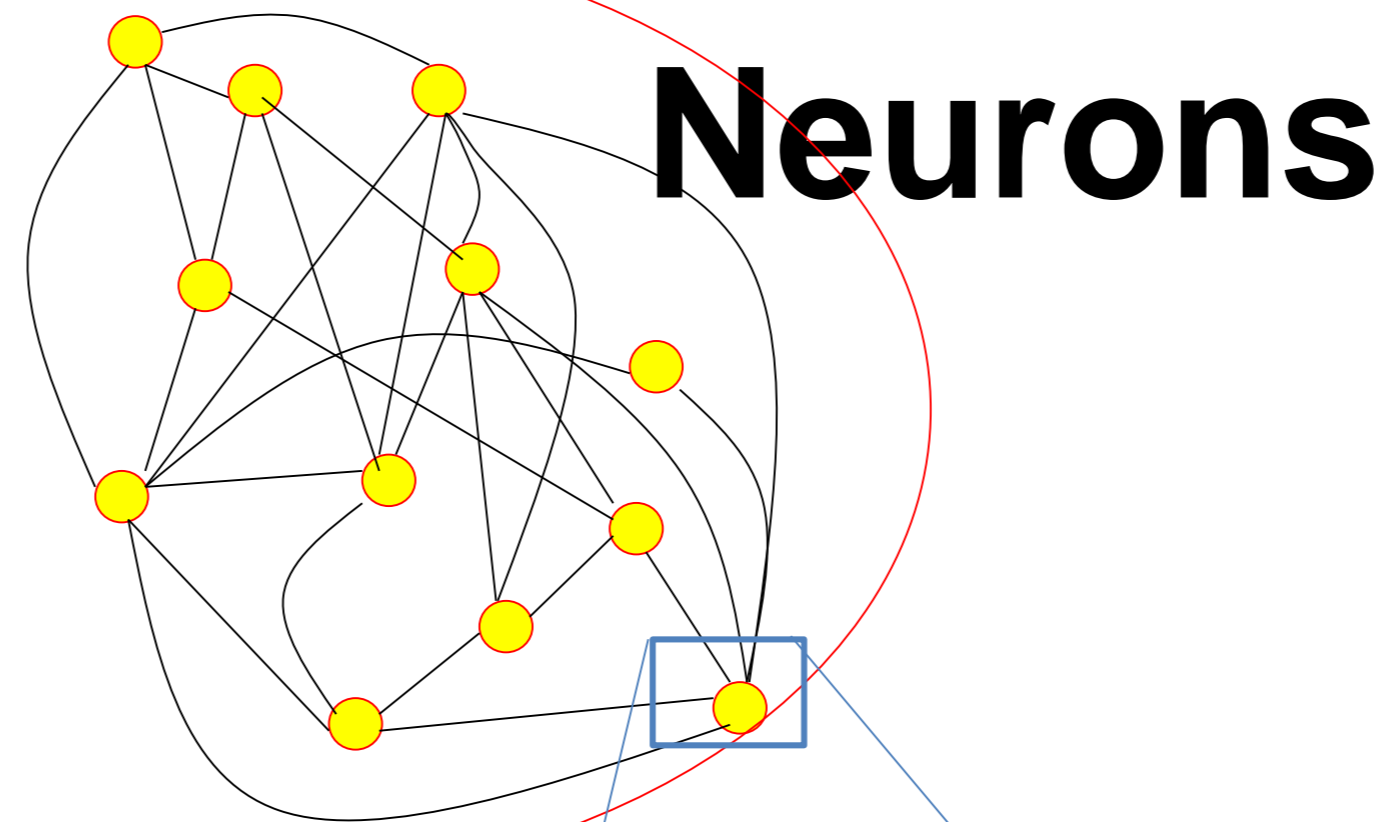
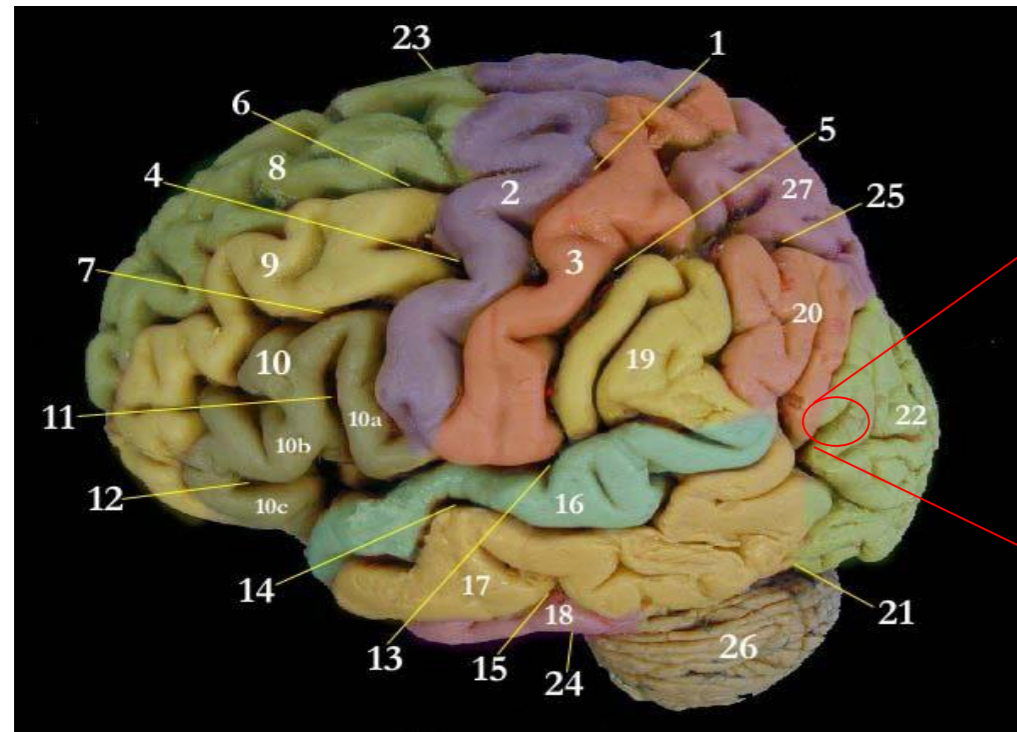
→ name of your mother

Remembering episodes

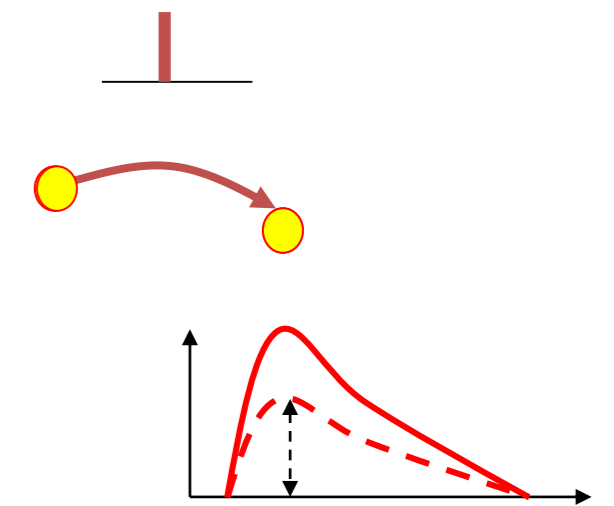
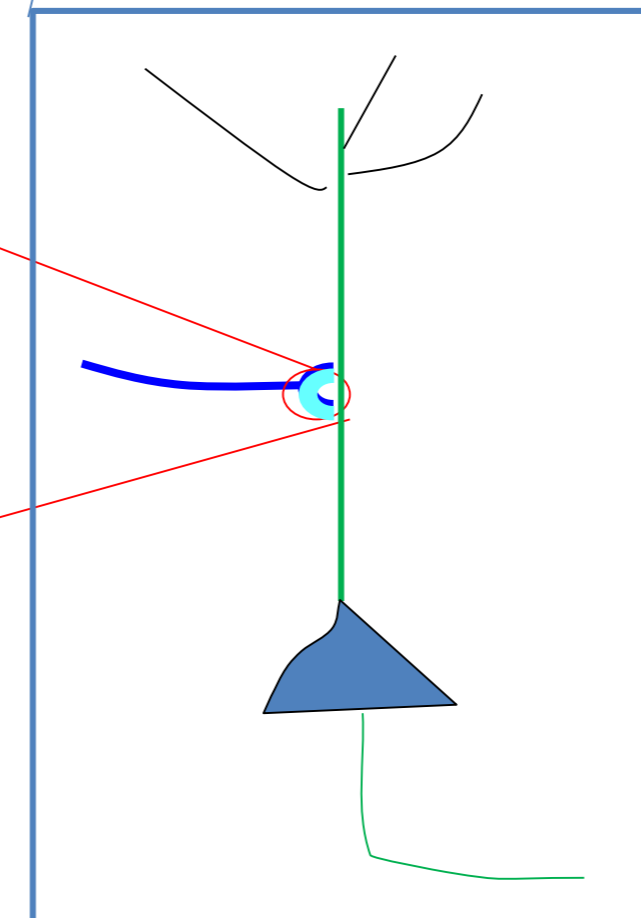
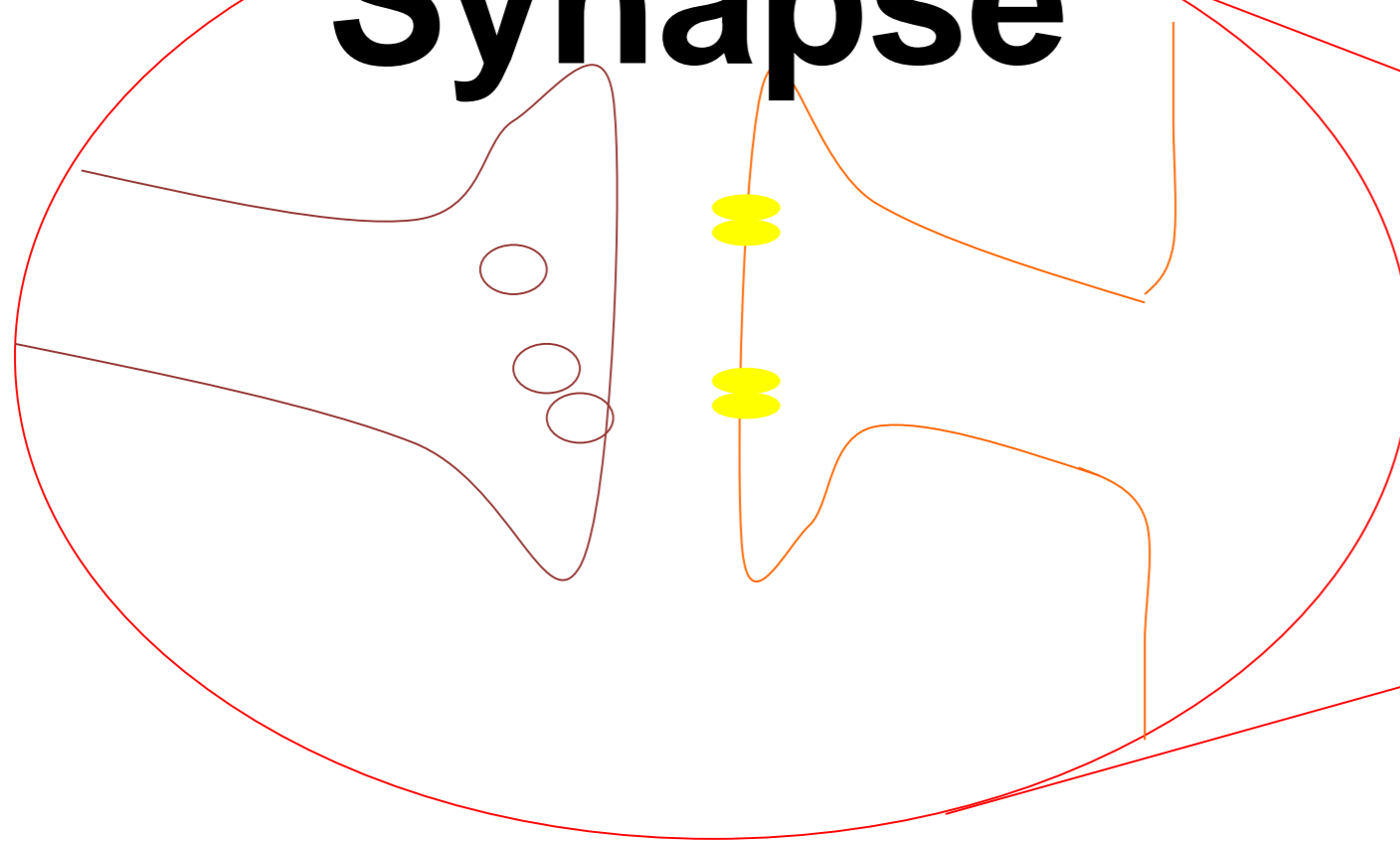
→ first day at EPFL

which parking spot?

1. Behavioral Learning – and synaptic plasticity

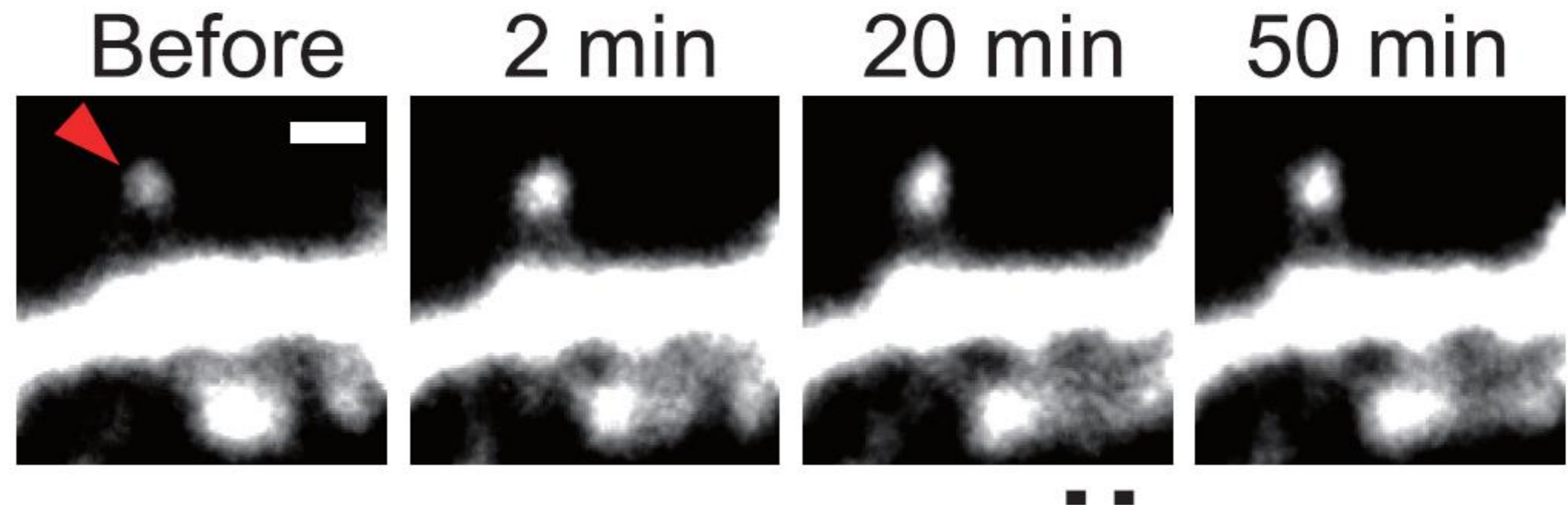
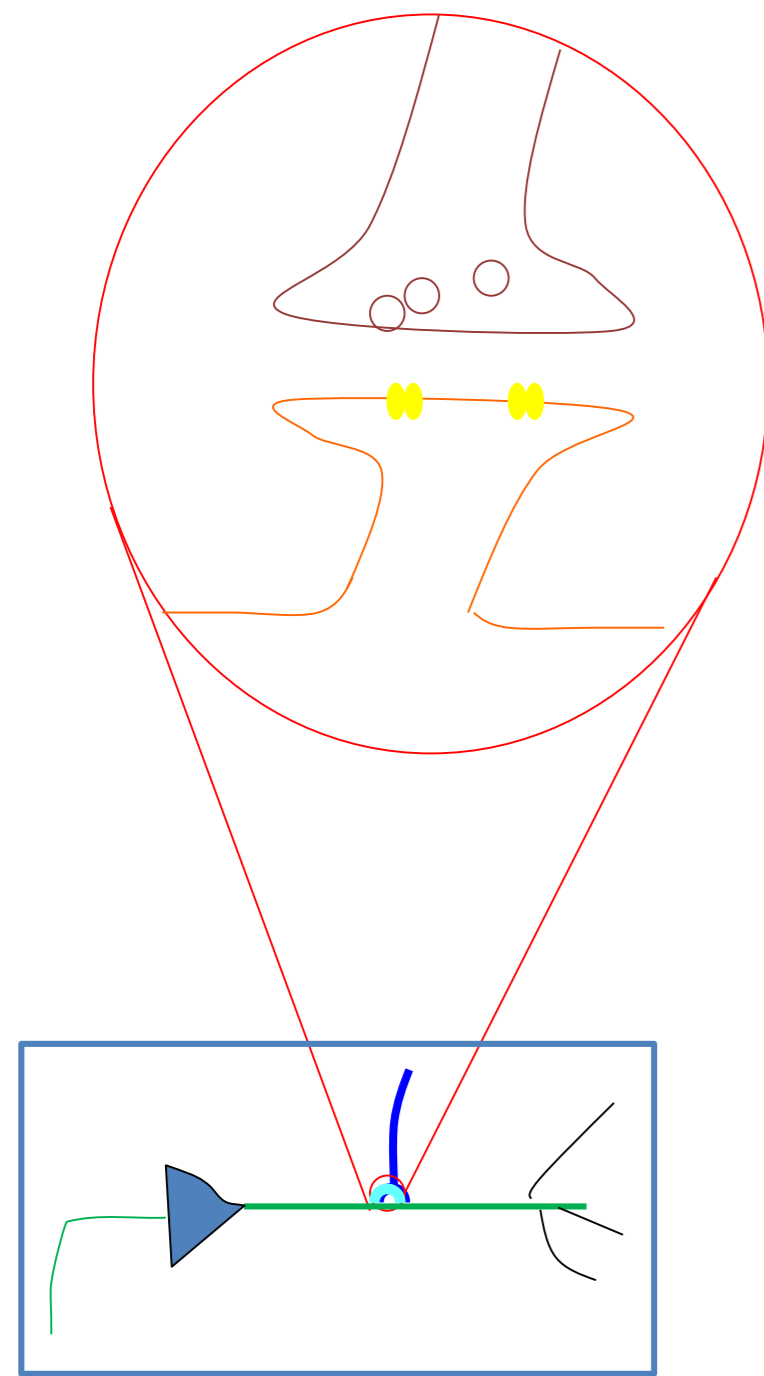


Synapse



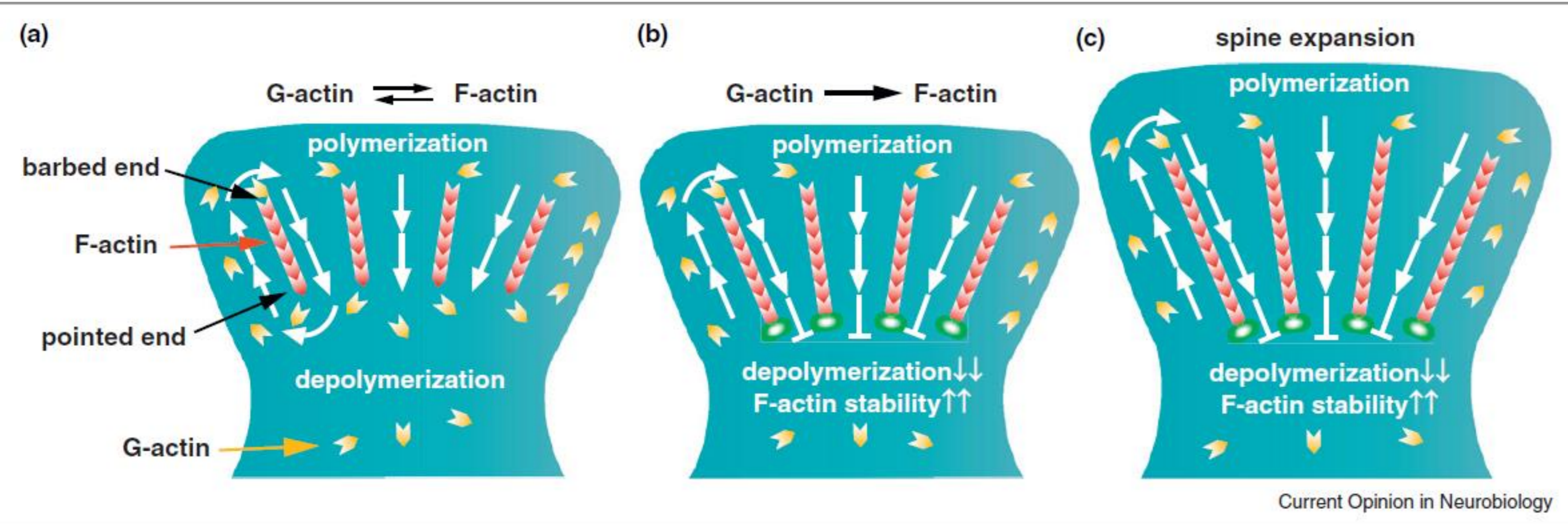
Synaptic Plasticity = Change in Connection Strength

1. Synaptic plasticity – structural changes

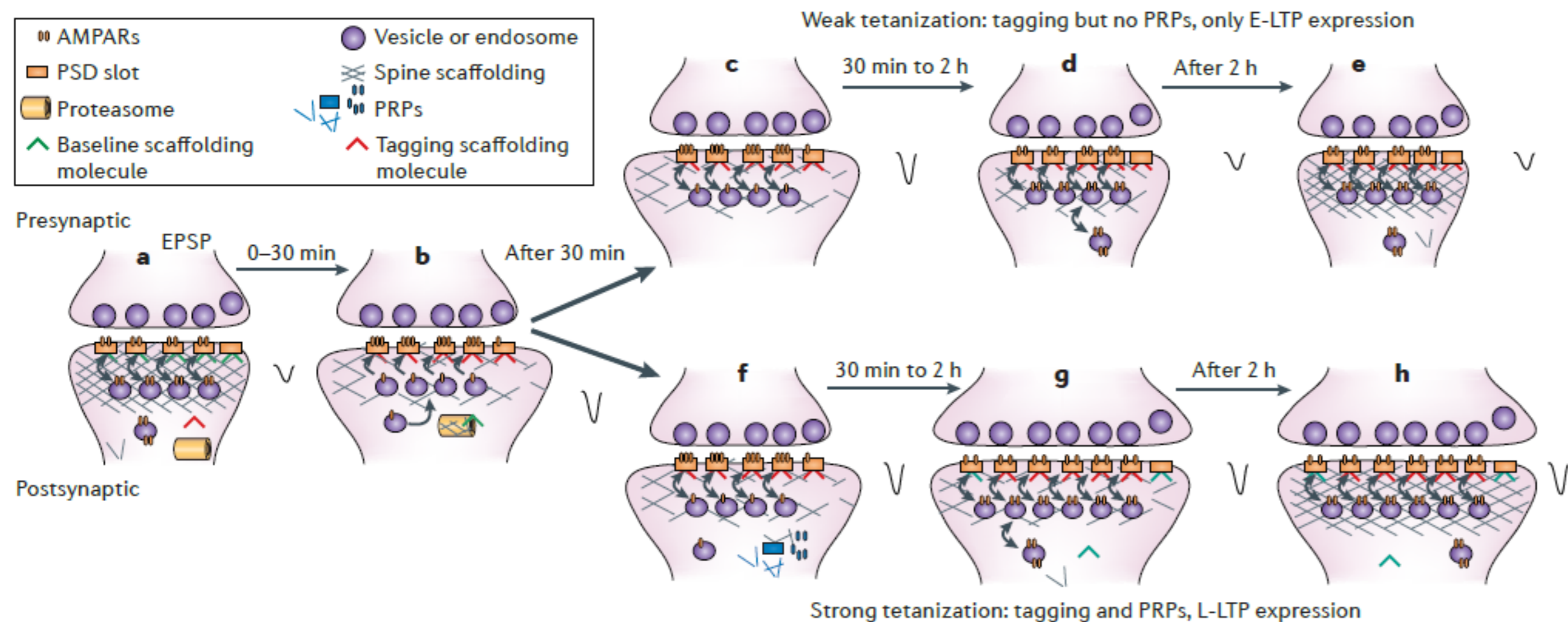


Yagishita et al.
Science, 2014

1. synaptic plasticity – molecular changes

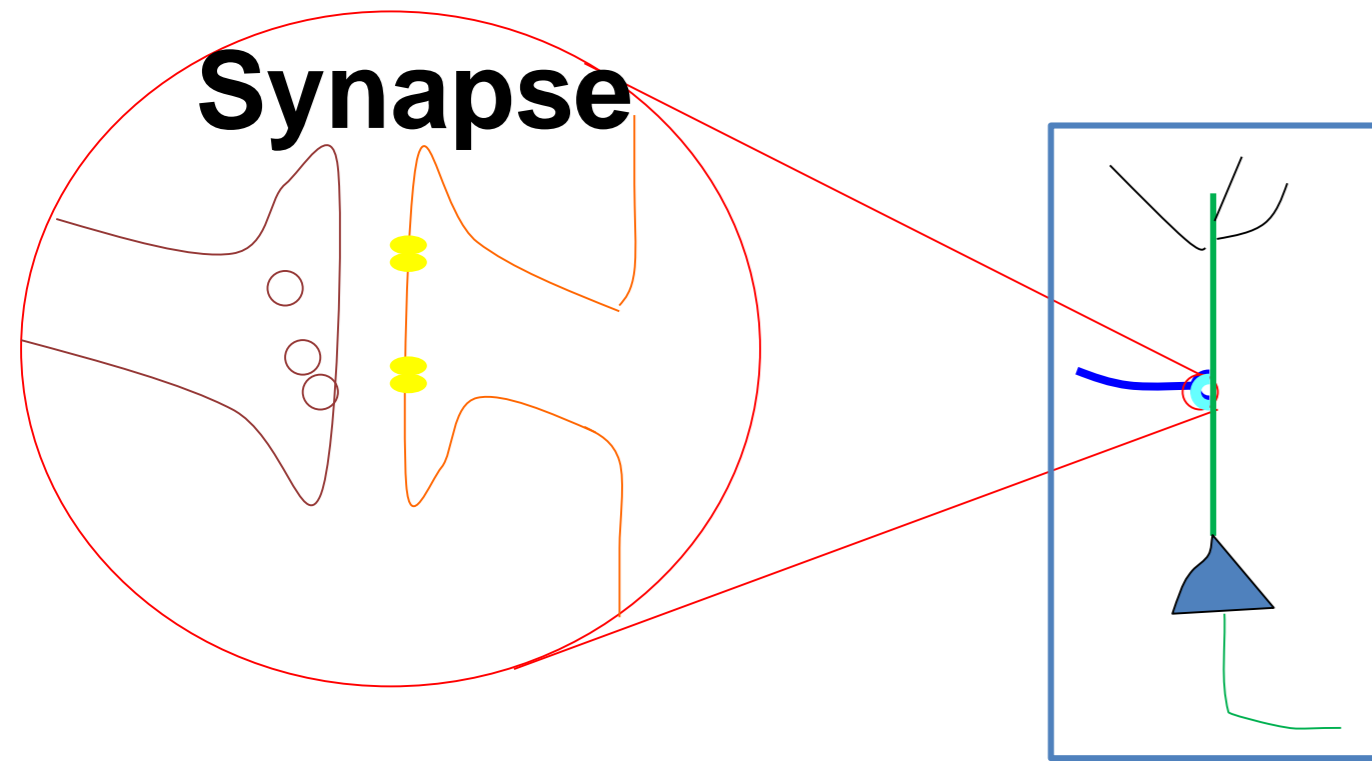


*Bosch et al. 2012,
 Curr. Opinion Neurobiol.*



*Redondo and Morris 2011,
 Nature Rev. Neurosci.*

1. synaptic plasticity – connections change

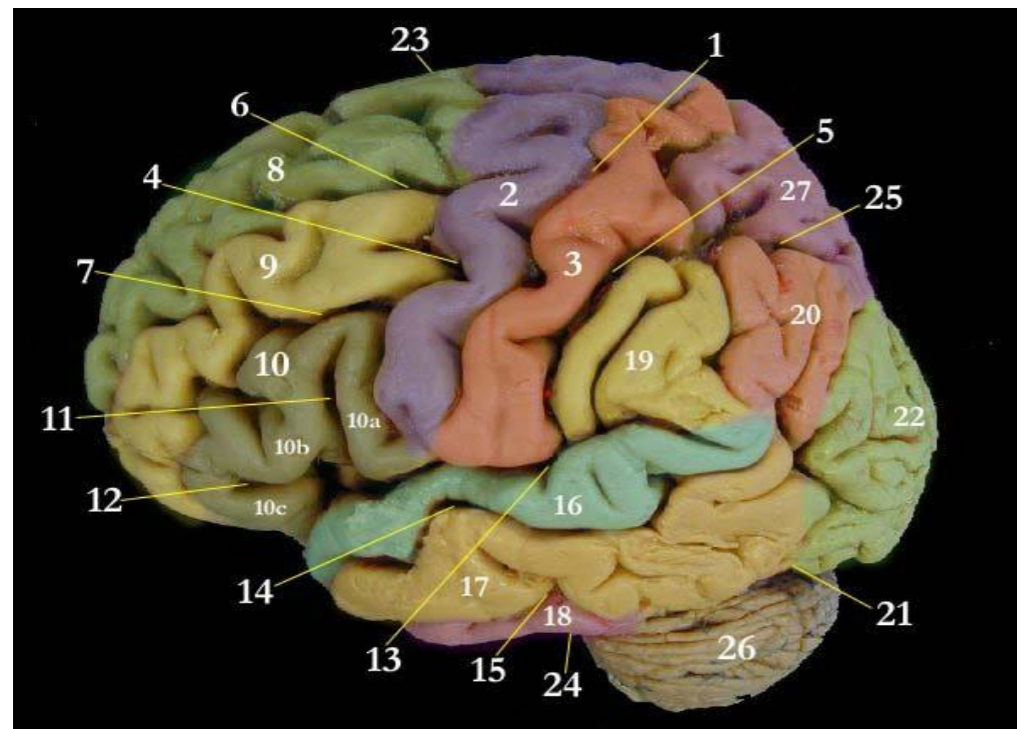


More space in cortex allocated
- musicians vs. non-musicians

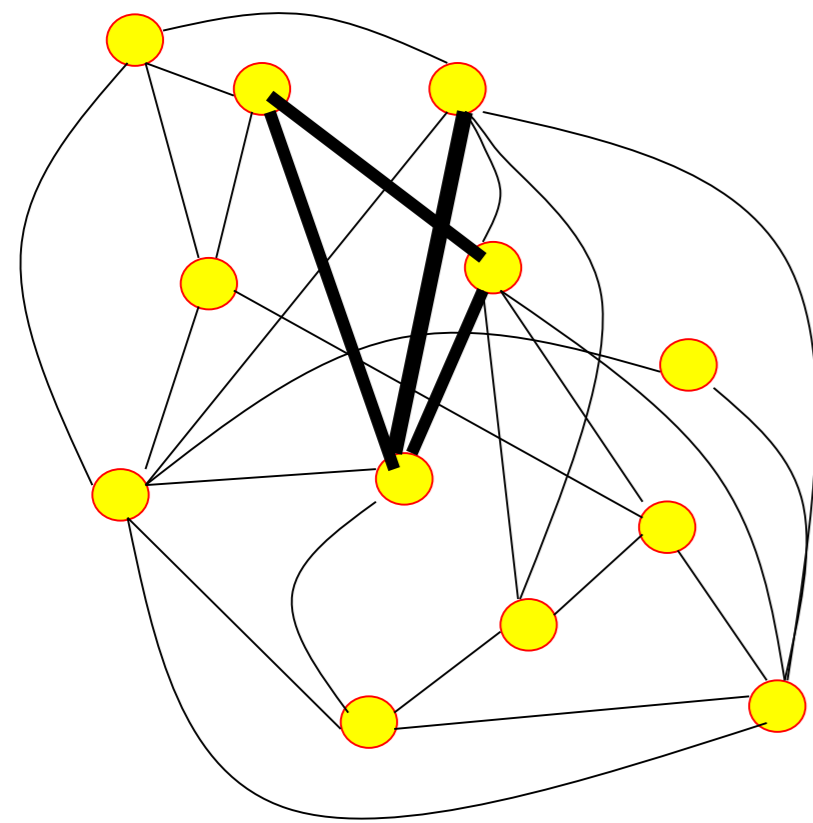
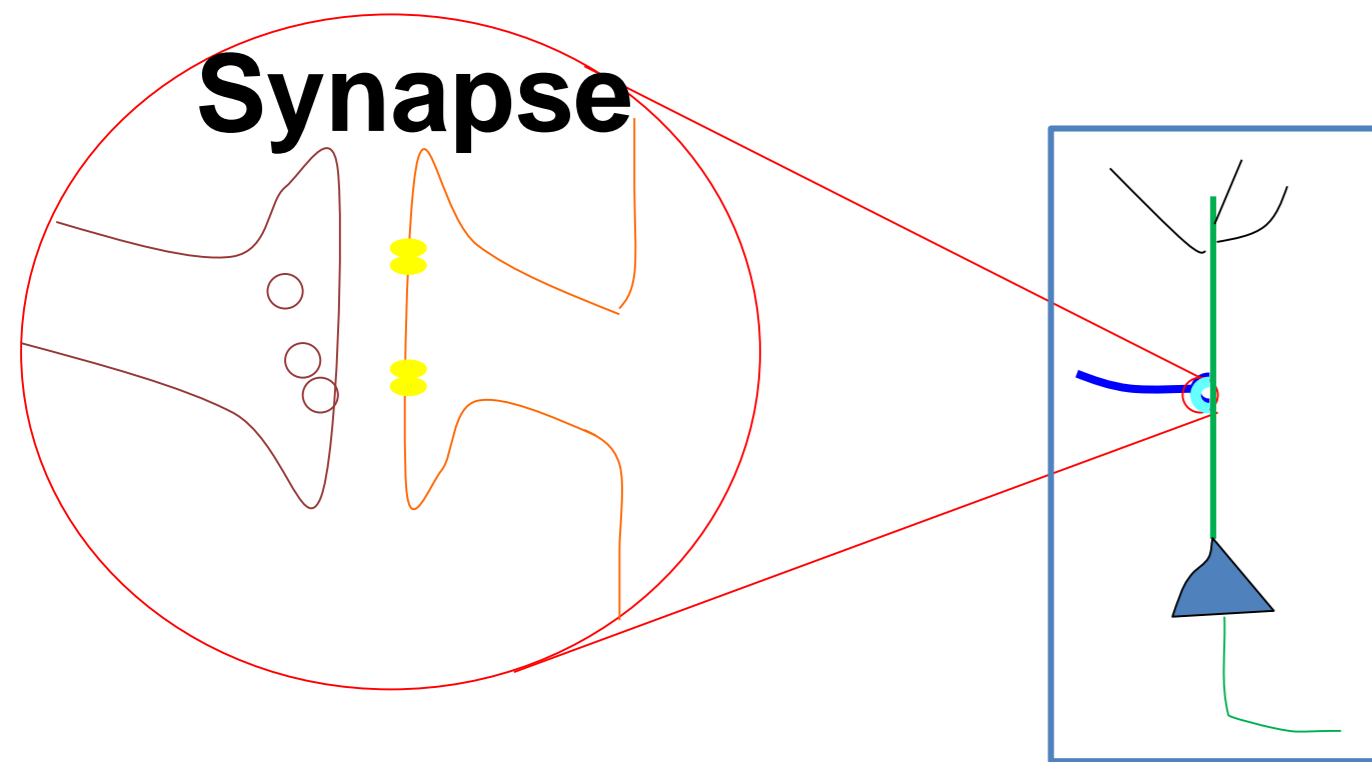
Amunts et al. Human Brain Map. 1997
Gaser and Schlaug, J. Neurosci. 2003

More space in hippocampus allocated
- London taxi driver vs bus driver

Macquire et al. Hippocampus 2006



1. Synaptic plasticity



Should enable **Learning**

- adapt to the statistics of task and environments
(receptive fields, allocate space etc)
- memorize facts and episodes
- learn motor tasks

Should avoid:

- blow-up of activity **homeostasis**
- unnecessary use of energy

Aim: models that capture the essence

1. Synaptic plasticity: program for this week

-Hebbian Learning

- Experiments on synaptic plasticity
- Mathematical Formulations of Hebbian Learning
- Back to Attractor Memory Models

1. Synaptic plasticity: summary

Synaptic plasticity (= changes of synaptic contact points) are the basis of learning.

Learning is necessary for a variety of different tasks.

Learning leads to measurable changes in performance (you get better at a task) and to measurable changes in the brain.

As an example of a synaptic plasticity rule, we consider Hebbian learning first.

We start with some experimental data, before move on to the mathematical formulation.

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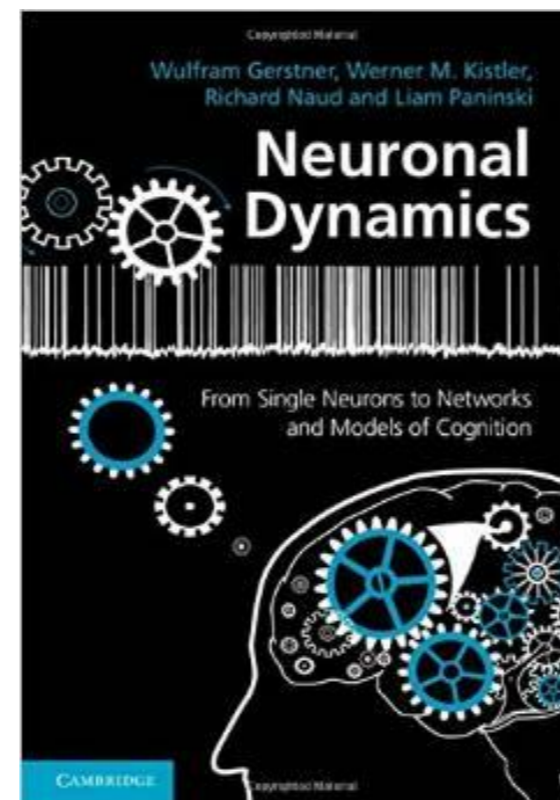
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5. Spiking Models of plasticity

2. Classification of synaptic changes: Short-term plasticity

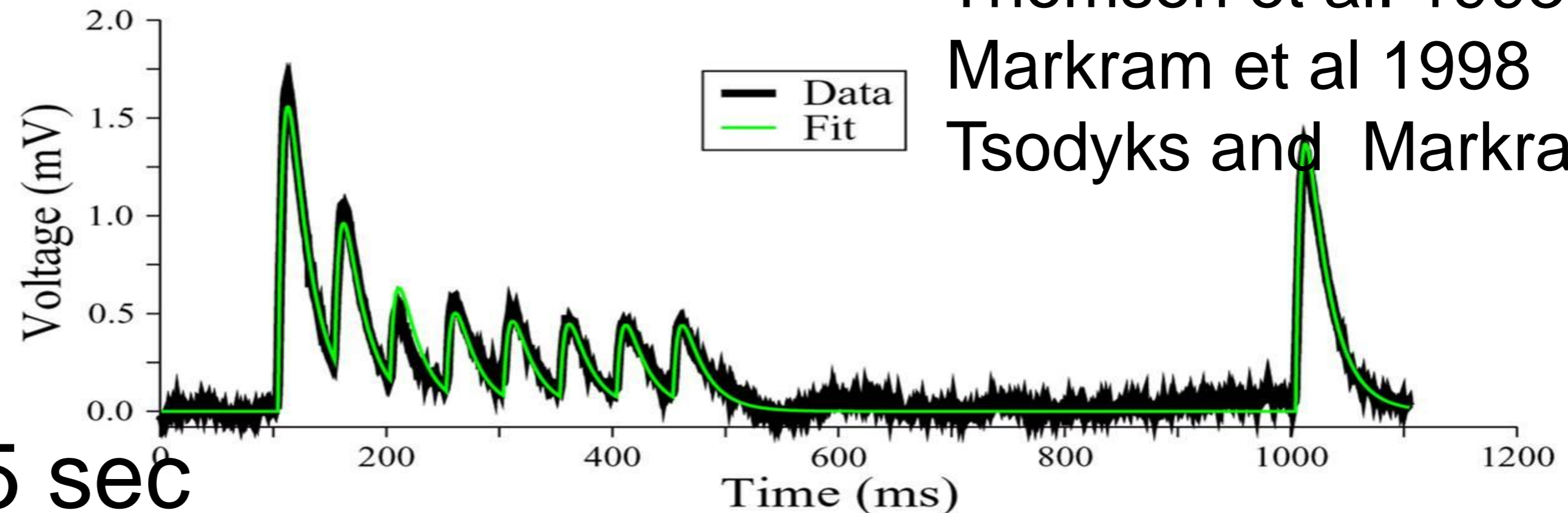


Short-term plasticity/fast synaptic dynamics

Thomson et al. 1993

Markram et al 1998

Tsodyks and Markram 1997



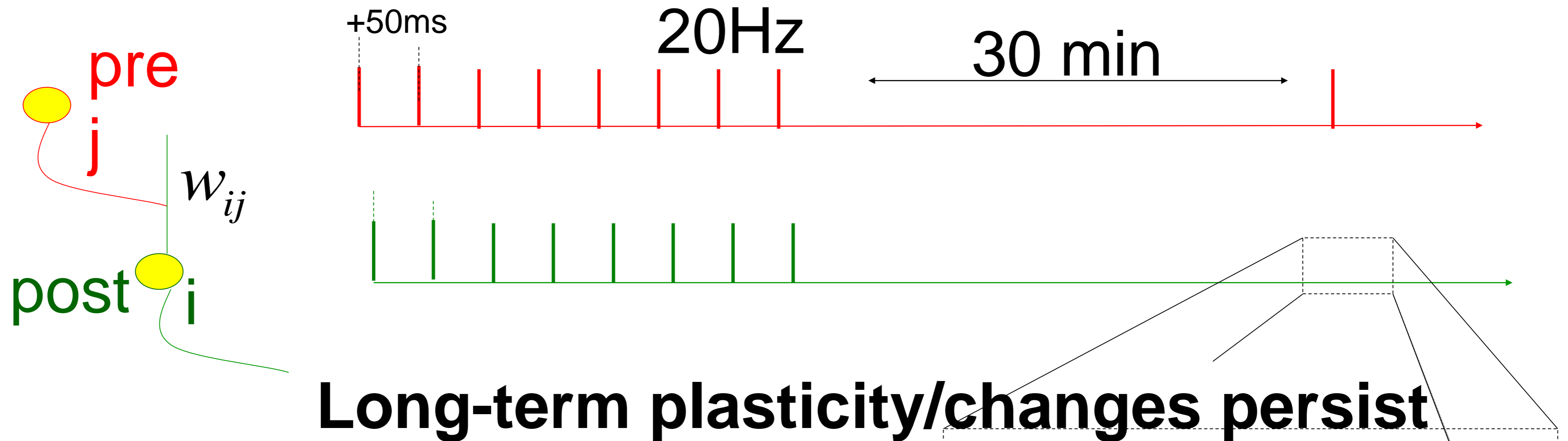
Changes

- induced over 0.5 sec
- recover over 1 sec

Data: Silberberg, Markram

Fit: Richardson (Tsodyks-Markram model)

2. Classification of synaptic changes: Long-term plasticity

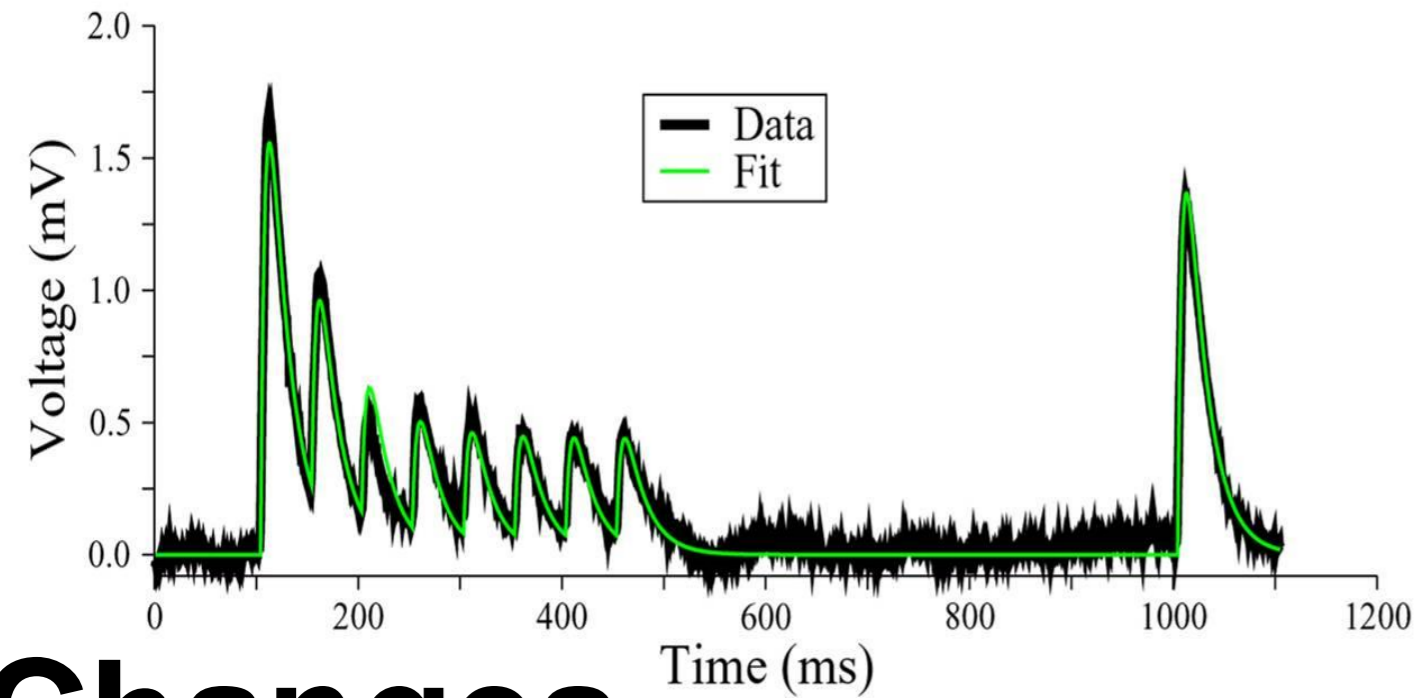


Changes

- induced over 3 sec (or longer?)
- persist over 1 – 10 hours

2. Classification of synaptic changes

Short-Term



Changes

- induced over 0.1-0.5 sec
- recover over 1 sec

Protocol

- presynaptic spikes

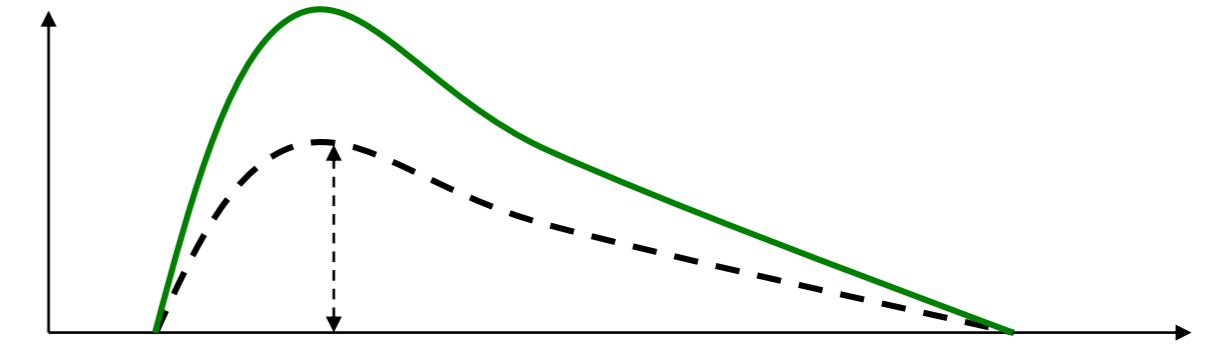
Model

- well established

(Tosdyks, Pawelzik, Markram
Abbott-Dayan)

vs/ Long-Term

LTP/LTD/Hebb



Changes

- induced over 0.5-5sec
- remains over hours

Protocol

- presynaptic spikes + ...

Model

- we will see

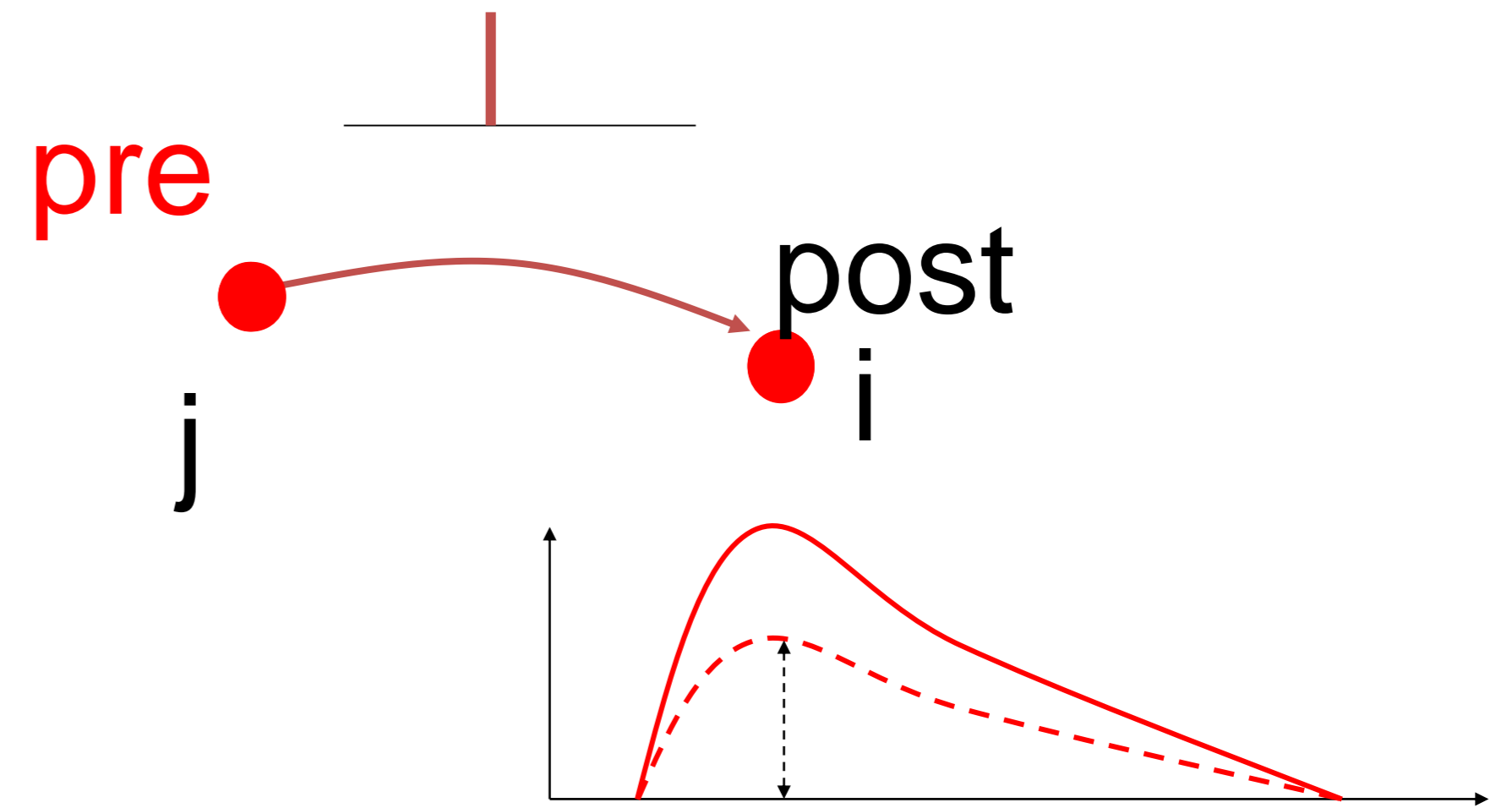
2. Classification of synaptic changes

Induction of changes

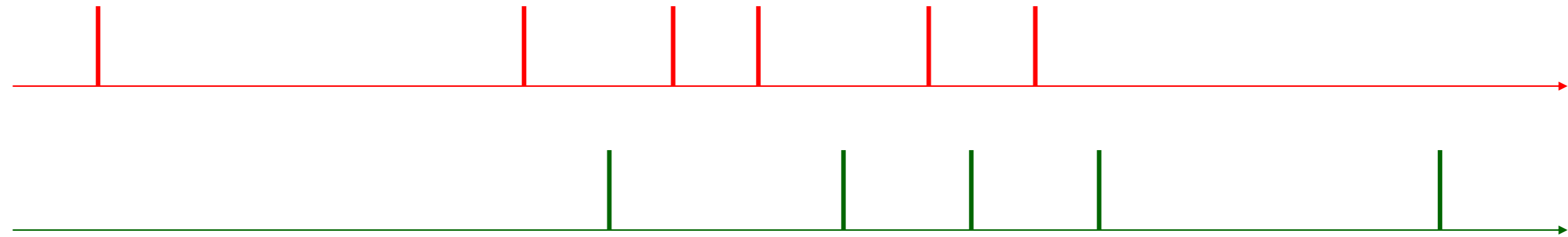
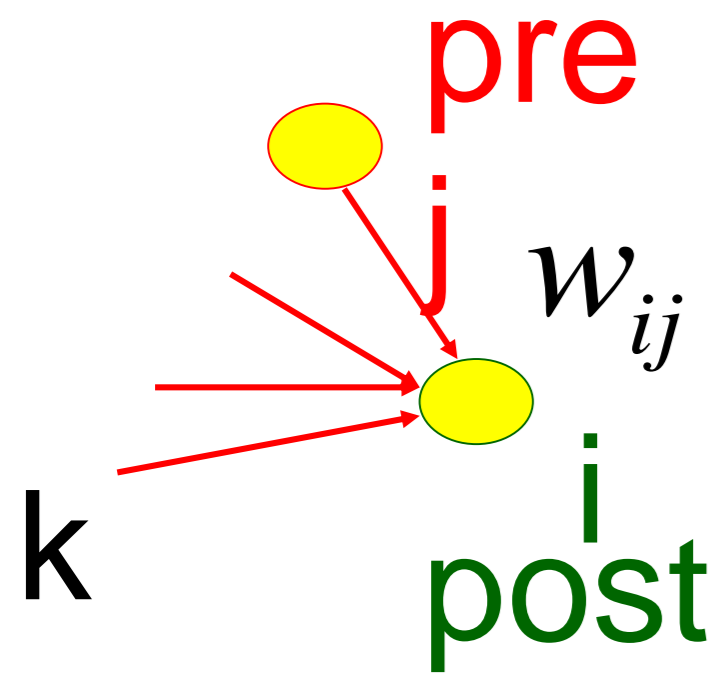
- fast (if stimulated appropriately)
- slow (homeostasis)

Persistence of changes

- long (LTP/LTD)
- short (short-term plasticity)



2. Review: Hebb rule



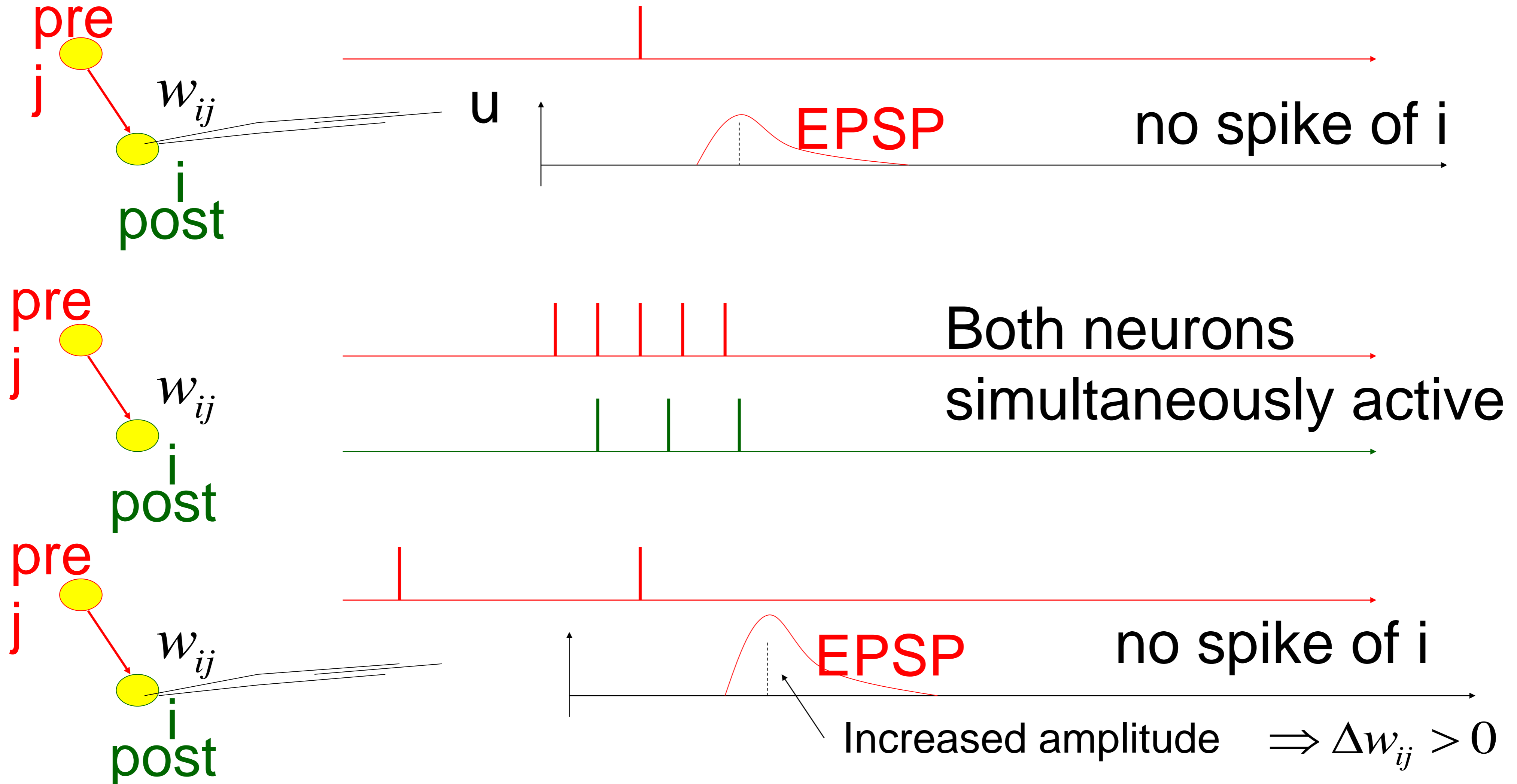
When an axon of cell **j** repeatedly or persistently takes part in firing cell **i**, then **j**'s efficiency as one of the cells firing **i** is increased

Hebb, 1949

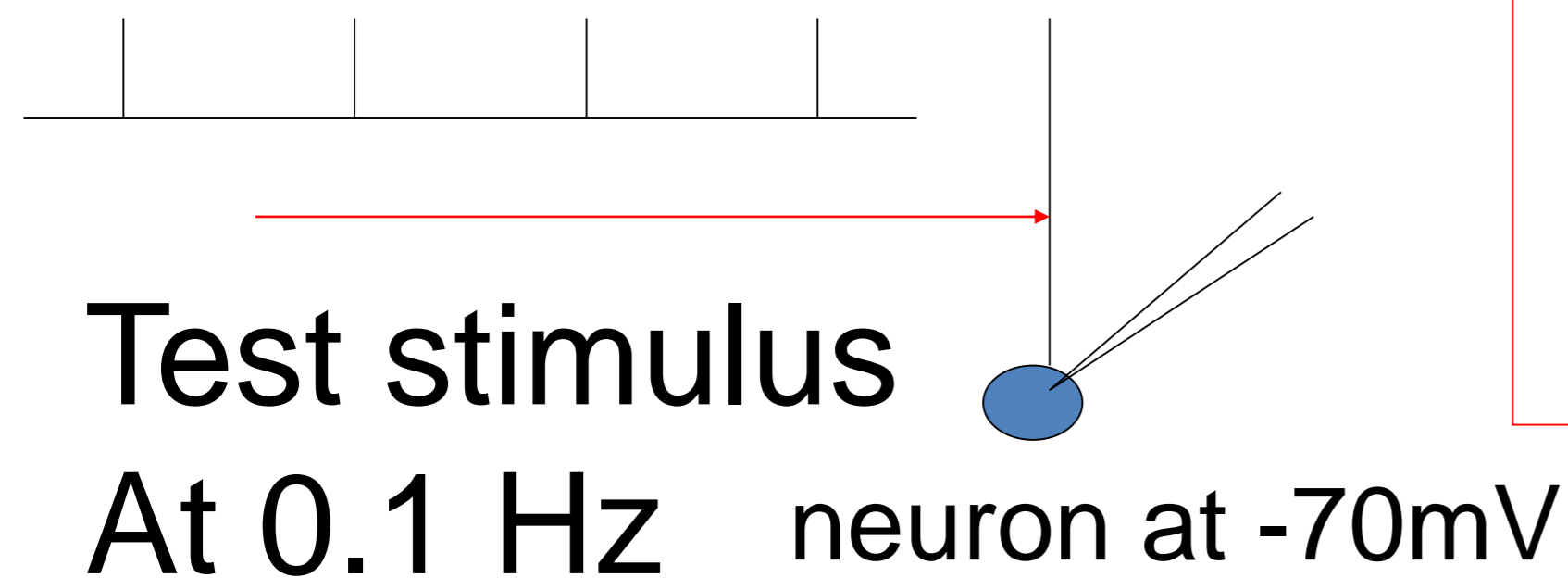
- local rule
- simultaneously active (correlations)

2. Synaptic plasticity: Long-Term Potentiation (LTP)

Hebbian Learning in experiments (schematic)



2. Classical paradigm of LTP induction – pairing



LTP induction:
tetanus at 100Hz



Standard LTP
PAIRING experiment

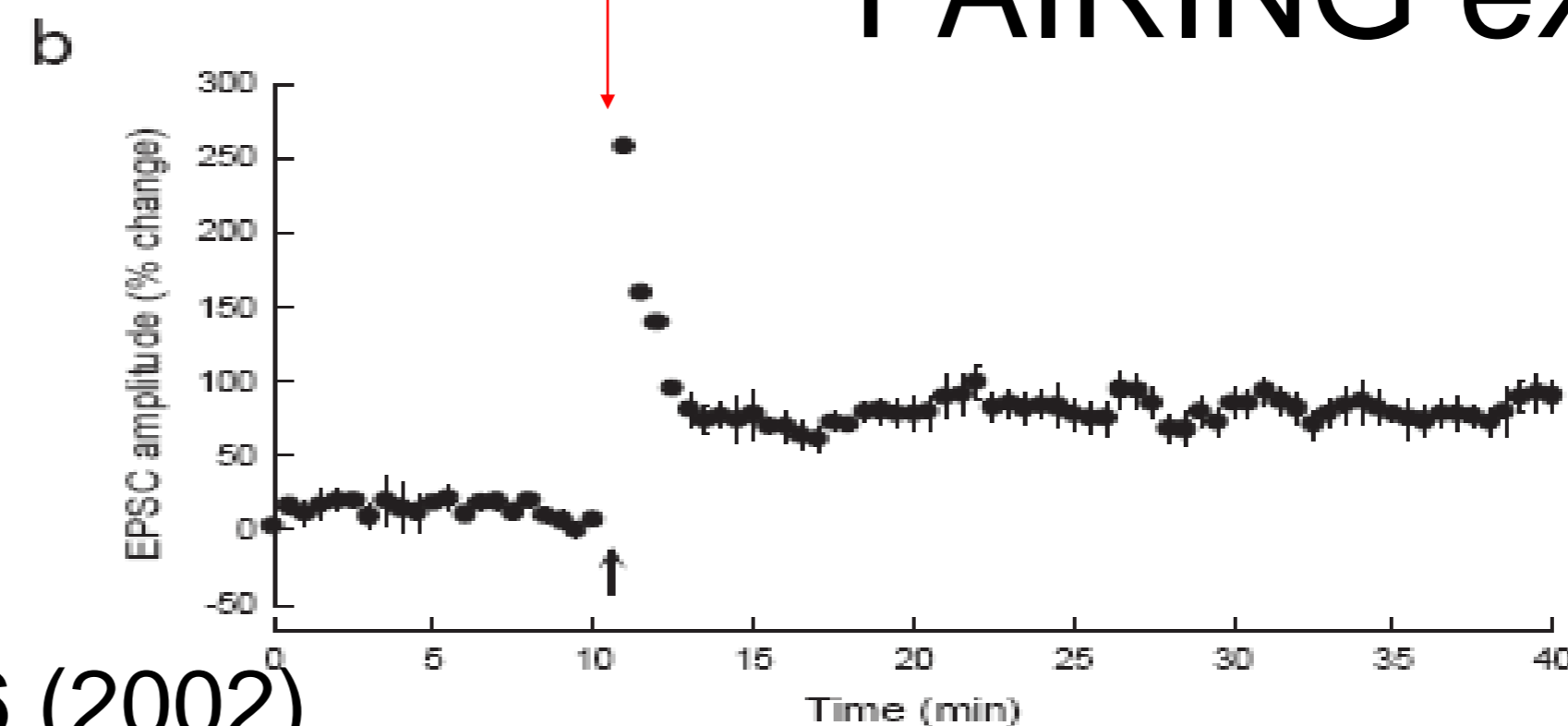
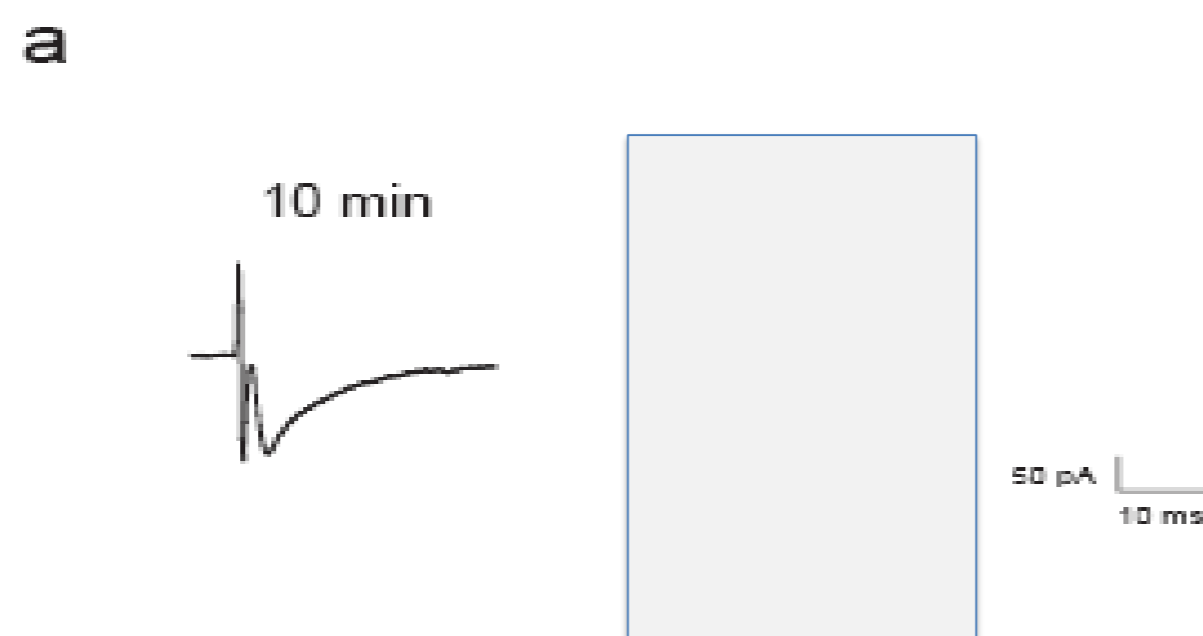
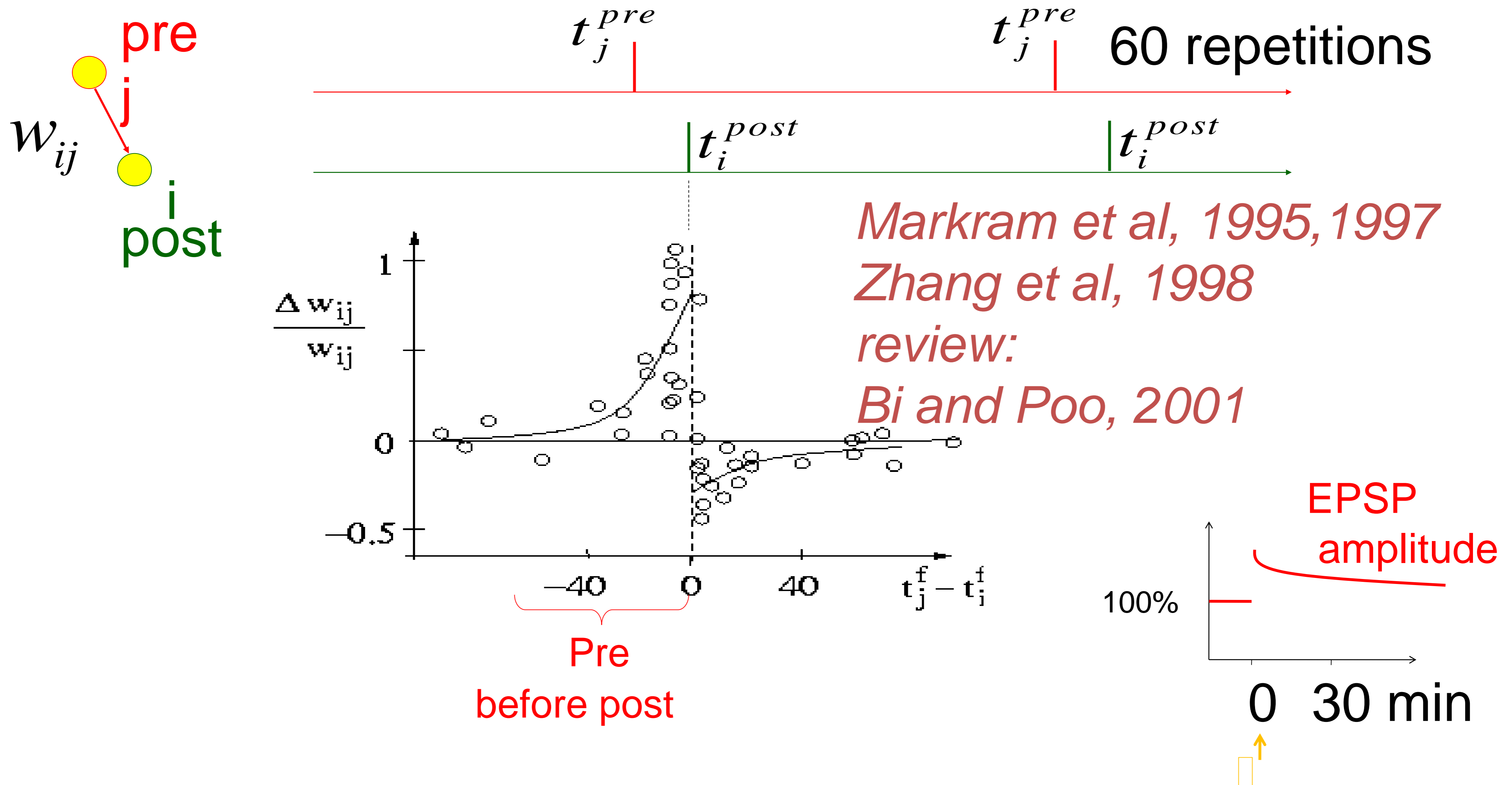


Fig. from Nature Neuroscience **5**, 295 - 296 (2002)

D. S.F. Ling, ... & Todd C. Sacktor

See also: Bliss and Lomo (1973), Artola, Brocher, Singer (1990), Bliss and Collingridge (1993)

2. Spike-timing dependent plasticity (STDP)



2. Classification of synaptic changes

Induction of changes

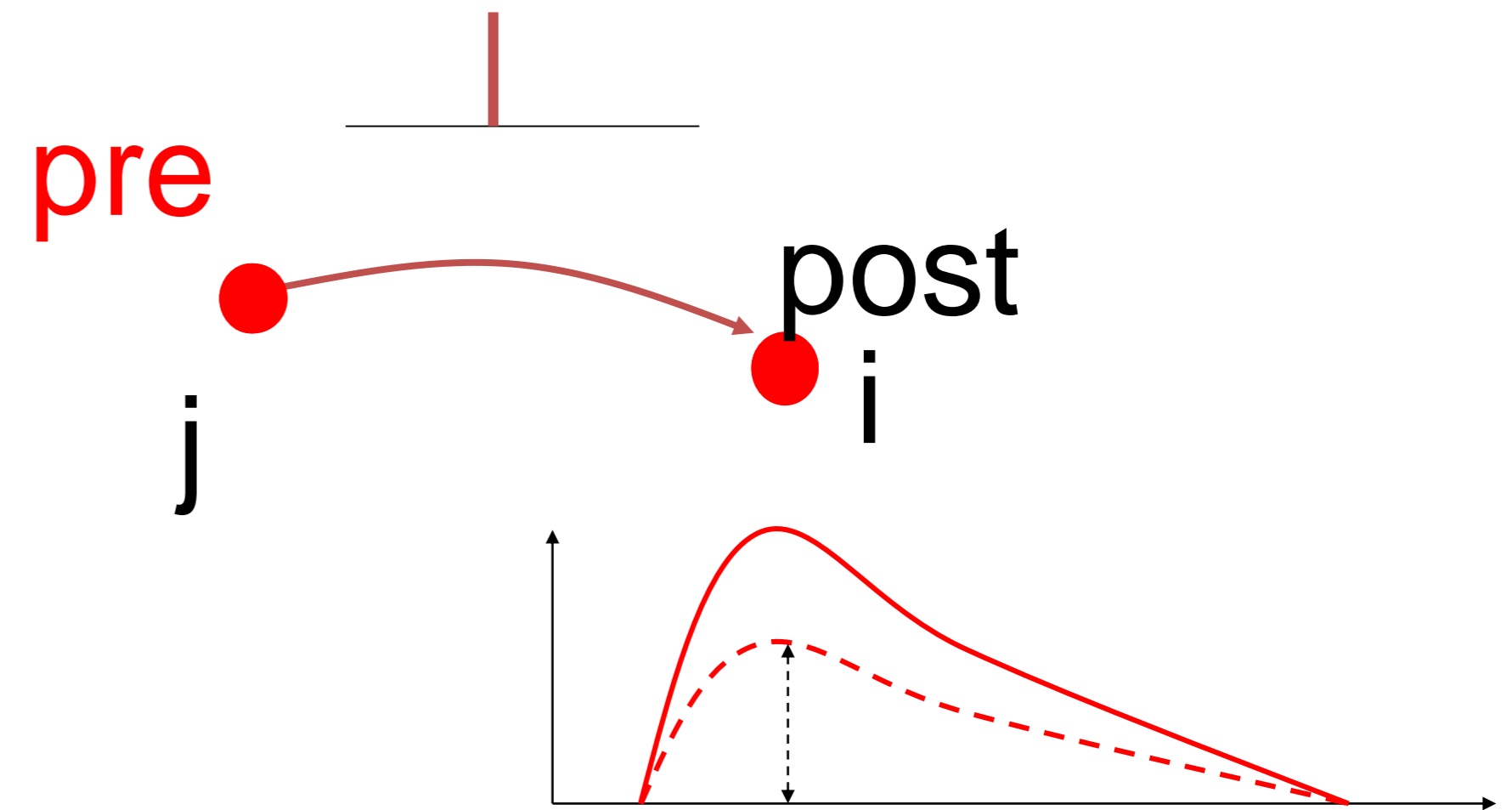
- fast (if stimulated appropriately)
- slow (homeostasis)

Persistence of changes

- long (LTP/LTD)
- short (short-term plasticity)

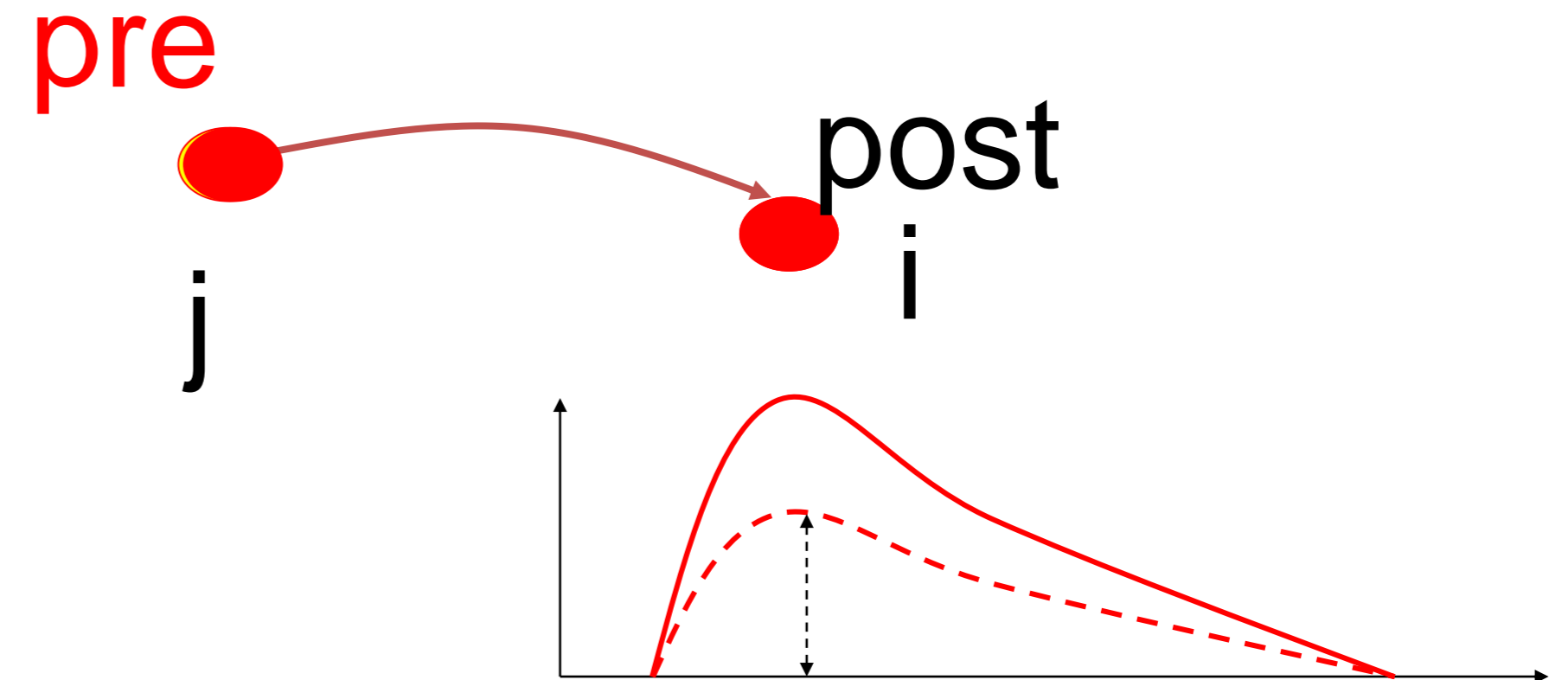
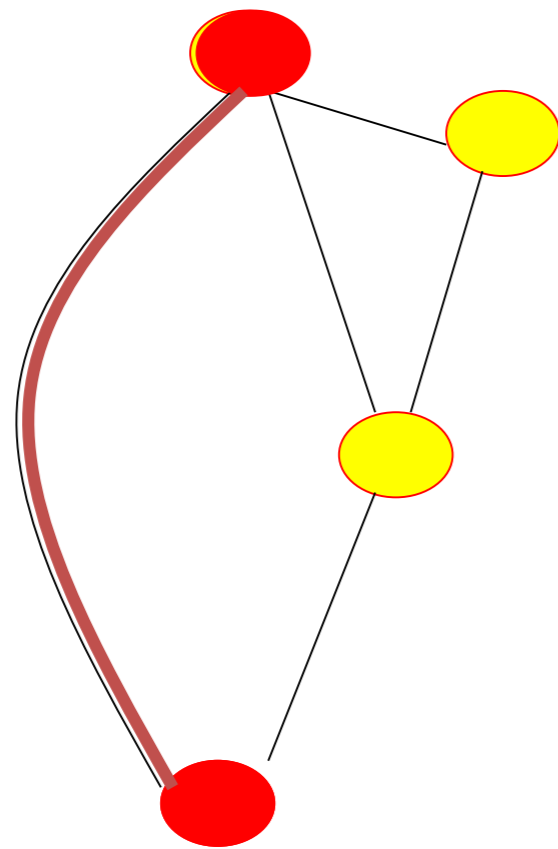
Functionality

- useful for learning a new behavior/forming new memories
- useful for development (wiring for receptive field development)
- useful for activity control in network: **homeostasis**
- useful for coding



2. Classification of synaptic changes: unsupervised learning

Hebbian Learning = unsupervised learning



$$w_{ij} \varepsilon(t - t_j^f)$$

$$\Delta w_{ij} \propto F(pre, post)$$

2.Limits of unsupervised learning

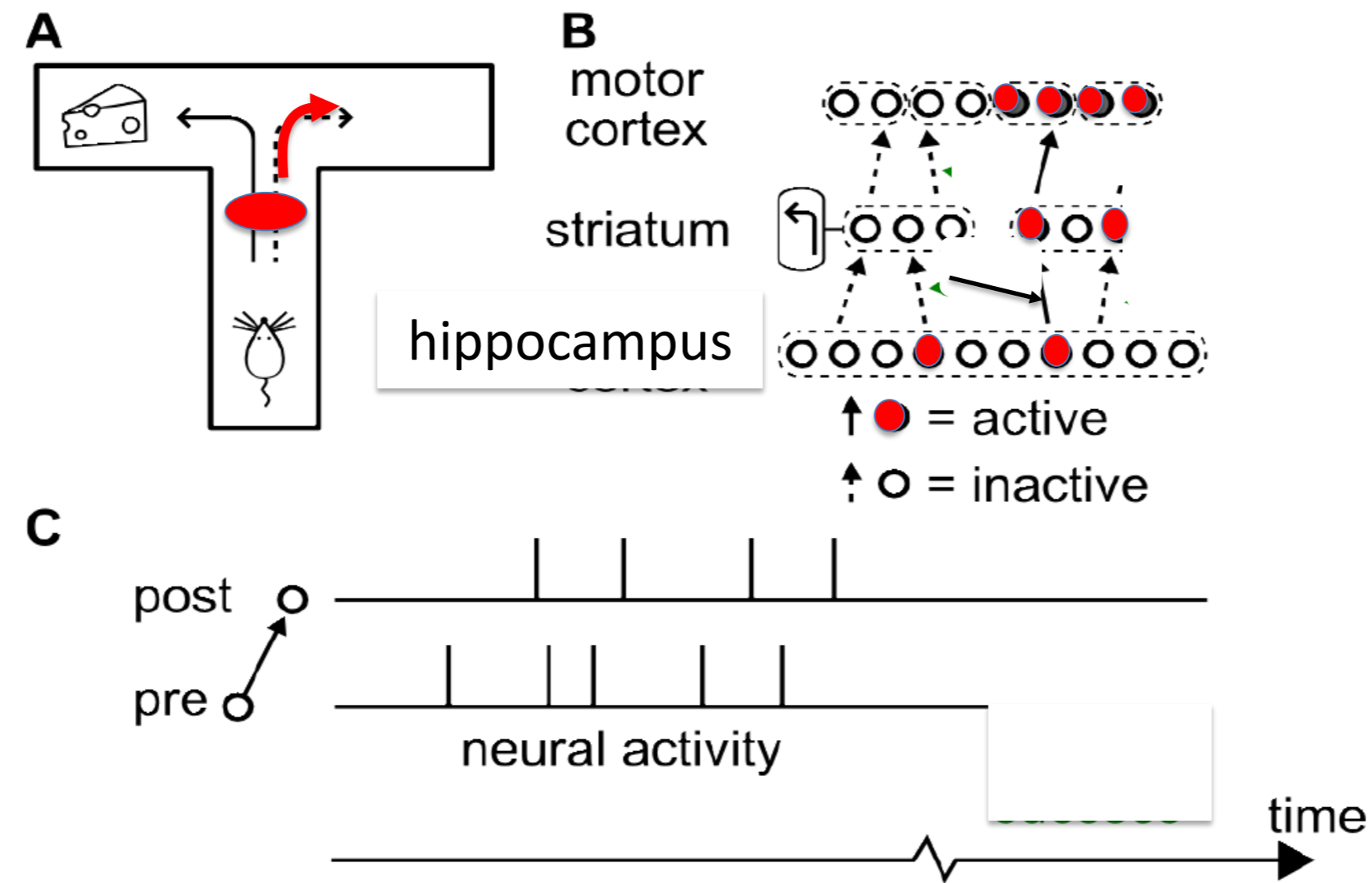
Is Hebbian Learning sufficient?

No!

Image: Gerstner et al. NEURONAL DYNAMICS,

Eligibility trace:
Synapse keeps memory of pre-post Hebbian events

Dopamine:
Reward/success

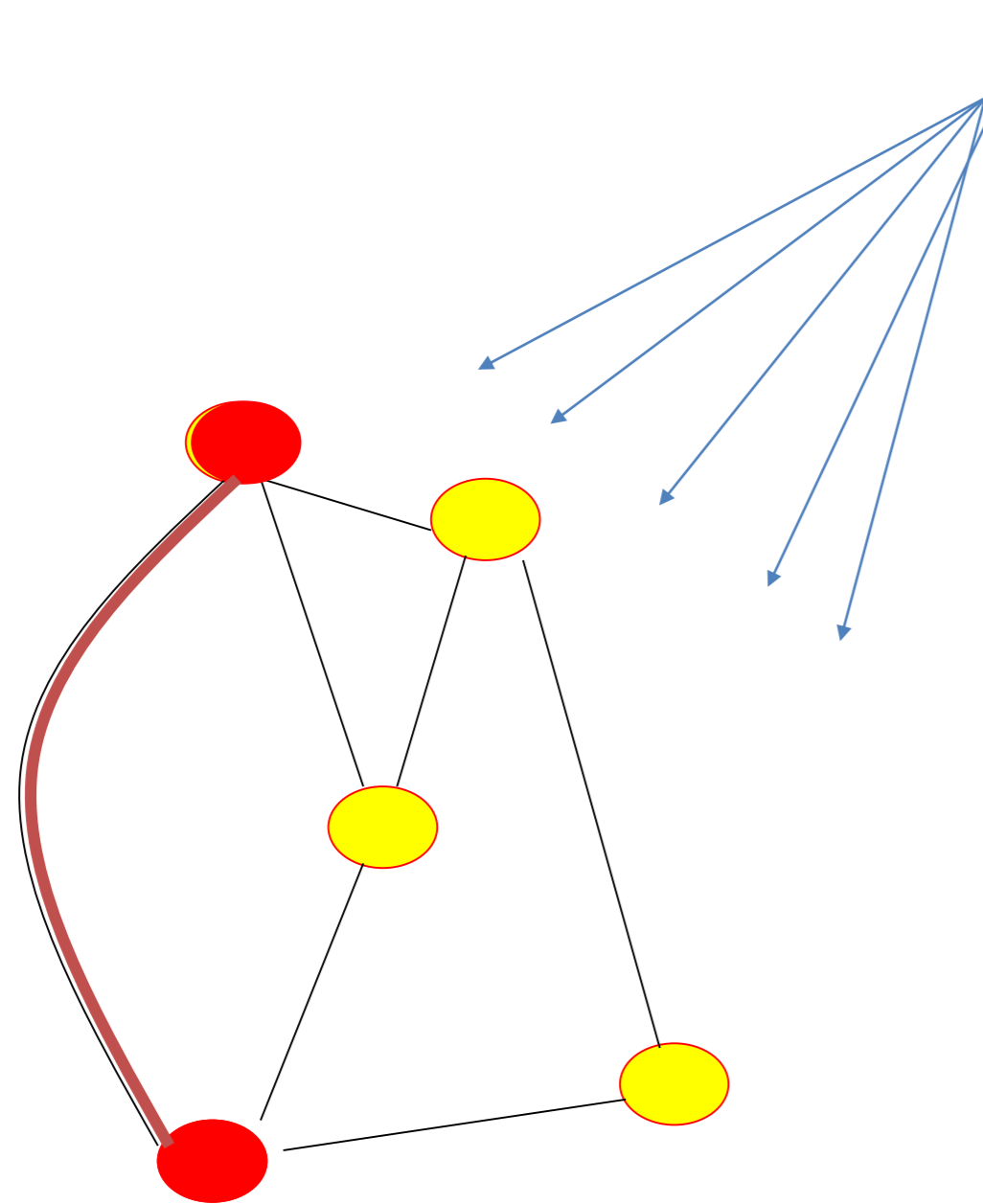


Schultz et al. 1997; Waelti et al., 2001;

→ Reinforcement learning: $\text{success} = \text{reward} - (\text{expected reward})$

TD-learning, SARSA, Policy gradient (book: Sutton and Barto, 1997)

2. Classification of synaptic changes: Reinforcement Learning



SUCCESS Reinforcement Learning
= reward + Hebb

$$\Delta w_{ij} \propto F(\textit{pre}, \textit{post}, \textit{SUCCESS})$$

↑
local

↑
global

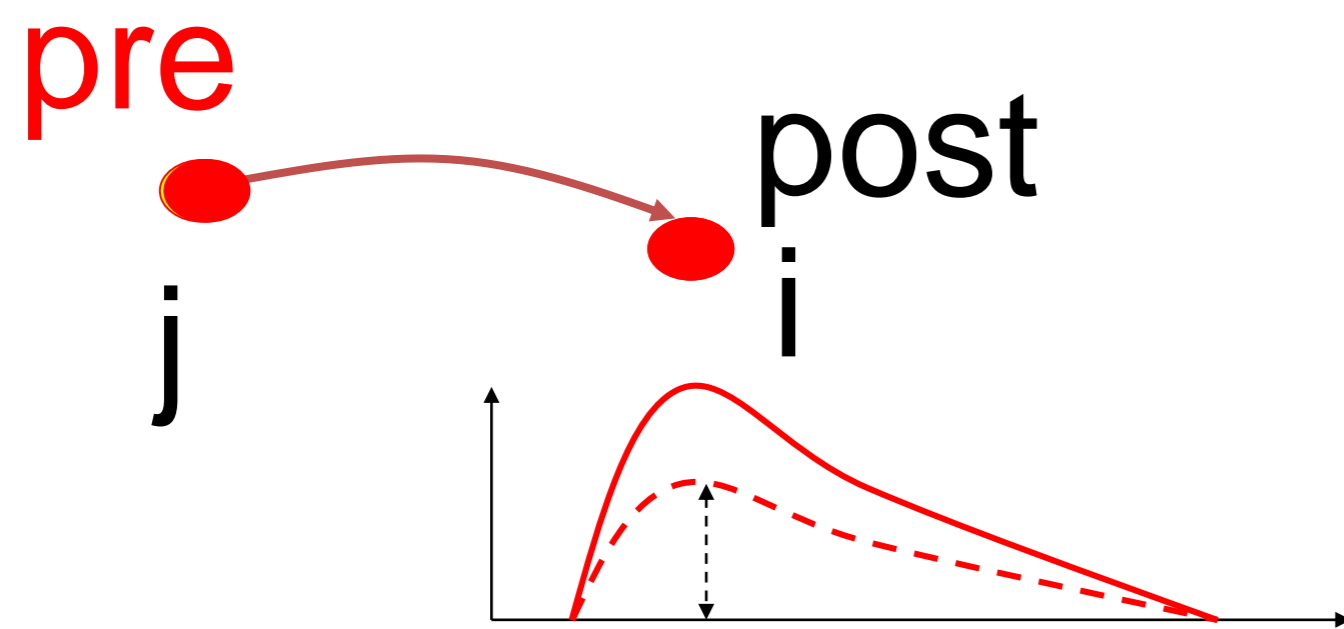
broadly diffused signal:
neuromodulator

2. Classification of synaptic changes

unsupervised vs reinforcement

LTP/LTD/Hebb Theoretical concept

- passive changes
- exploit statistical correlations

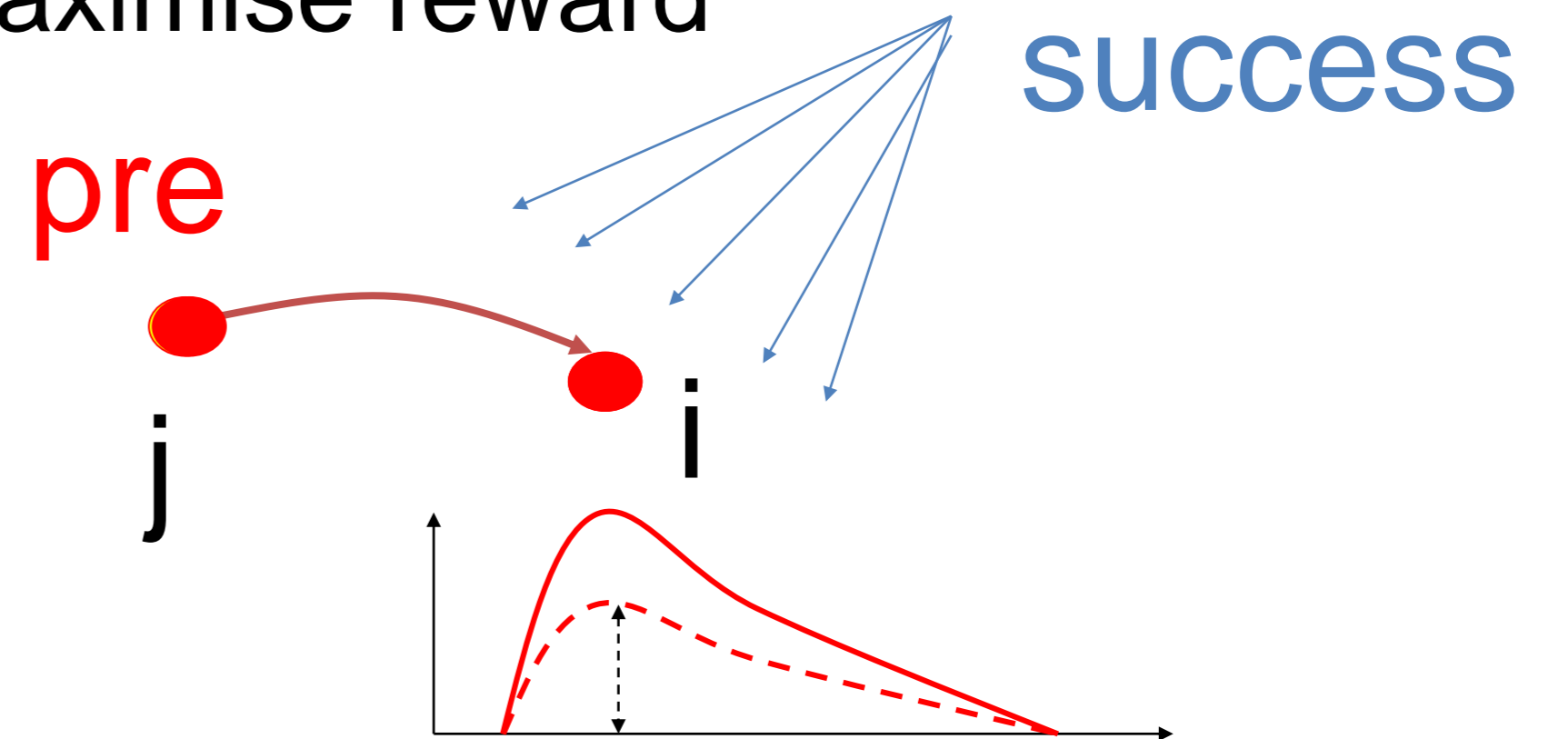


Functionality

- useful for development
(wiring for receptive field)

Reinforcement Learning Theoretical concept

- conditioned changes
- maximise reward

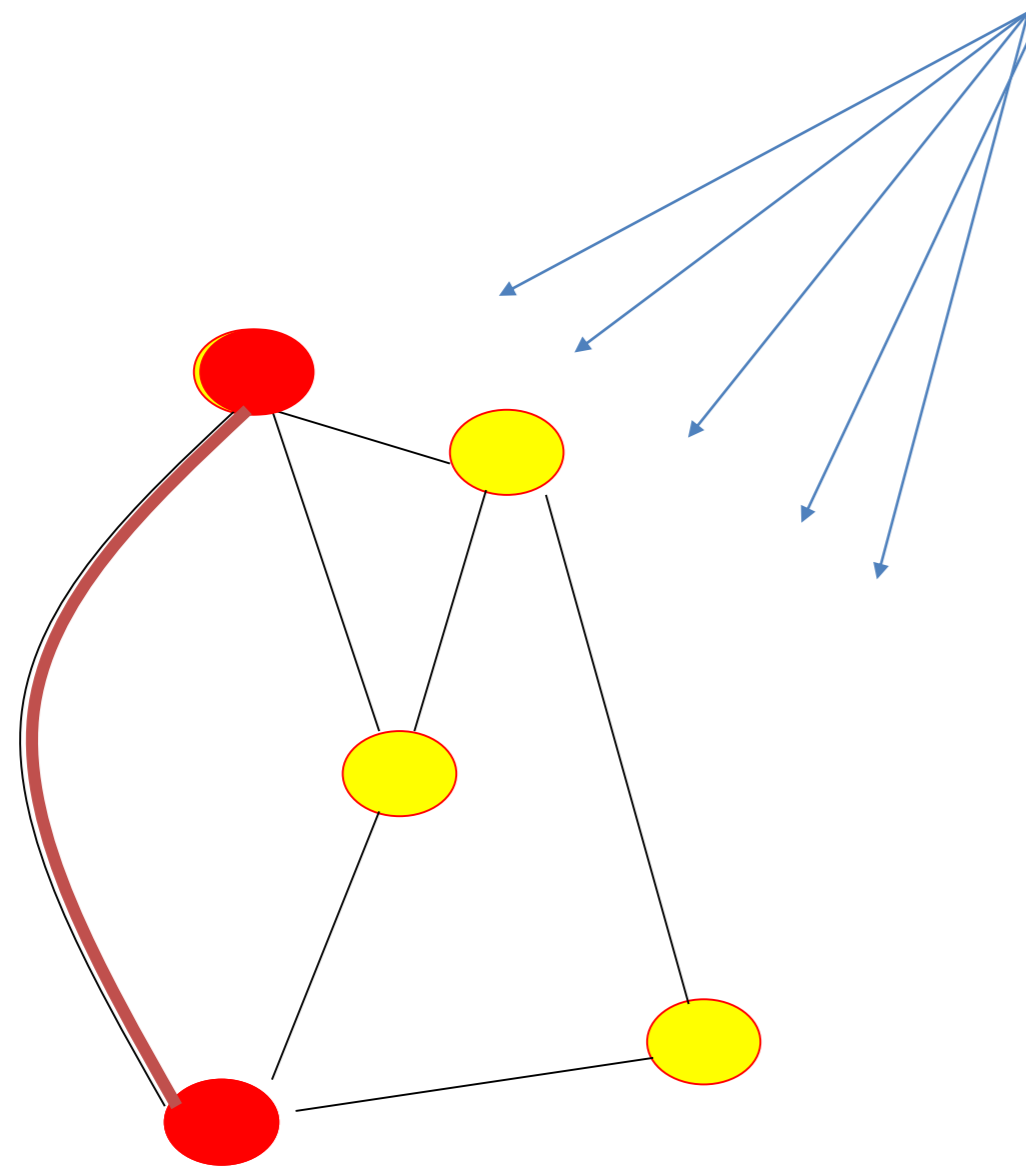


Functionality

- useful for learning
a new behavior

2. Three-factor rule of Hebbian Learning

= Hebb-rule gated by a neuromodulator



Neuromodulators: Interestingness, surprise;
attention; novelty

$$\Delta w_{ij} \propto F(\underset{\substack{\uparrow \\ \text{local}}}{pre}, \underset{\substack{\uparrow \\ \text{local}}}{post}, \underset{\substack{\uparrow \\ \text{global}}}{MOD})$$

local

global

Neuromodulator projections

- 4 or 5 neuromodulators
- near-global action

Dopamine/reward/TD:
Schultz et al., 1997,
Schultz, 2002

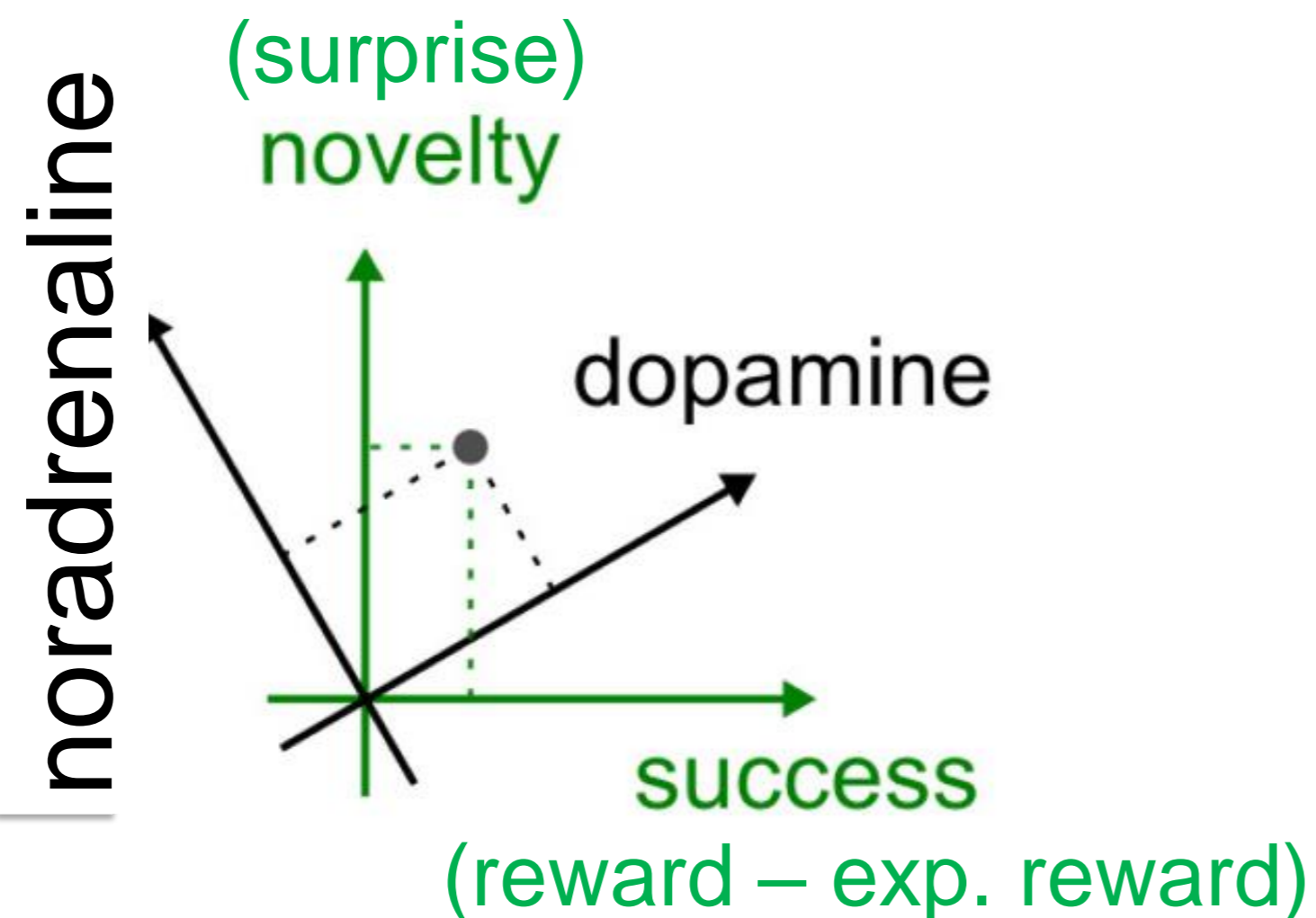
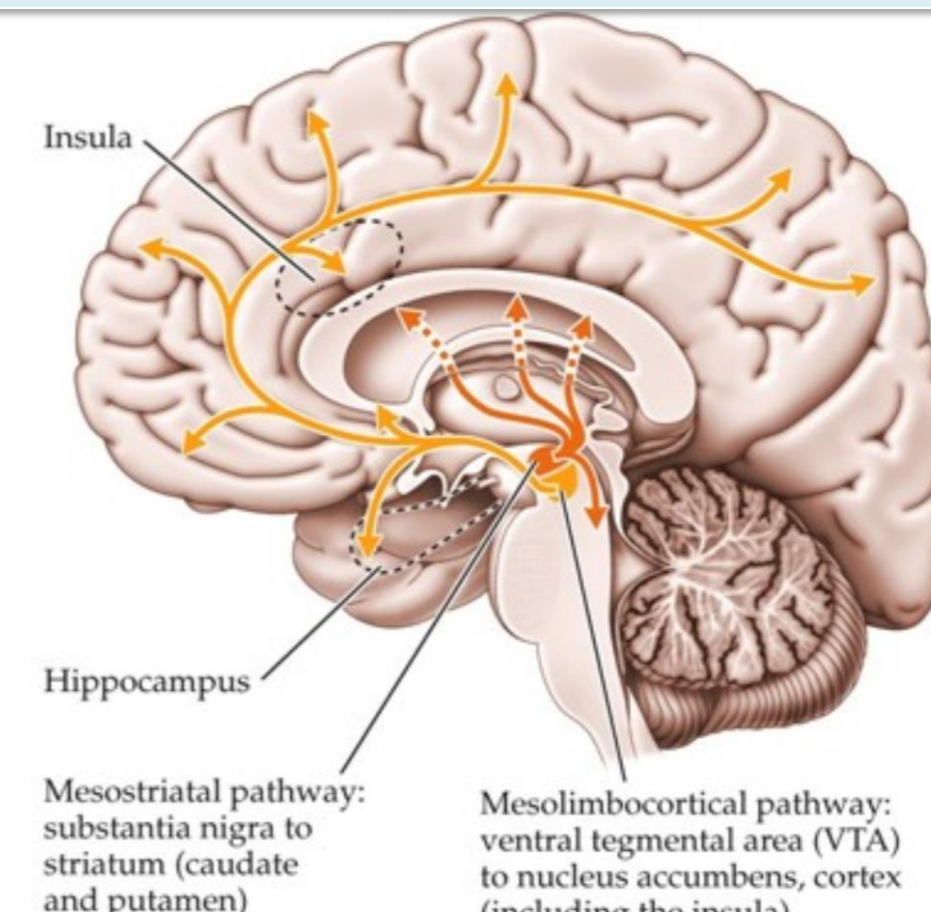


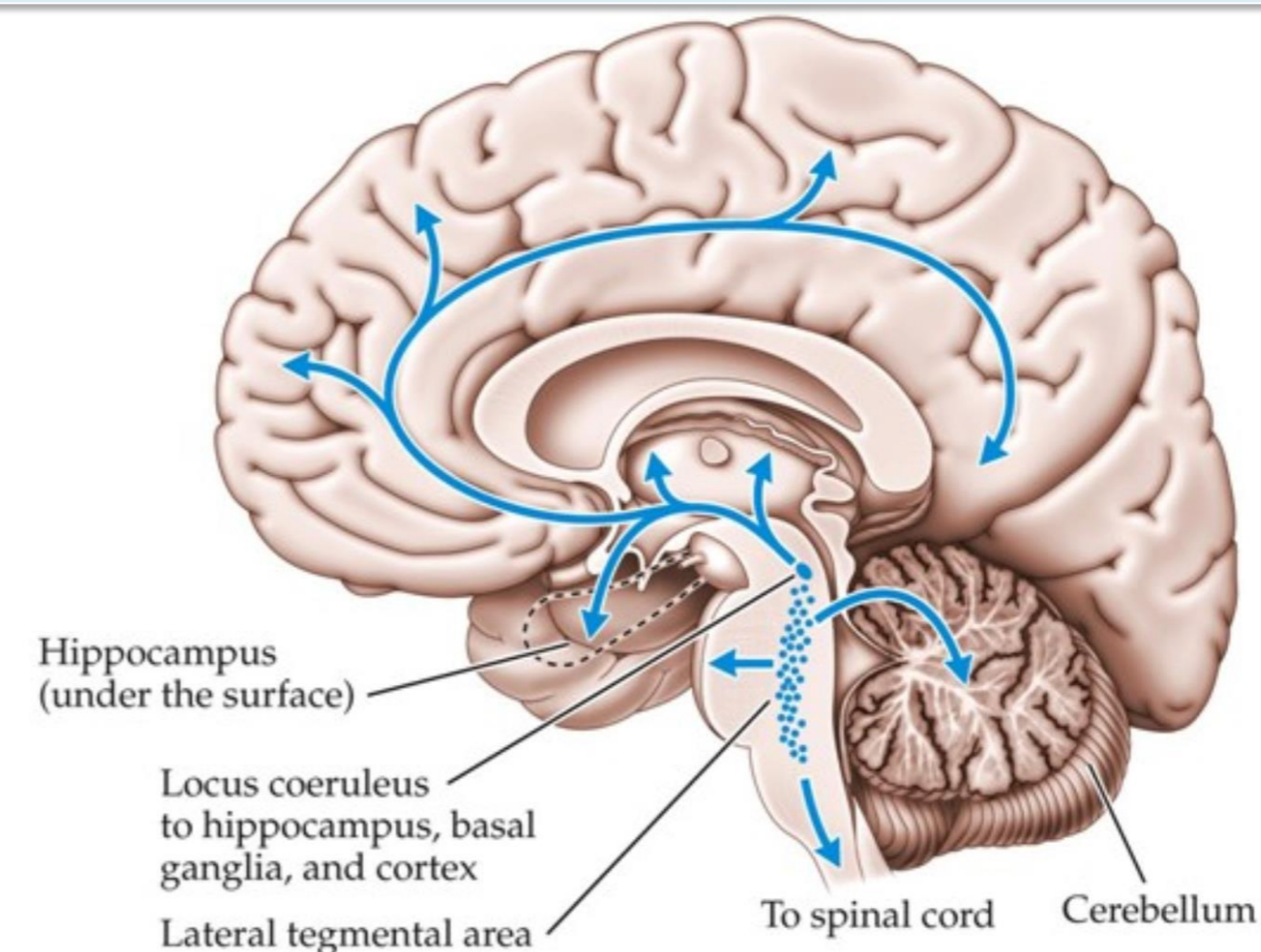
Image:
Fremaux and Gerstner, Frontiers (2016)

Image: *Biological Psychology, Sinauer*

Dopamine

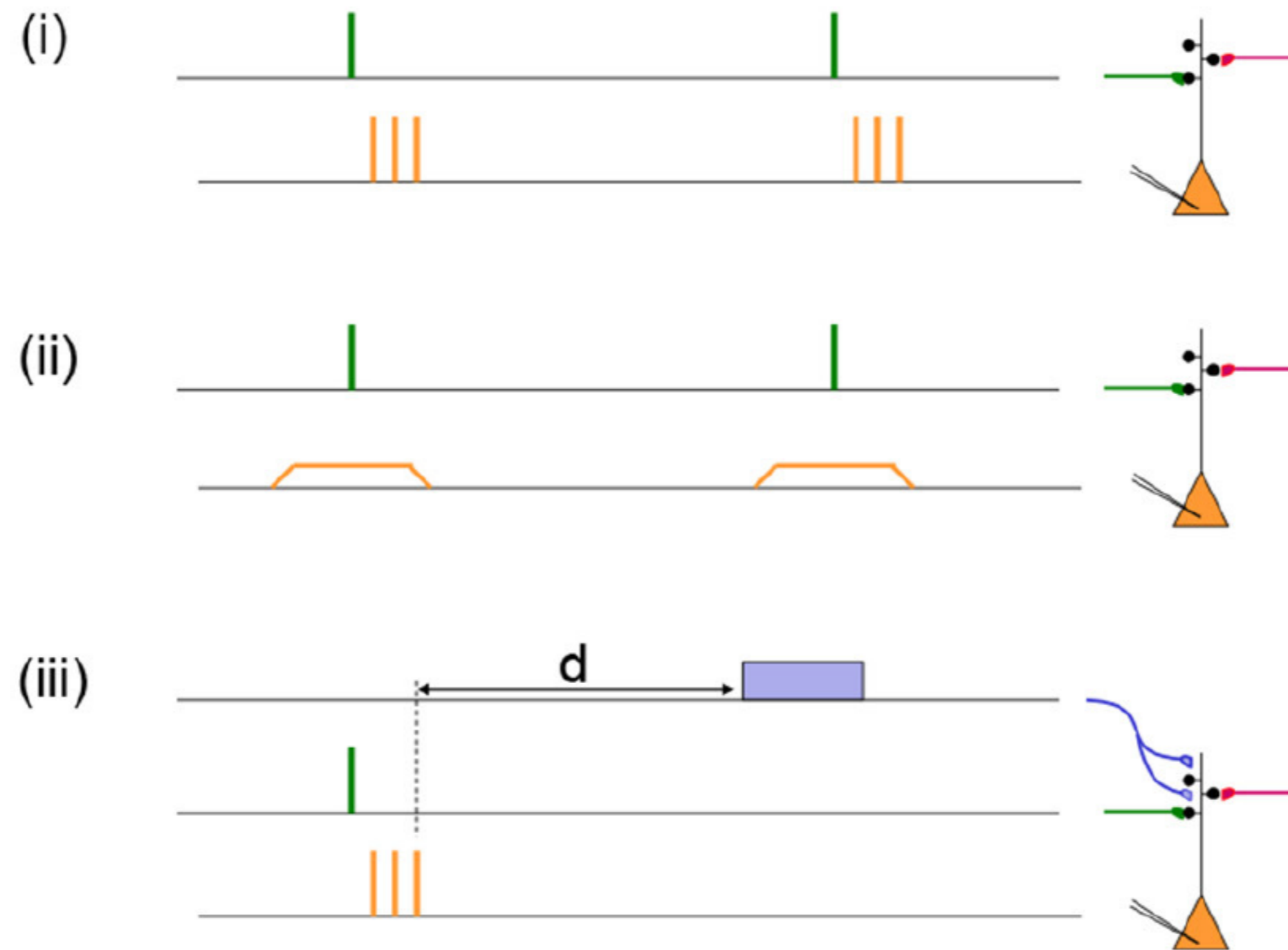


Noradrenaline



BIOLOGICAL PSYCHOLOGY 7e, Figure 4.5

2-factor versus 3-factor rules



Hebbian:
'post' = spikes

Hebbian:
'post' = voltage

3-factor

3-factor = Hebbian combined with
(potentiall delayed) Neuromodulator:

2. Summary: Classification of synaptic changes

Several categories can be used to classify synaptic changes:

- 1) Do changes last for a long time (hours: Long-Term Potentiation) or do they decay rapidly back to baseline (around a second: Short-Term Potentiation).
- 2) Do changes depend mainly on presynaptic and postsynaptic activity (Hebbian learning/2-factor rule), or also on the presence of a neuromodulator (three-factor rule).
- 3) Learning paradigm: is the learning scenario just exploiting input statistics (unsupervised learning/no teacher, no reward); or does it also involve notions of 'reward' or 'success (reinforcement learning)

Quiz 1. Synaptic Plasticity and Learning Rules

Long-term potentiation

- has an acronym LTP
- takes more than 10 minutes to induce
- lasts more than 30 minutes
- depends on presynaptic activity, but not on state of postsynaptic neuron

Short-term potentiation

- has an acronym STP
- takes more than 10 minutes to induce
- lasts more than 30 minutes
- depends on presynaptic activity, but not on state of postsynaptic neuron

Learning rules

- Hebbian learning depends on presynaptic activity and on state of postsynaptic neuron
- Reinforcement learning depends on neuromodulators such as dopamine indicating reward

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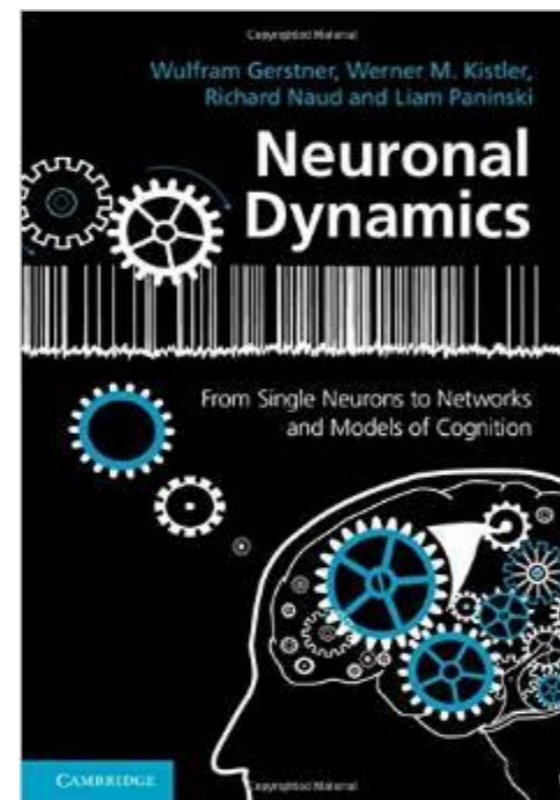
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See Week X on MOODLE or See week 3 on:

<http://lcn.epfl.ch/~gerstner/NeuronalDynamics-MOOC1.html>

Synapses, dendrites and the cable equation

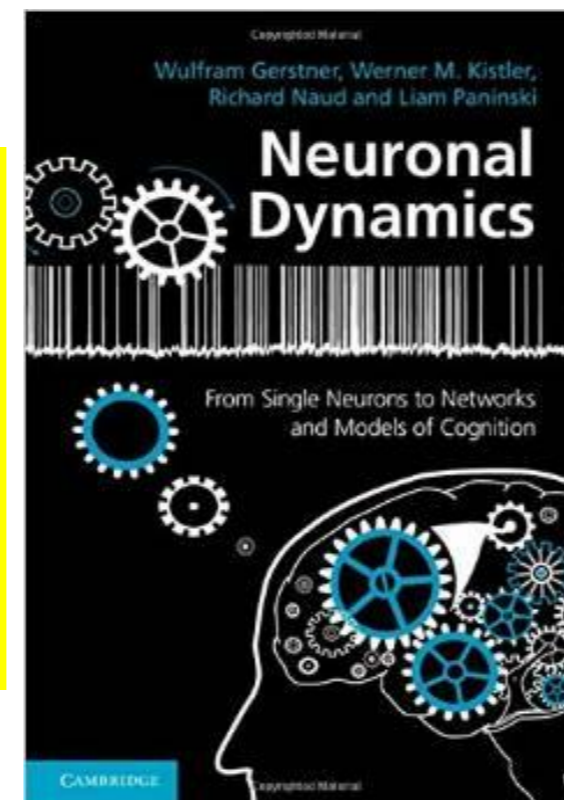
Part 1 - [Synapses \(15 min\)](#)

Part 2 - [Synaptic short term plasticity \(9 min\)](#)

https://www.youtube.com/watch?v=iEz_SUsJMJ8

Reading for STP:
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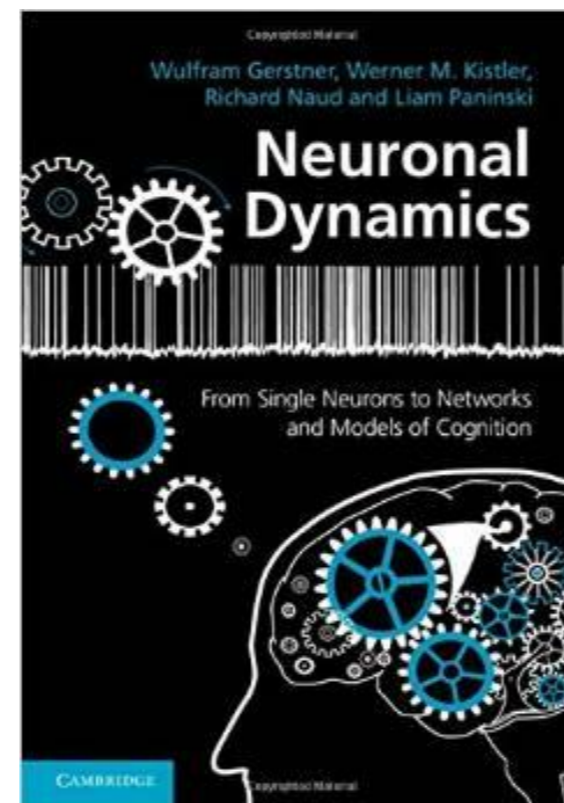
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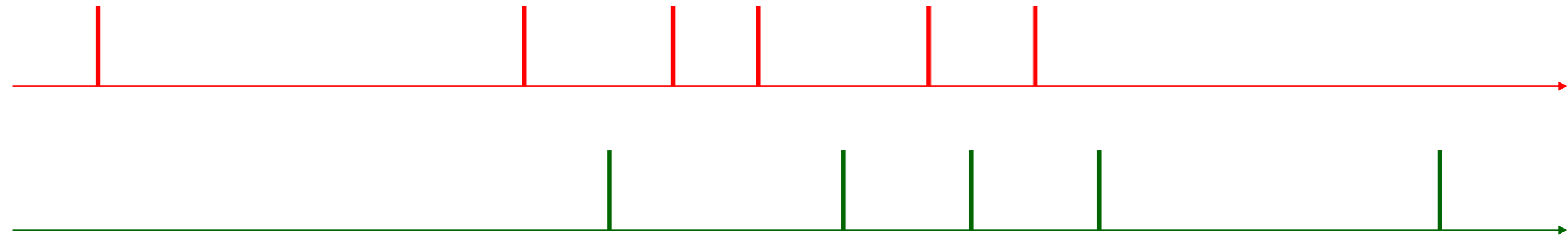
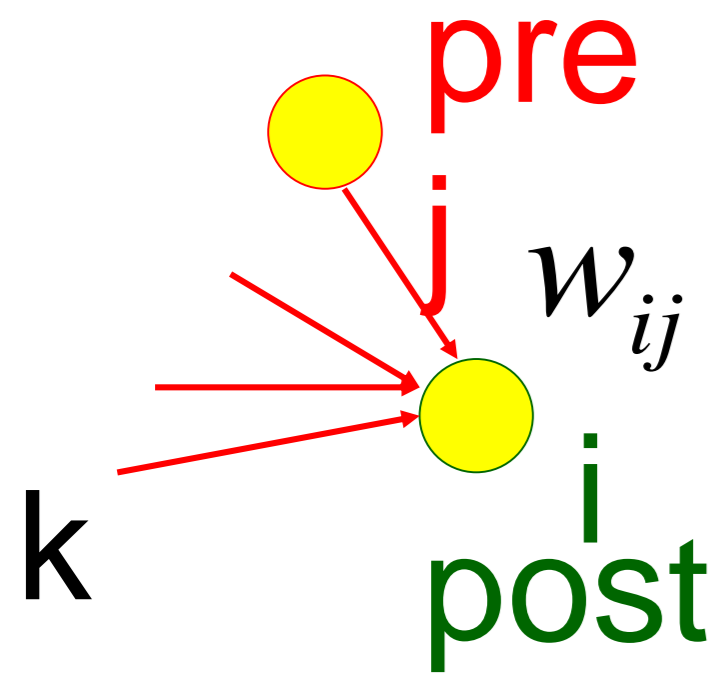
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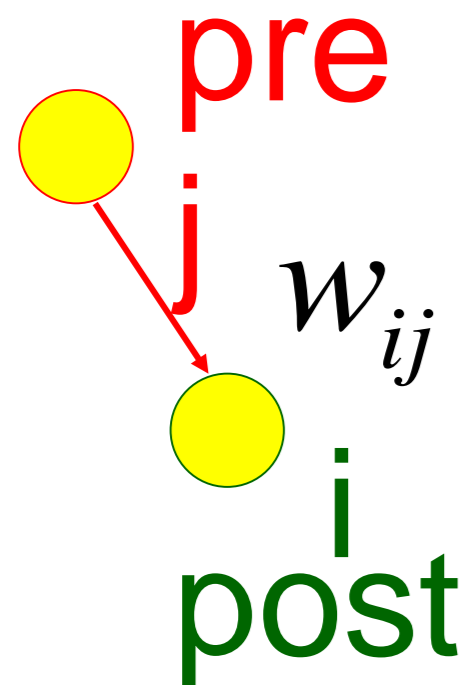
Hebb, 1949

- local rule
- simultaneously active (correlations)

Rate model:

active = high rate = many spikes per second

4. Rate-based Hebbian Learning



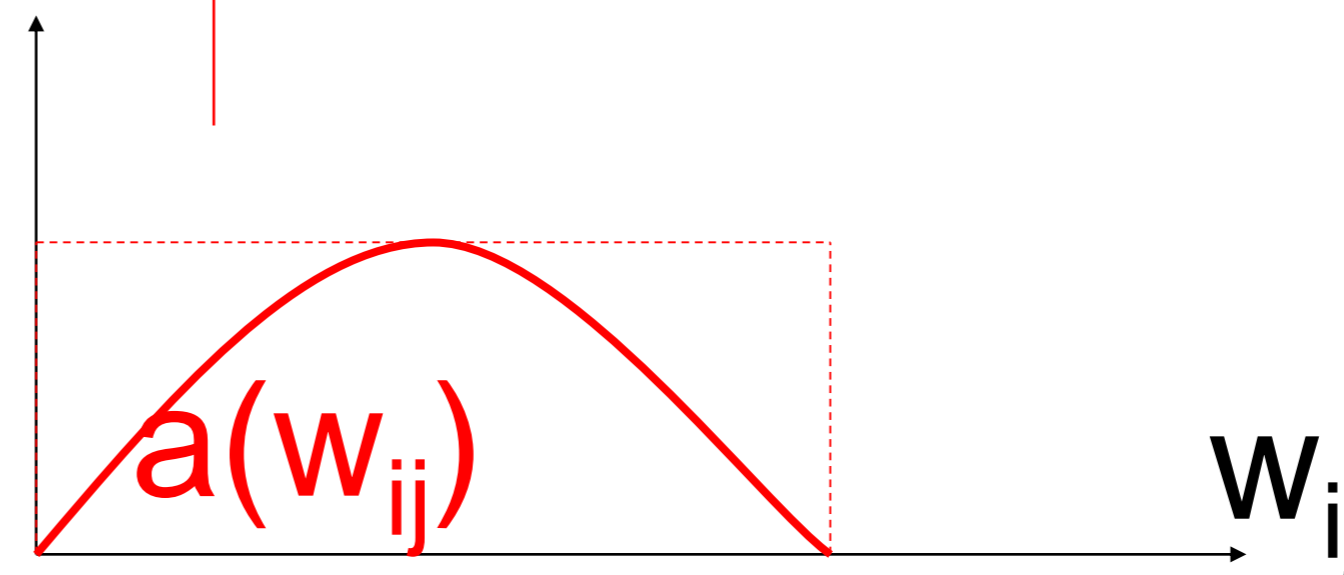
Local rule:

$$\frac{d}{dt} w_{ij} = F(w_{ij}, MOD; v_j^{pre}, v_i^{post})$$

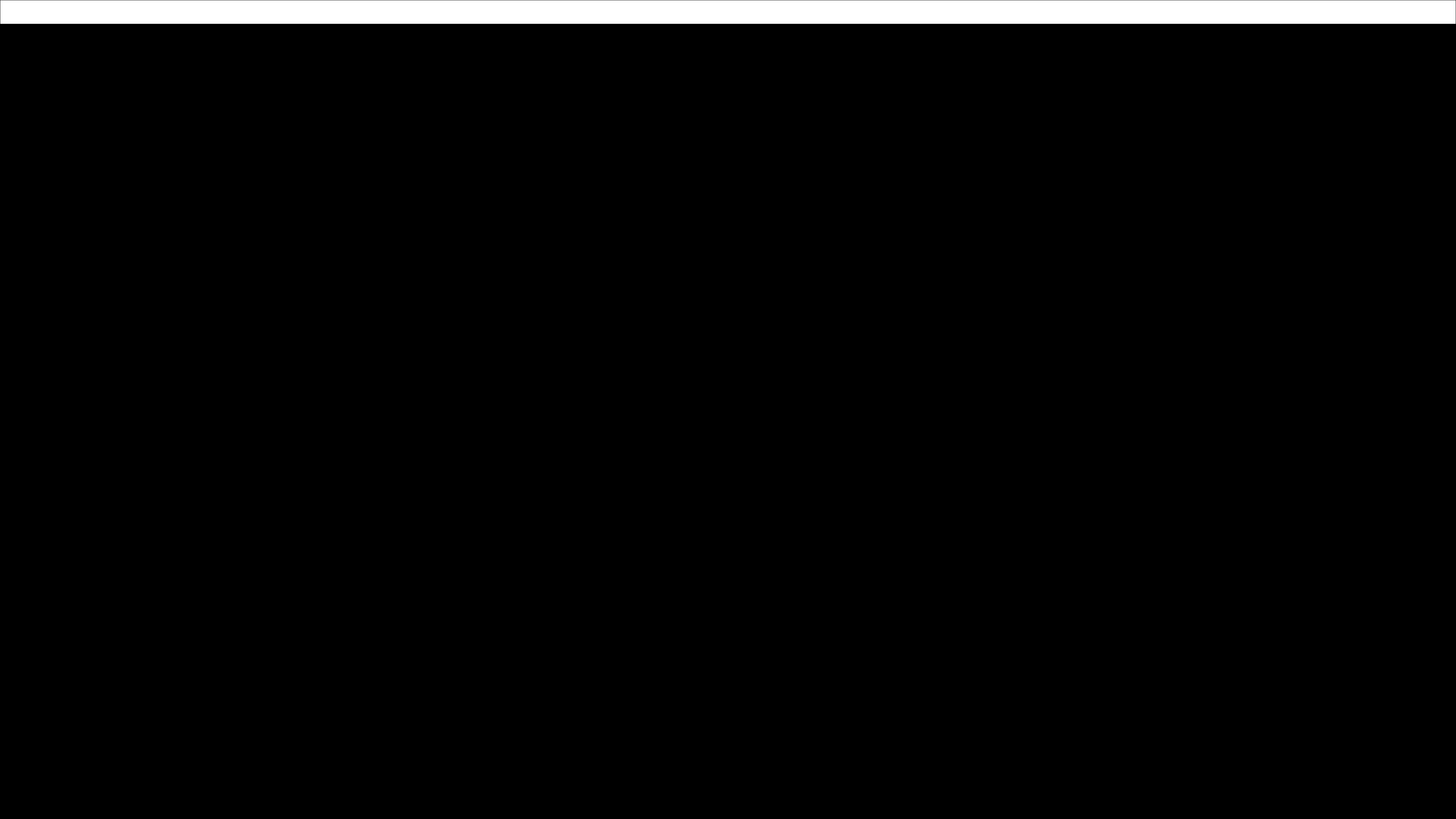
Taylor expansion:

$$\frac{d}{dt} w_{ij} = a_0 + a_1^{pre} v_j^{pre} + a_1^{post} v_i^{post} + a_2^{corr} v_j^{pre} v_i^{post} + \dots$$

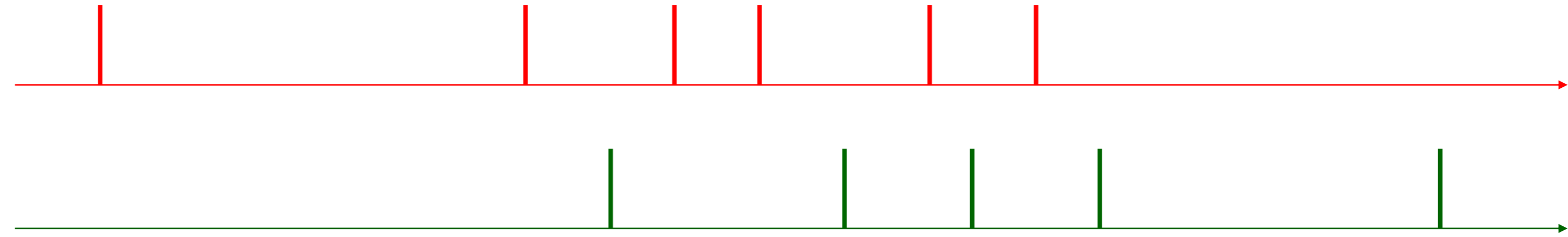
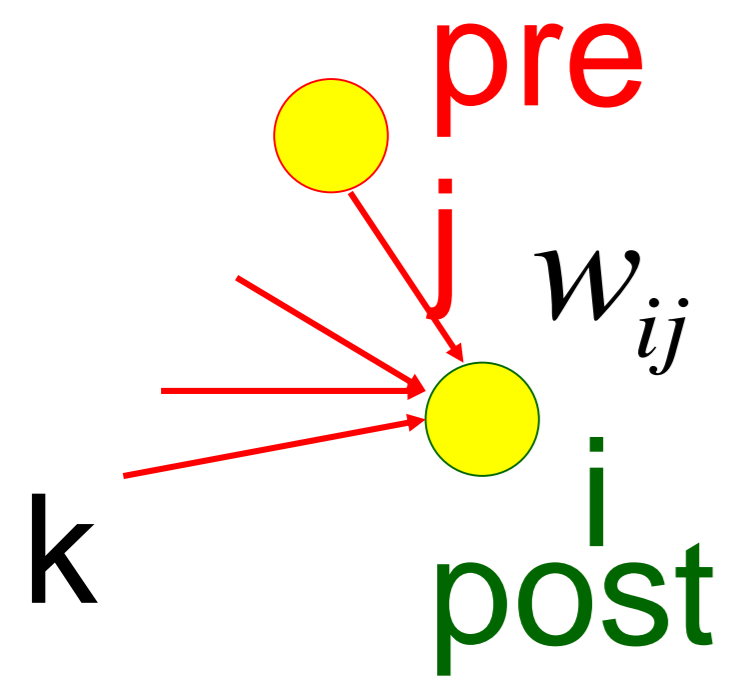
$$a = a(w_{ij})$$



Blackboard1



4. Rate-based Hebbian Learning

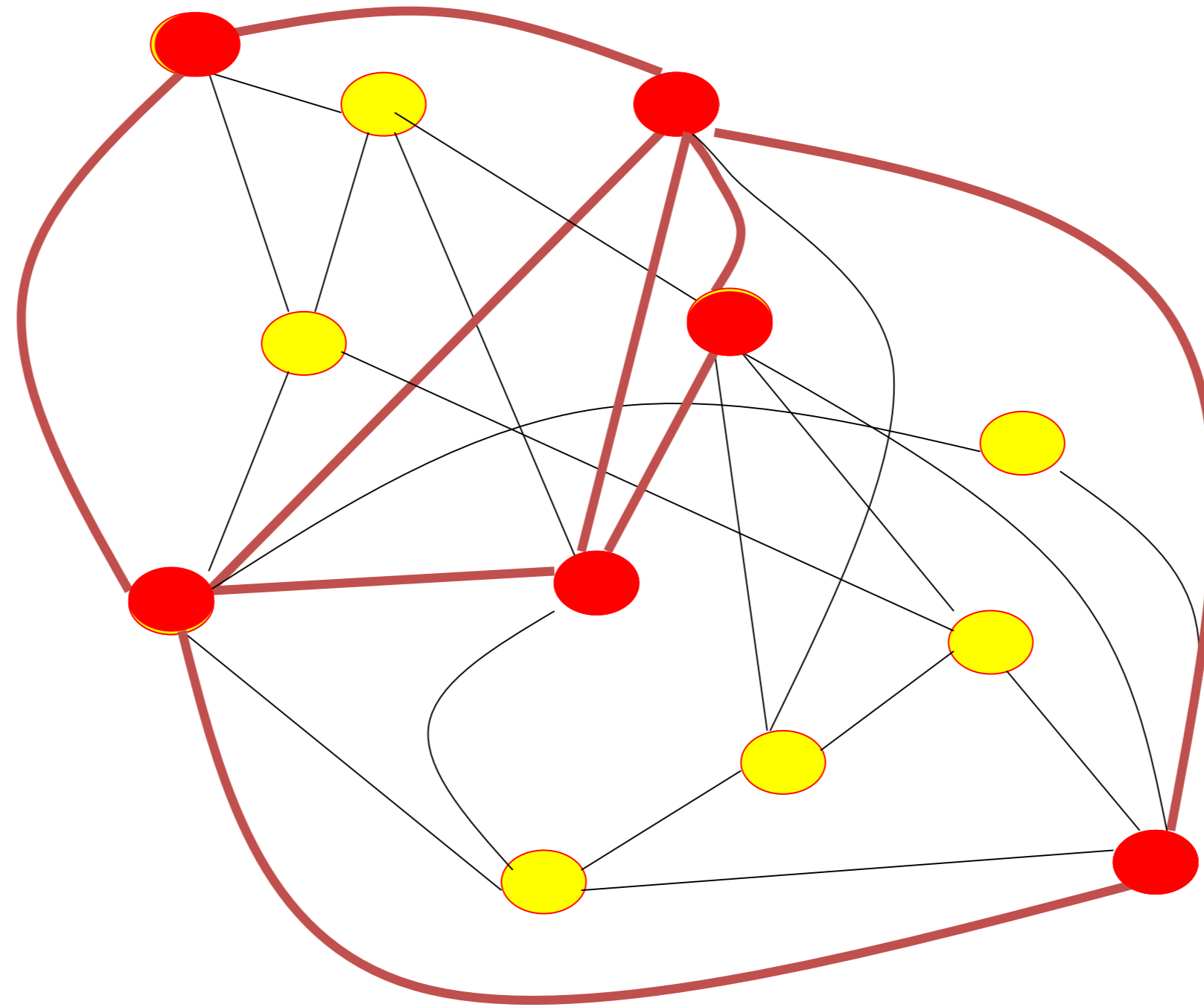


pre
post

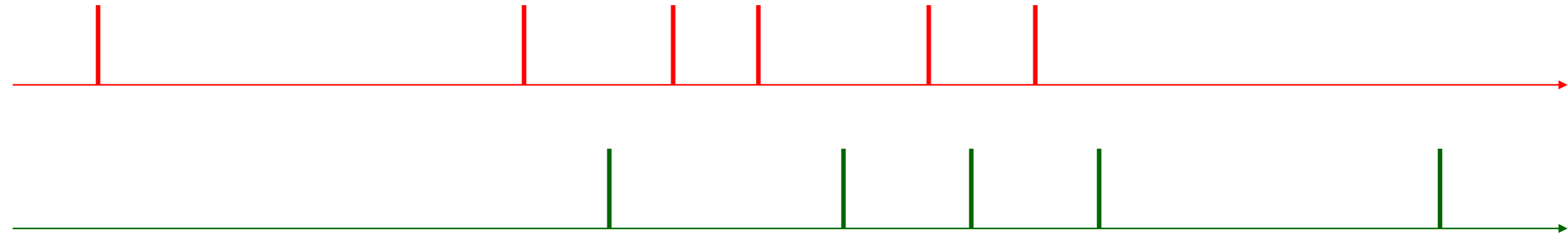
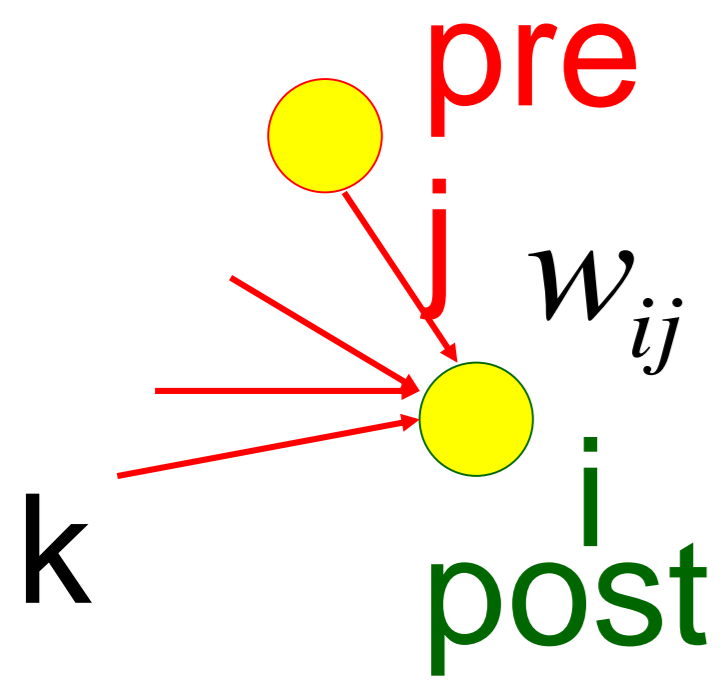
on	off	on	off
on	on	off	off
+	0	0	0

$$\frac{d}{dt} w_{ij} = a_2^{corr} v_j^{pre} v_i^{post}$$

Review from week 5: Hebbian Learning



4. Rate-based Hebbian Learning



pre
post

$$\frac{d}{dt} w_{ij} = a_2^{corr} v_j^{pre} v_i^{post}$$

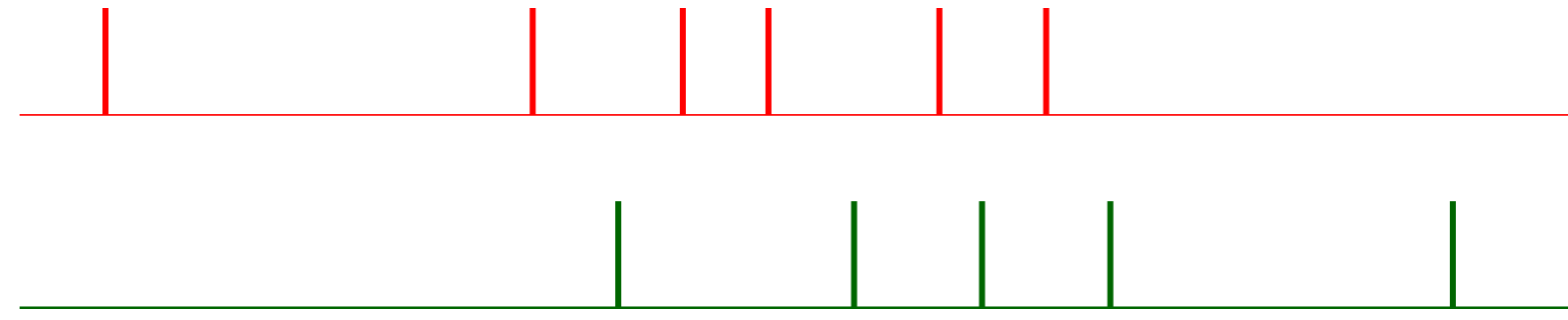
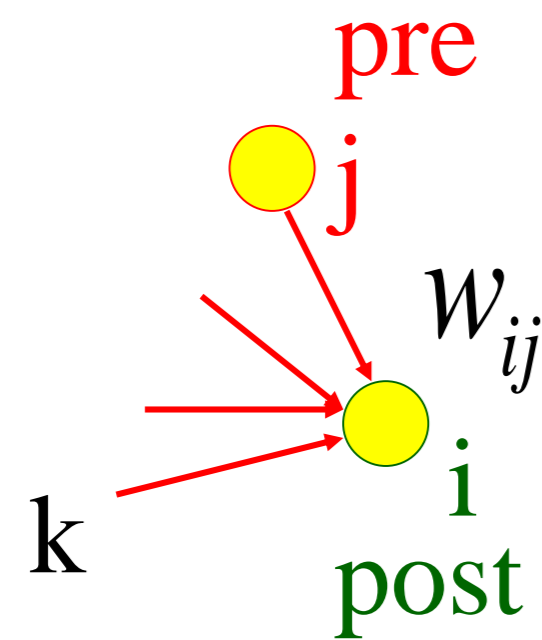
$$\frac{d}{dt} w_{ij} = a_2^{corr} v_j^{pre} v_i^{post} - c$$

$$\frac{d}{dt} w_{ij} = a_2^{corr} v_j^{pre} (v_i^{post} - \mathcal{G})$$

$$\frac{d}{dt} w_{ij} = a_2^{corr} (v_j^{pre} - \mathcal{G})(v_i^{post} - \mathcal{G})$$

on	off	on	off
on	on	off	off
+	0	0	0
+	-	-	-
+	0	-	0
+	-	-	+

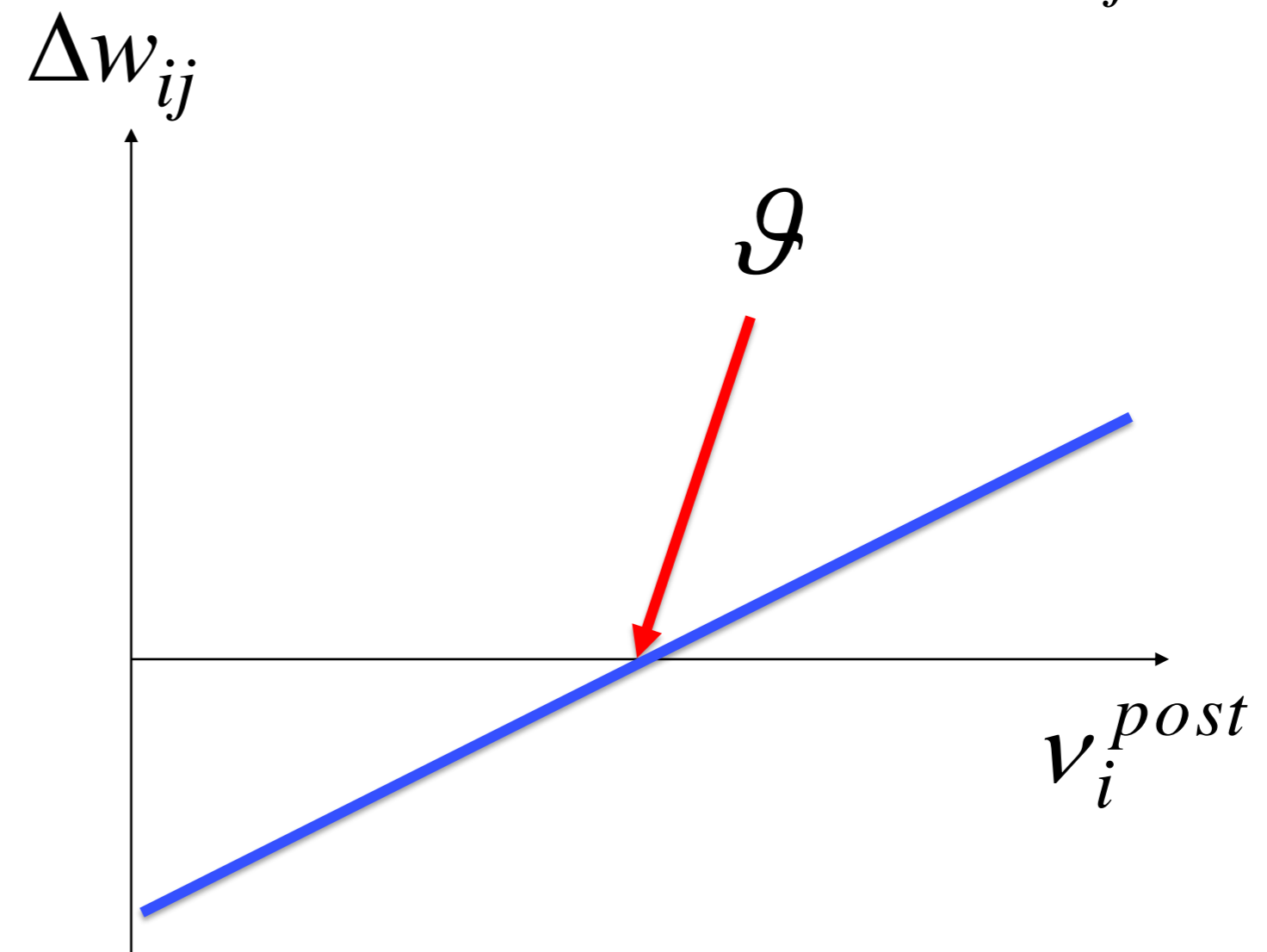
4. Presynaptically gated plasticity rule



presynaptically gated

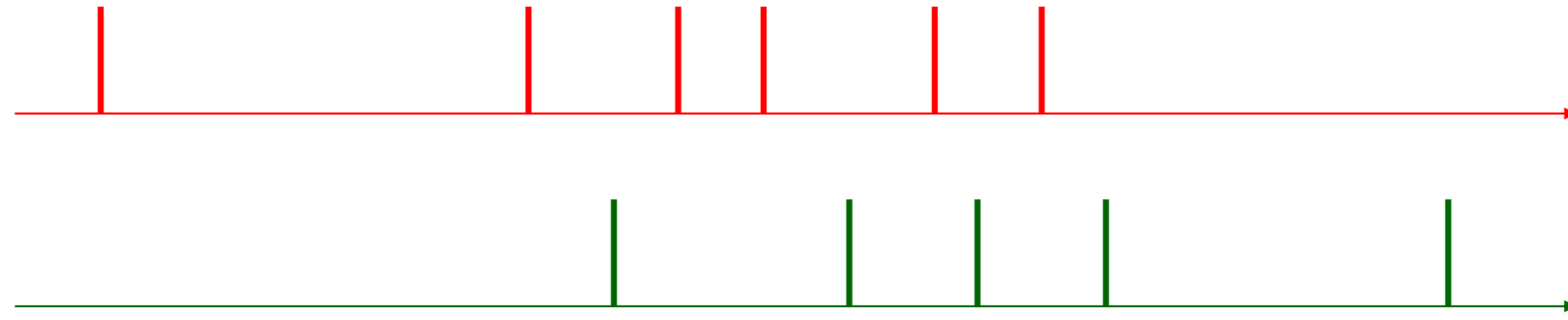
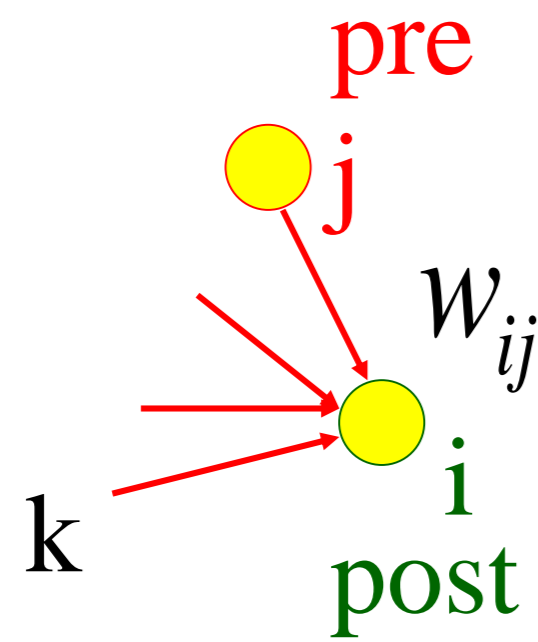
$$\frac{d}{dt} w_{ij} = a_2^{corr} (v_i^{post} - \mathcal{G}) v_j^{pre}$$

Assume activity $v_j^{pre} > 0$



4. Bienenstock-Cooper-Munro rule

*Bienenstock, Cooper
Munro, 1982*



presynaptically gated

$$\frac{d}{dt} w_{ij} = a_2^{corr} (v_i^{post} - \mathcal{G}) v_j^{pre}$$

BCM: 3rd order ('triplet')

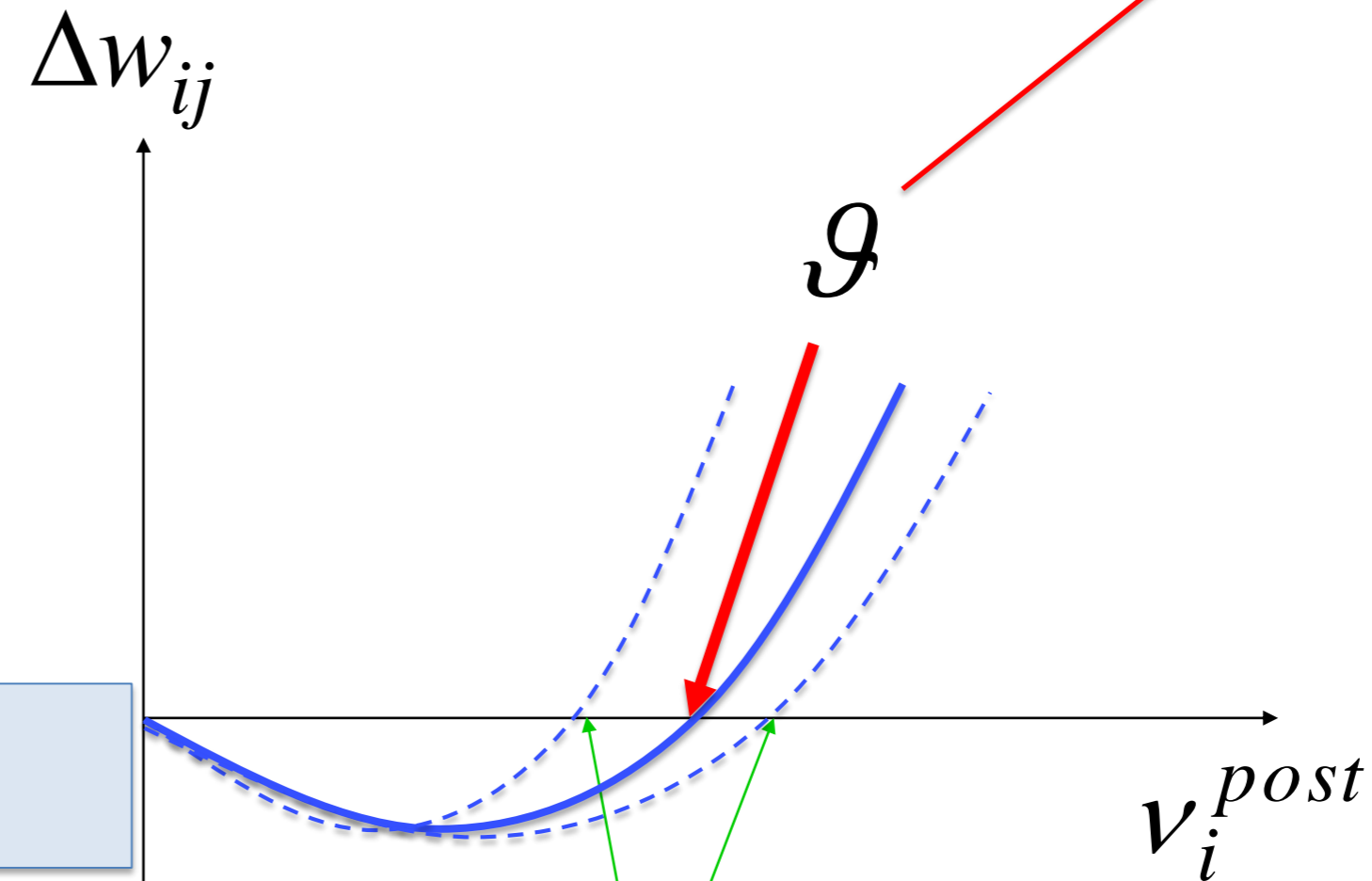
$$\frac{d}{dt} w_{ij} = b v_i^{post} (v_i^{post} - \mathcal{G}) v_j^{pre}$$

Map to
Taylor expansion?

triplet

pair

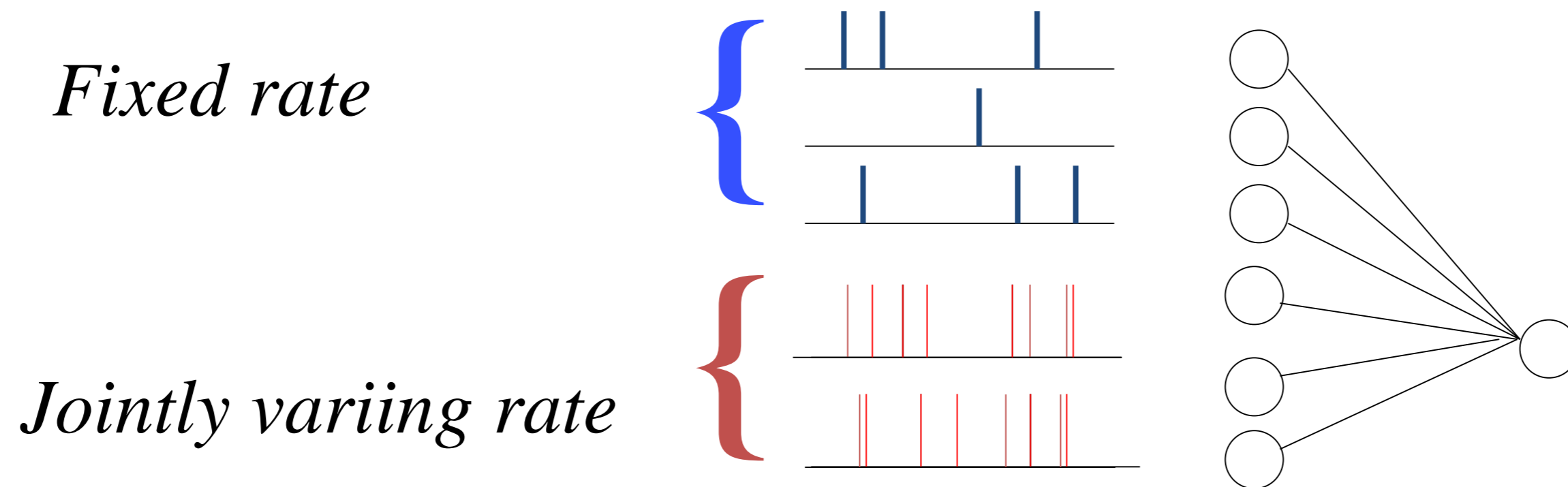
$$\frac{d}{dt} w_{ij} = b v_i^{post} (v_i^{post} - \mathcal{G}) v_j^{pre}$$



assume
 $v_j^{pre} > 0$

Homeostasis $\mathcal{G} = f(\bar{v}_i^{post})$

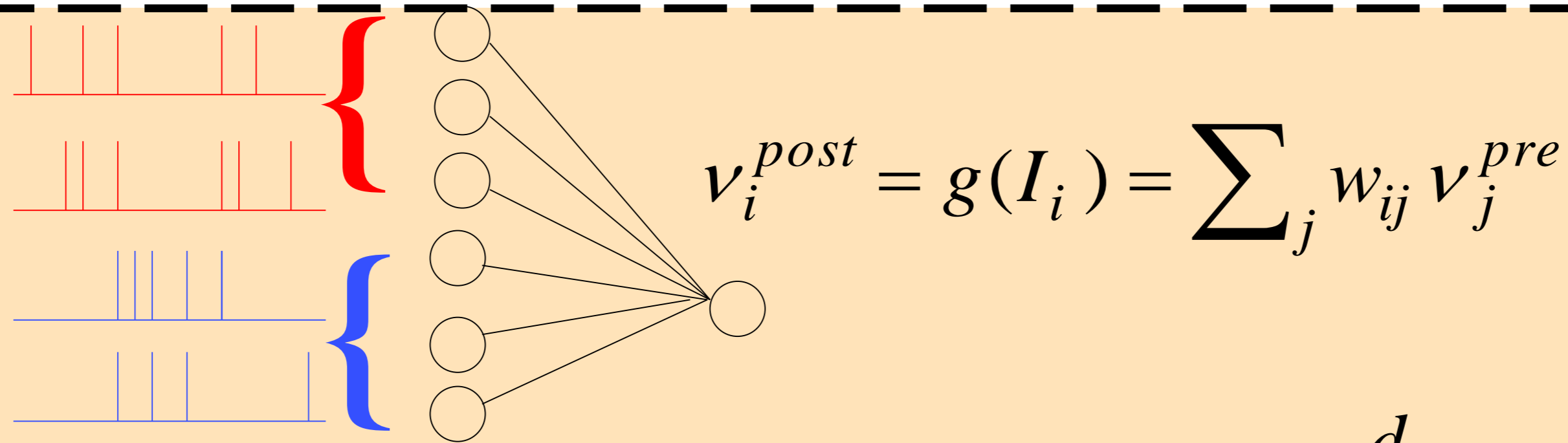
4. Functional Consequence of Hebbian Learning



Hebbian Learning detects correlations in the input

→ **Development of Receptive Fields**
(see also course:
Unsupervised and Reinforcement Learning)

Exercise 1 now: Bienenstock-Cooper-Munro



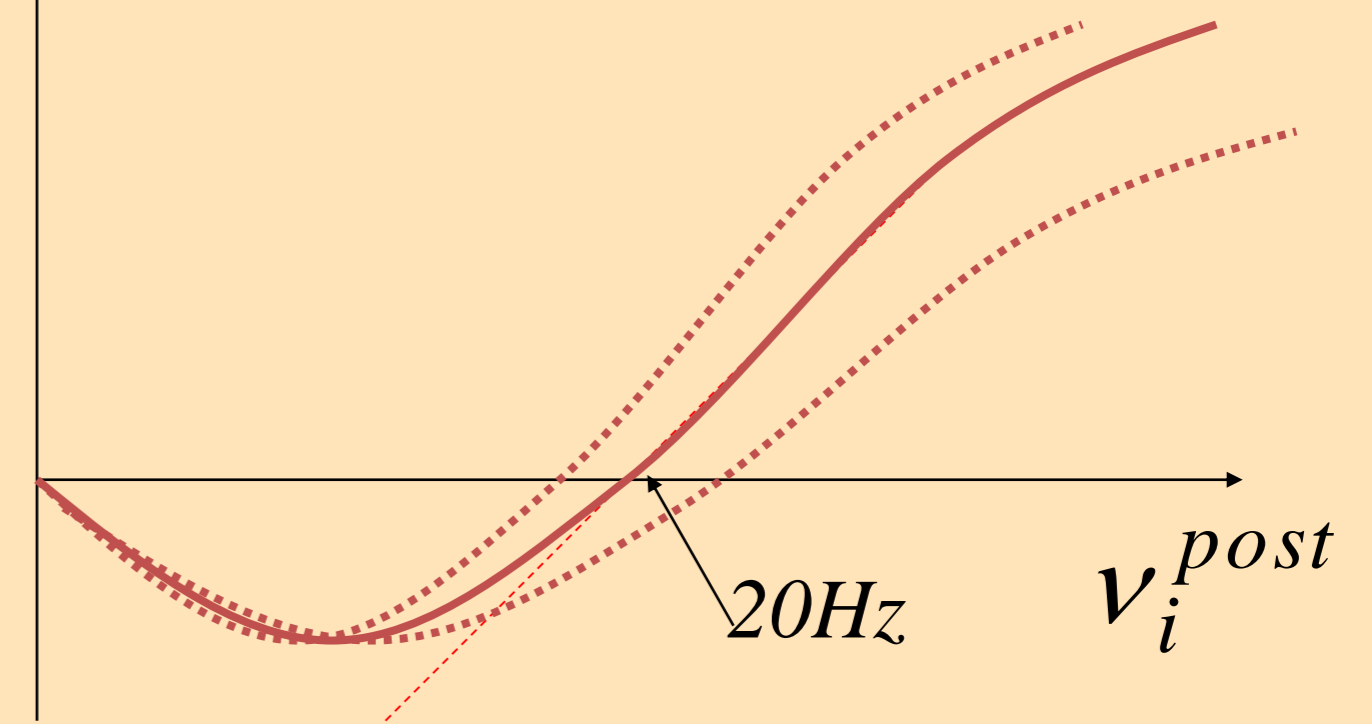
Take 8 minutes =
Discussion of ex
At 10:20

BCM rule

$$\frac{d}{dt} w_{ij} = a_2^{corr} \Phi(v_i^{post} - \mathcal{G}) v_j^{pre}$$

20Hz

$$\frac{d}{dt} w_{ij} \text{ if } v_j^{pre} > 0$$



Assume 2 groups of 10 neurons each. All weights equal 1.

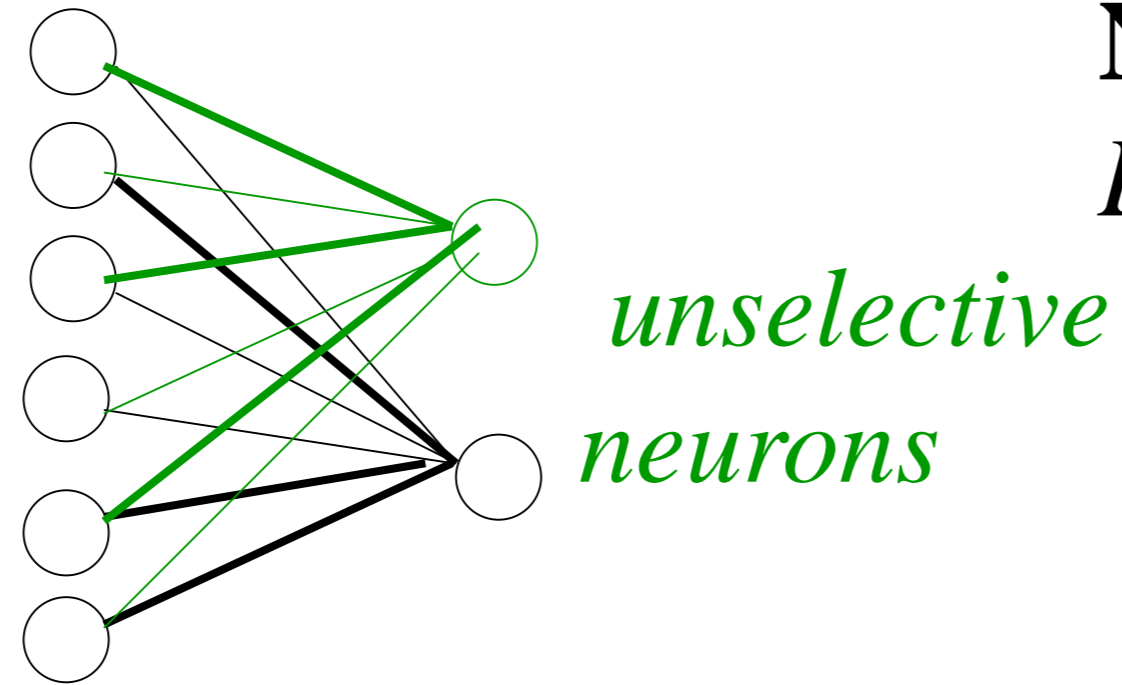
a) Group 1 fires at 3 Hz, then group 2 at 1 Hz. What happens?

b) Group 1 fires at 3 Hz, then group 2 at 2.5 Hz. What happens?

c) As in b, but make theta a function of the averaged rate. What happens?

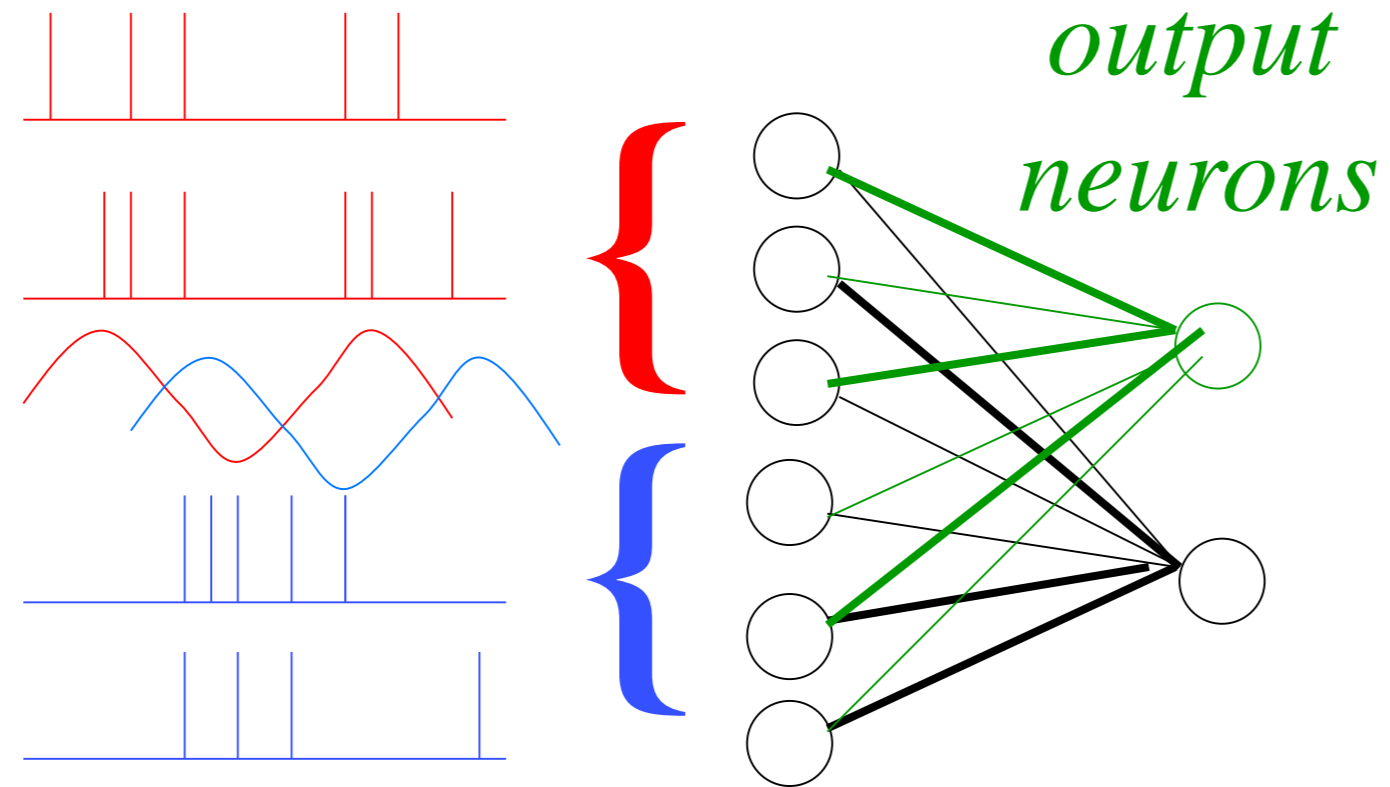
4. Synaptic Changes for Development of Cortex

Initial:
random
connections

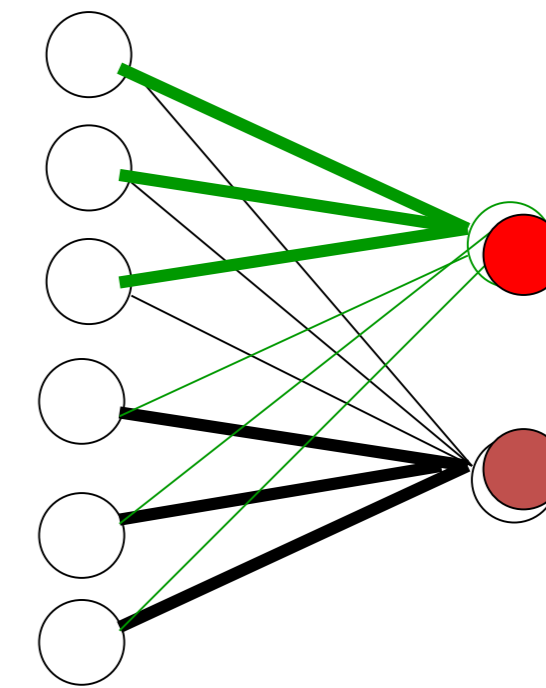


**BCM leads to specialized
Neurons (developmental learning);**
Bienenstock et al. 1982

Development and learning rules:
Willshaw & Malsburg, 1976
Linsker, 1986
K.D. Miller et al., 1989



Correlated input



output neurons specialize:
Receptive fields

4. Models for Hebbian Long-Term-Plasticity

- Many 'Hebbian' rules
- LTP and LTD
- Can describe RF development
- BCM is a well-known example
- Competition: some synapses grow at the expense of others

4. Summary: Models for Hebbian Learning

- Hebbian learning refers to a family of learning rules, rather than one specific rule.
- Rules can be classified by mapping them to a Taylor expansion.
- Terms with a negative coefficient induce long-term depression (LTD).
- A clever combination of LTP and LTD can explain the development of receptive fields (RF).
- A clever combination of LTP and LTD leads to competition: some synapses grow at the expense of others. A well-known example of a Hebbian rule is the Bienenstock-Cooper-Munro (BCM) rule

Biological Modeling of Neural Networks

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Week 13

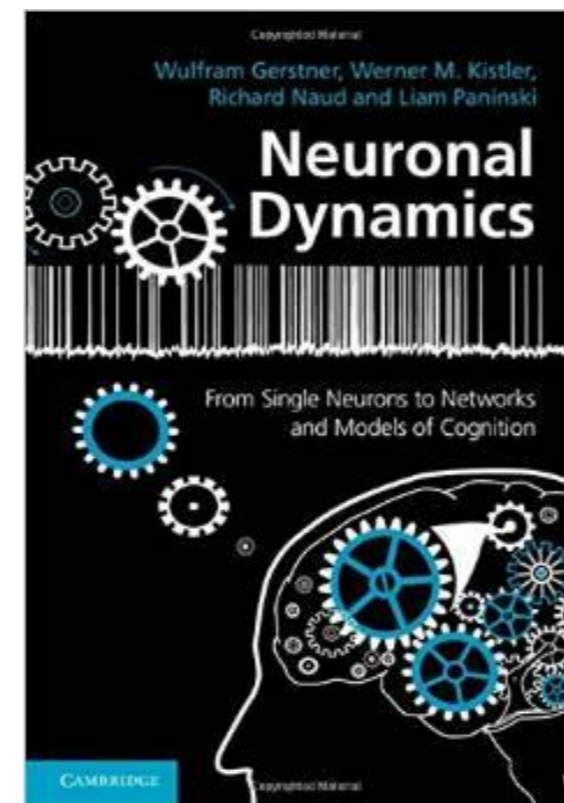
Synaptic plasticity and Learning

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Reading for plasticity:
NEURONAL DYNAMICS
- Ch. 19.1-19.3

Cambridge Univ. Press



1. Synaptic plasticity

motivation and aims

2. Classification of plasticity

short-term vs. long-term

unsupervised vs. reward modulated

3. Model of short-term plasticity

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- Bienenstock-Cooper-Munro rule

5. Spiking Models of plasticity

- STDP as Hebbian learning

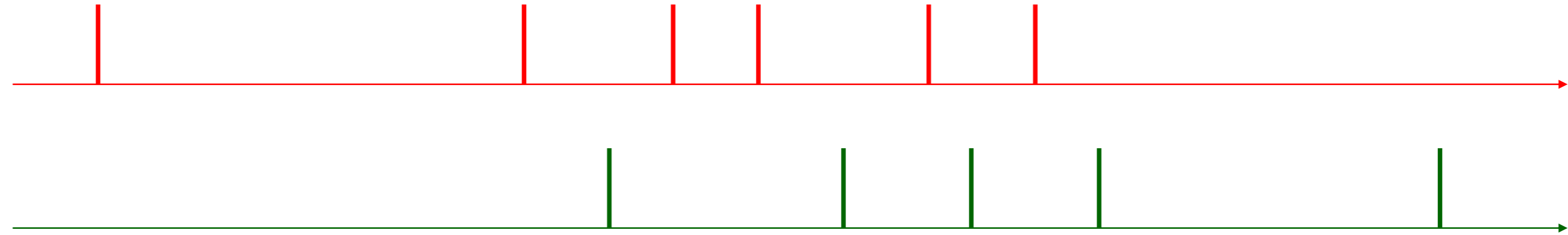
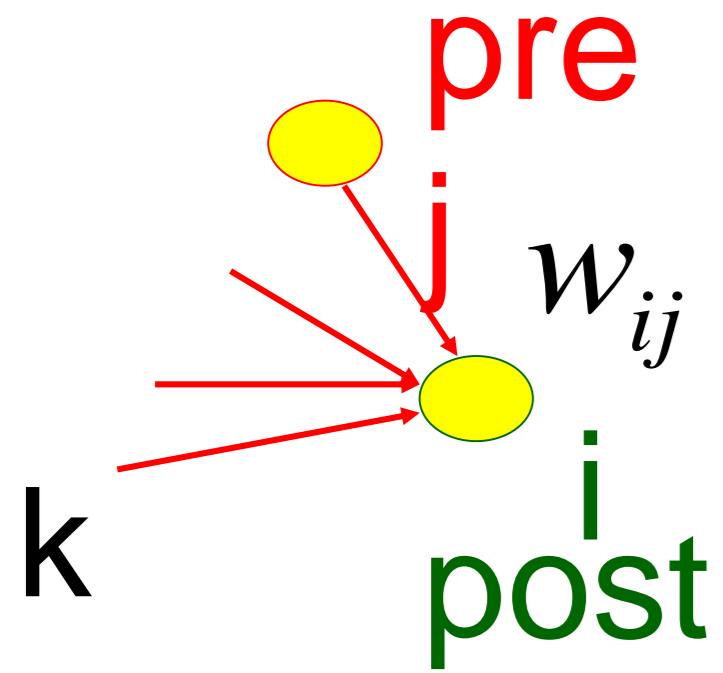
- Model of STDP: synaptic traces

6. From STDP to rate models

7. Triplet STDP model

8. Online learning of memories

5. Hebbian Learning



When an axon of cell **j** repeatedly or persistently takes part in firing cell **i**, then **j**'s efficiency as one of the cells firing **i** is increased

Hebb, 1949

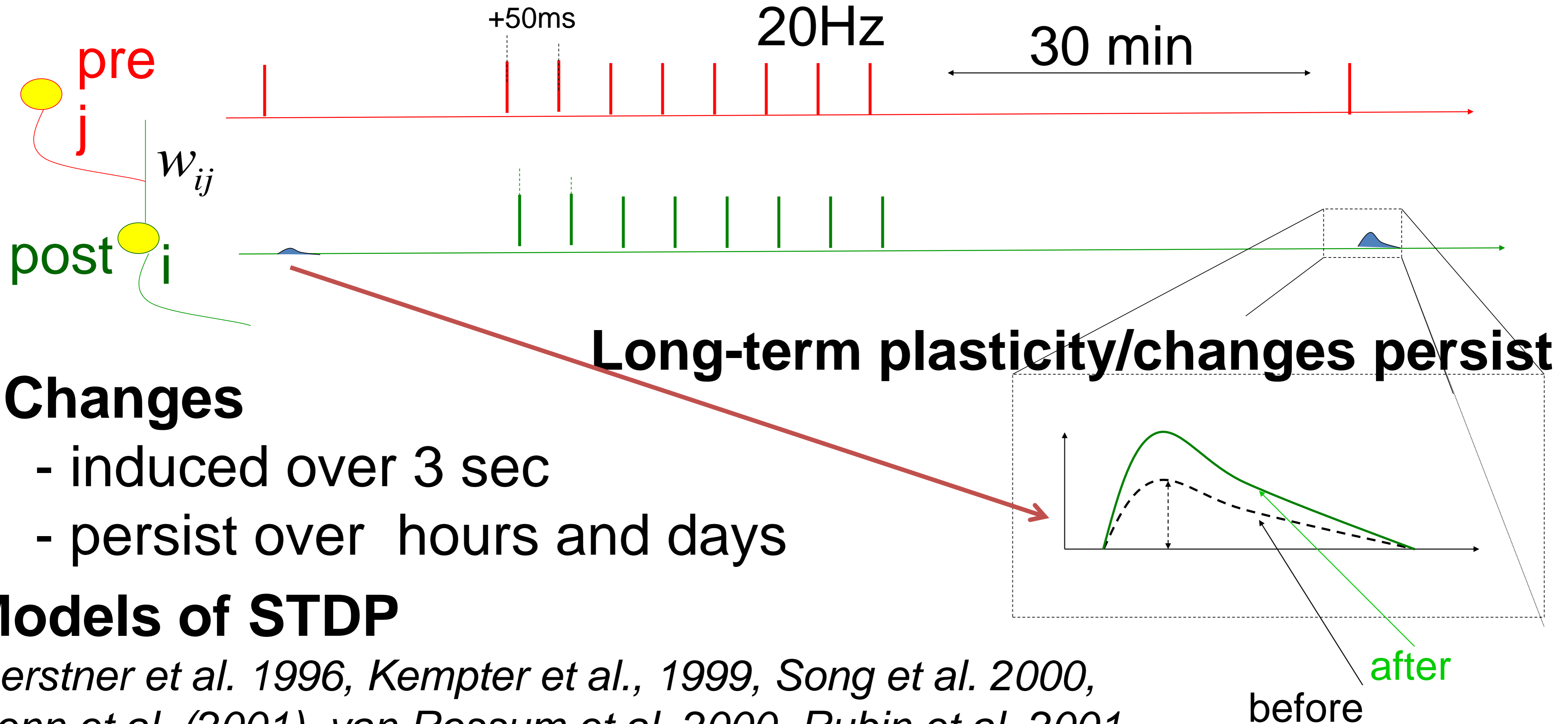
Experiments: Bliss and Lomo 1973, Levy and Stewart, 1983, ...

Markram et al. 1997, Bi and Poo, 1998, ...

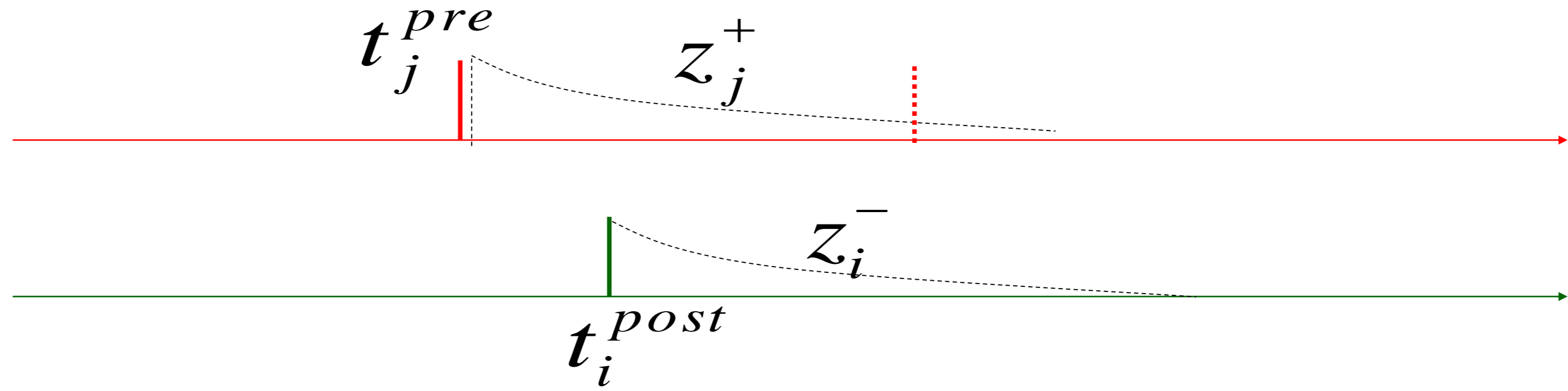
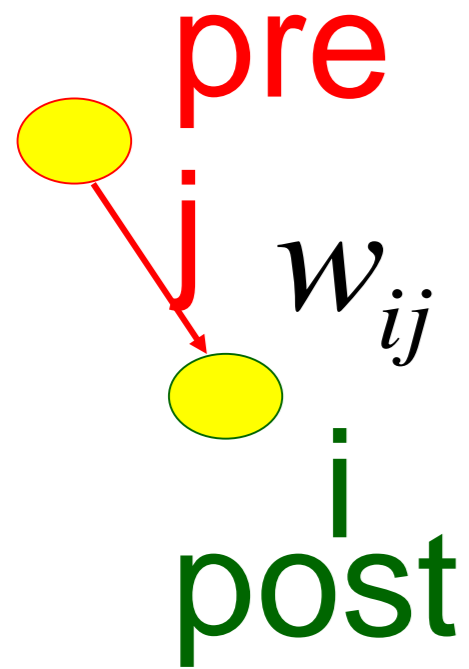
Reviews: Bliss and Collingridge, 1993, Sjostrom et al. 2008...

Markram et al. 2011, ...

5. STDP as Hebbian Learning



5. Spike-timing dependent plasticity: 'traces' for STDP



$$\tau_+ \frac{d}{dt} z_j^+ = -z_j^+ + \delta(t - t_j^{pre})$$

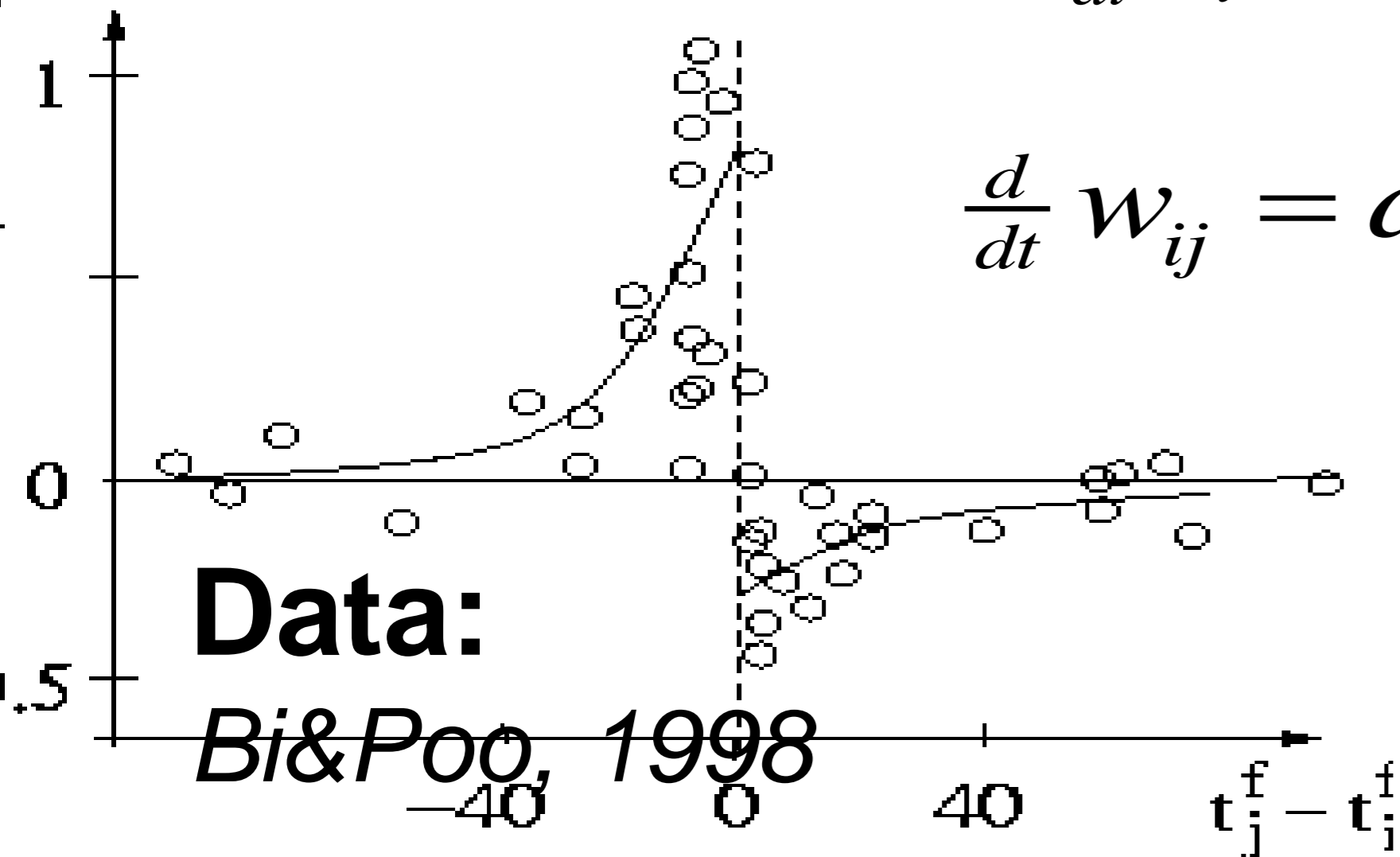
jump at presyn. spike

$$\tau_- \frac{d}{dt} z_i^- = -z_i^- + \delta(t - t_i^{post})$$

jump at postsyn. spike

$$\frac{d}{dt} w_{ij} = a(w_{ij}) z_j^+ \delta(t - t_i^{post}) - b(w_{ij}) z_i^- \delta(t - t_j^{pre})$$

pre-before-post
post-before-pre



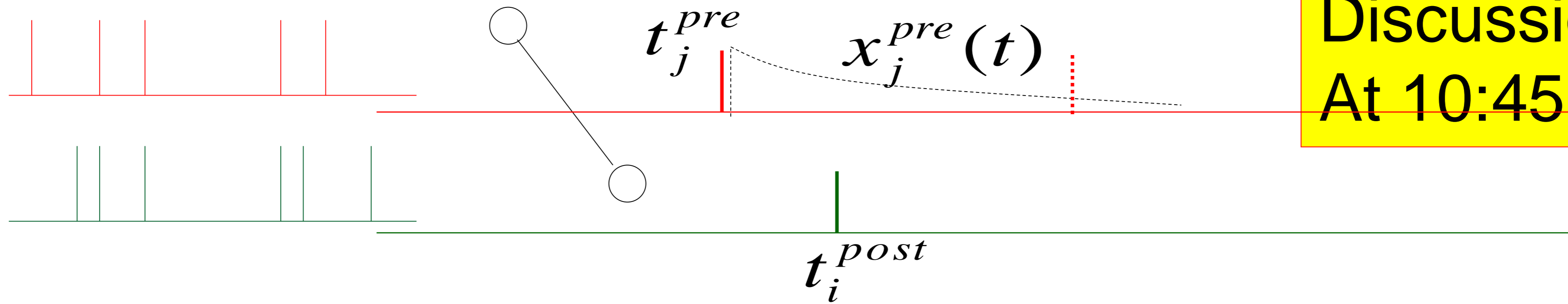
→ **Simple STDP model**

(Gerstner et al. 1996,

Song-Miller-Abbott 2000, etc)

Exercise STDP now:

Take 8 minutes =
Discussion of ex
At 10:45



- What is the shape of the STDP window?
- calculate the effect of one pair of spikes
- calculate the effect of many pairs of spikes

5. Summary: Spike-timing dependent plasticity (STDP)

STDP is a form of Hebbian learning induced by spikes. For a phenomenological model, we can take the view that each spike arriving at the presynaptic terminal leaves a trace at the synapse (e.g., amount of glutamate in the synaptic cleft, or bound to the postsynaptic receptor). If a spike of the postsynaptic neuron coincides with the trace left by the presynaptic spike, a change happens (proportional to the momentary value of the trace).

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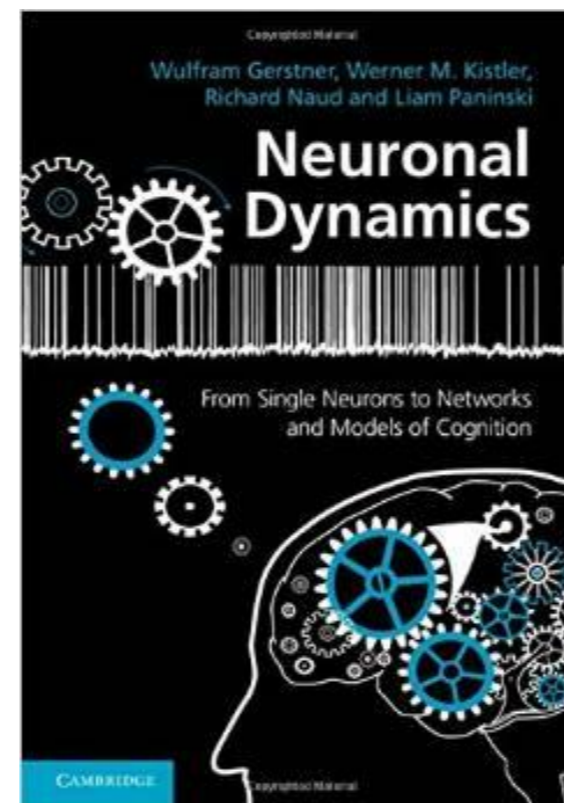
Synaptic plasticity and Learning

Wulfram Gerstner

EPFL, Lausanne, Switzerland

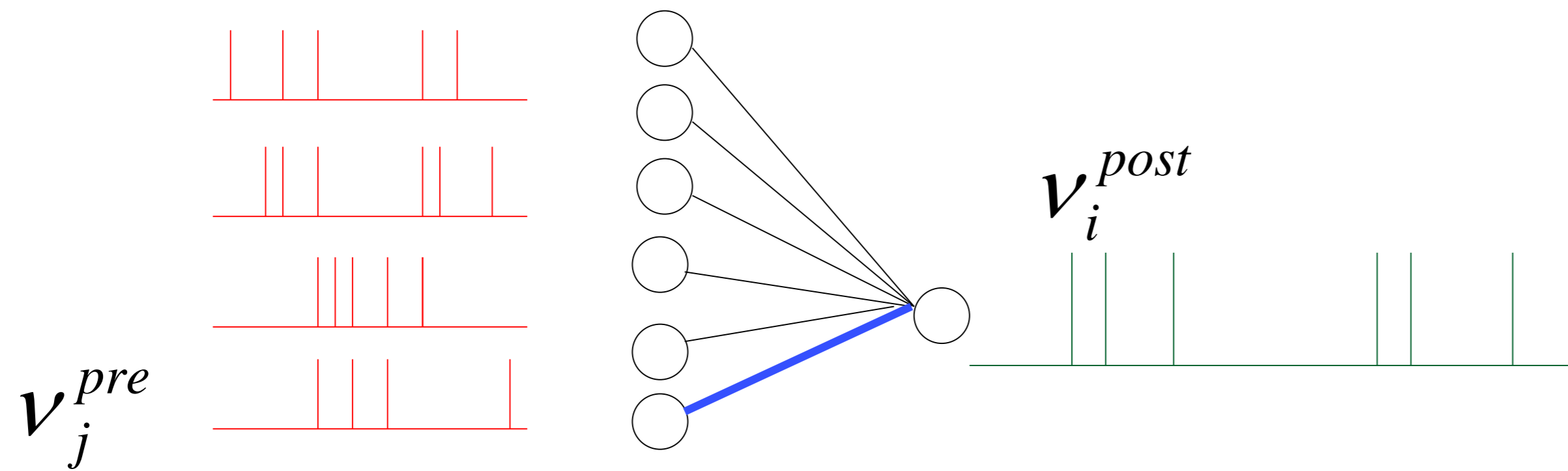
Reading for plasticity:
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- Ch. 19.1-19.3

Cambridge Univ. Press



- 1. Synaptic plasticity**
motivation and aims
- 2. Classification of plasticity**
short-term vs. long-term
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- 3. Model of short-term plasticity**
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 - Hebbian learning rules
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- 5. Spiking Models of plasticity**
 - STDP as Hebbian learning
 - Model of STDP: synaptic traces
- 6. From STDP to rate models**
- 7. Triplet STDP model**
- 8. Online learning of memories**

6. from STDP to rate models

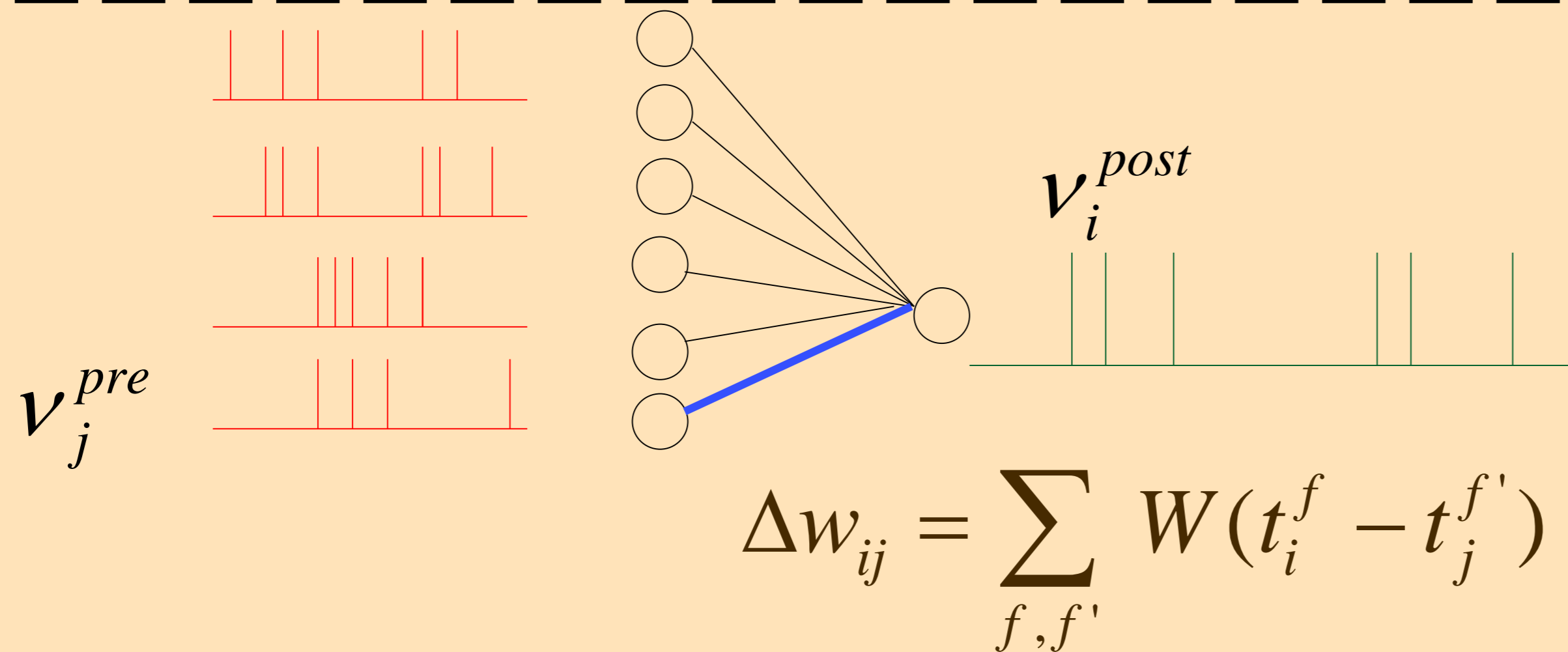


$$\Delta w_{ij} = \sum_{f, f'} W(t_i^f - t_j^{f'})$$

$$\frac{1}{T} \Delta w_{ij} = \frac{1}{T} \int_0^T \int_{-\infty}^{\infty} W(s) S_i(t) S_j(t+s) ds$$

Exercise 4a: STDP to rate now:

Take 8 minutes =
Discussion of ex
At 11:20



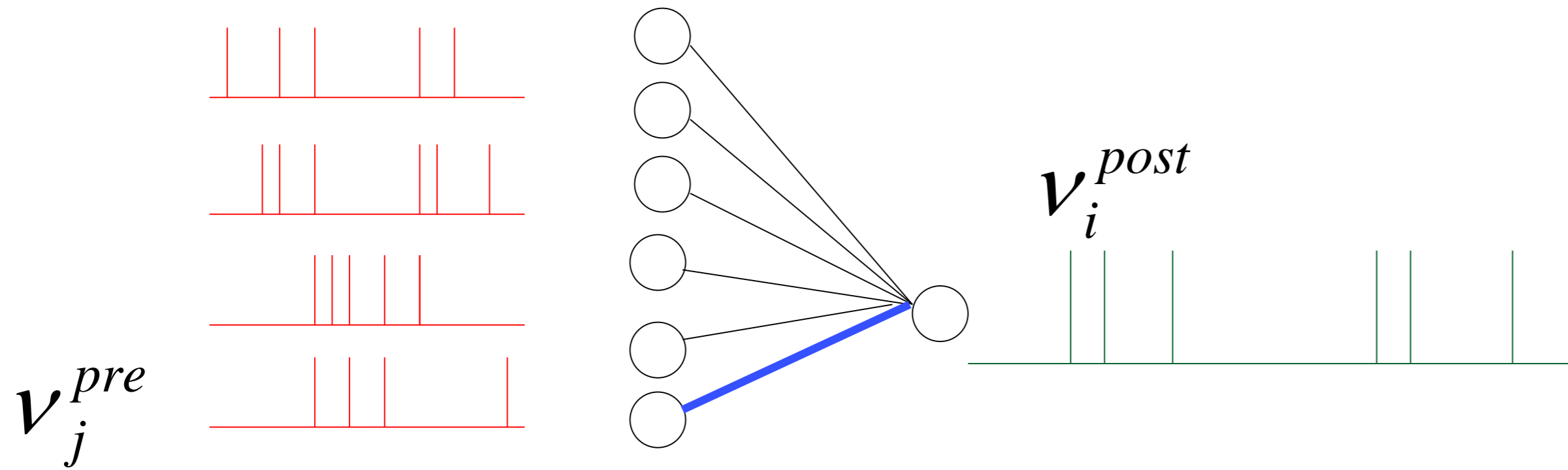
Assume presynaptic spikes are generated by Poisson process
with rate v_j^{pre}

Assume postsynaptic spikes are generated by Poisson process
with rate v_i^{post}

What is the expected change of weights in a time T ?

$$(T \gg \tau^{LTP}, \tau^{LTD})$$

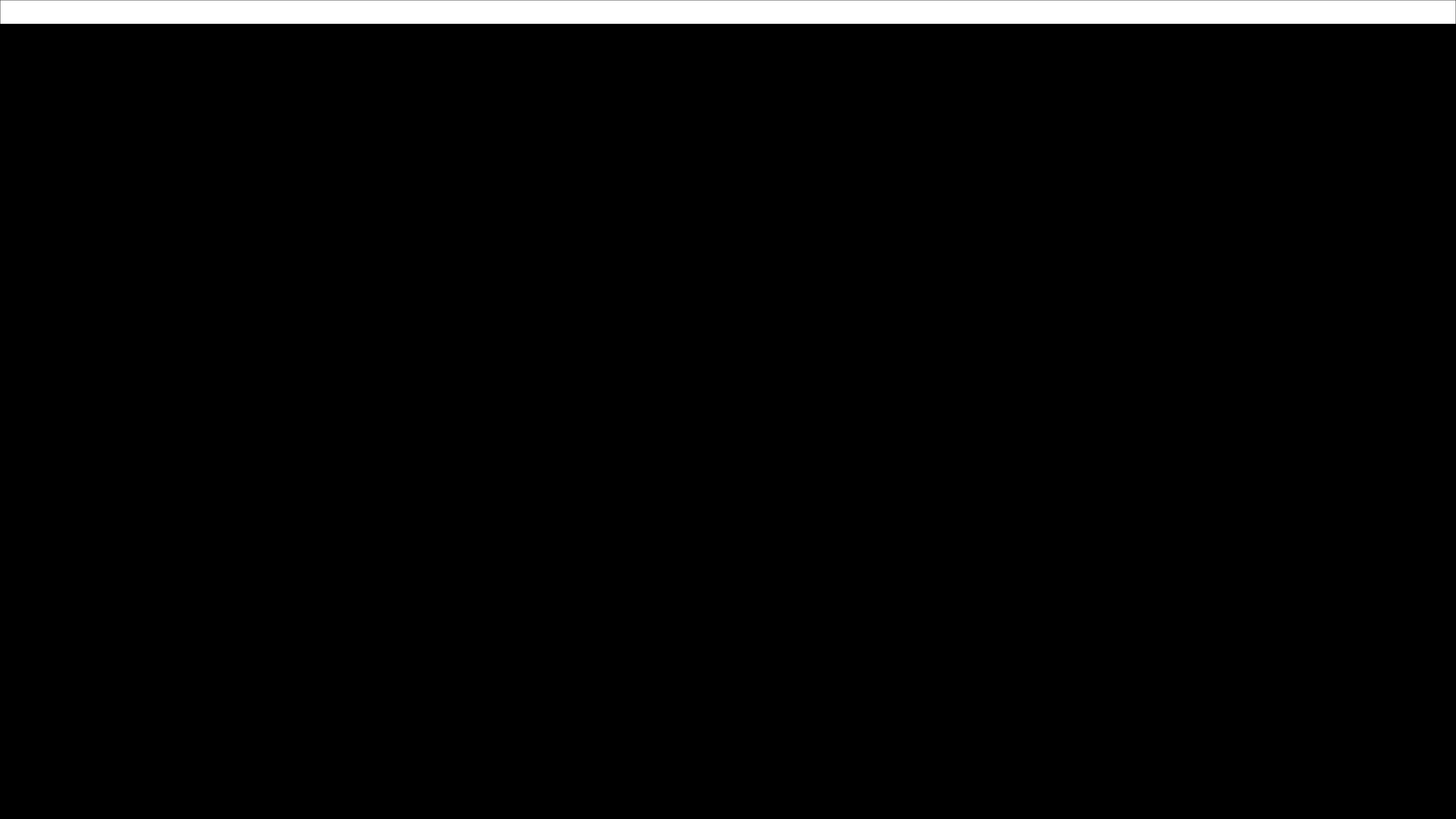
6. from STDP to rate models



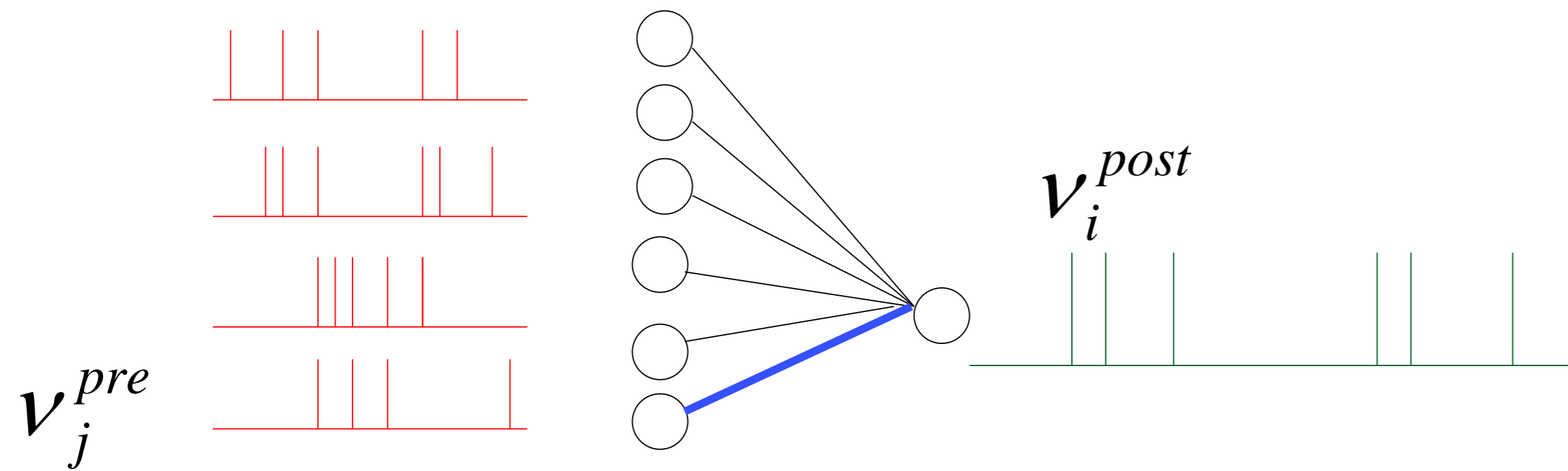
$$\Delta w_{ij} = \sum_{f, f'} W(t_i^f - t_j^{f'})$$

$$\frac{1}{T} \Delta w_{ij} = \frac{1}{T} \int_0^T \int_{-\infty}^{\infty} W(s) S_i(t) S_j(t+s) ds$$

Blackboard2



6. from STDP to rate models



$$\Delta w_{ij} = \sum_{f, f'} W(t_i^f - t_j^{f'})$$

$$\frac{1}{T} \Delta w_{ij} = \frac{1}{T} \int_0^T \int_{-\infty}^{\infty} W(s) S_i(t) S_j(t+s) ds$$

$$\frac{d}{dt} w_{ij} = S_i(t) \int_0^{\infty} W_+(-s) S_j(t-s) ds + S_j(t) \int_0^{\infty} W_-(s) S_i(t-s) ds$$

6. Summary: from STDP to rate models

In an STDP model, changes of synapses depend on the exact timing of pre- and postsynaptic spikes.

However, if we assume that both presynaptic and postsynaptic spike trains are generated by a homogeneous Poisson Process (with stationary firing rates ν_i and ν_j), we can translate the effect induced by STDP after many spikes into an equivalent rate model by evaluation the expected change.

The standard STDP window gives then a rate model

$$c \nu_i \nu_j$$

where c is the integral over the STDP window.

Expectations and Correlations of Poisson spike train:

see week 11.2 or

Watch video 'Membrane Potential fluctuations' on:

<http://lcn.epfl.ch/~gerstner/NeuronalDynamics-MOOC1.html>

direct link:

<https://www.youtube.com/watch?v=YTQqOyrFQQ4>

Computational Neuroscience: Neuronal Dynamics of Cognition

EPFL

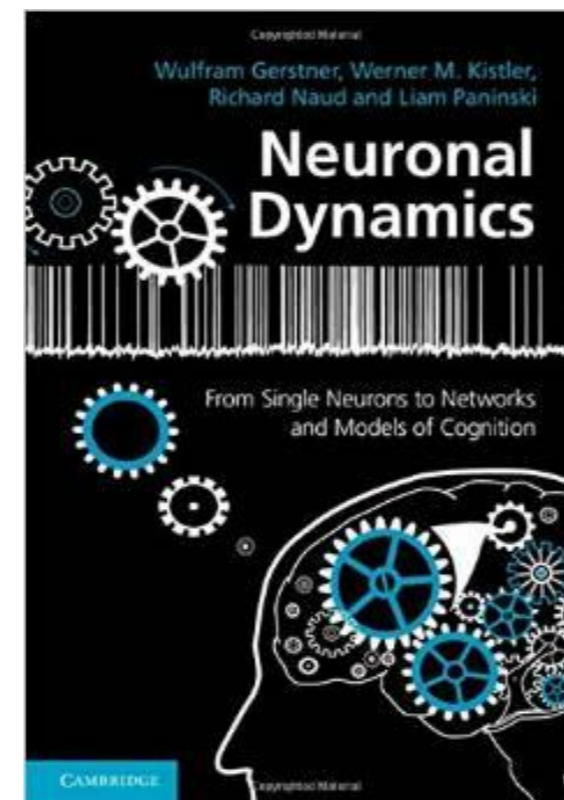
Synaptic Plasticity and Learning

Wulfram Gerstner

EPFL, Lausanne, Switzerland

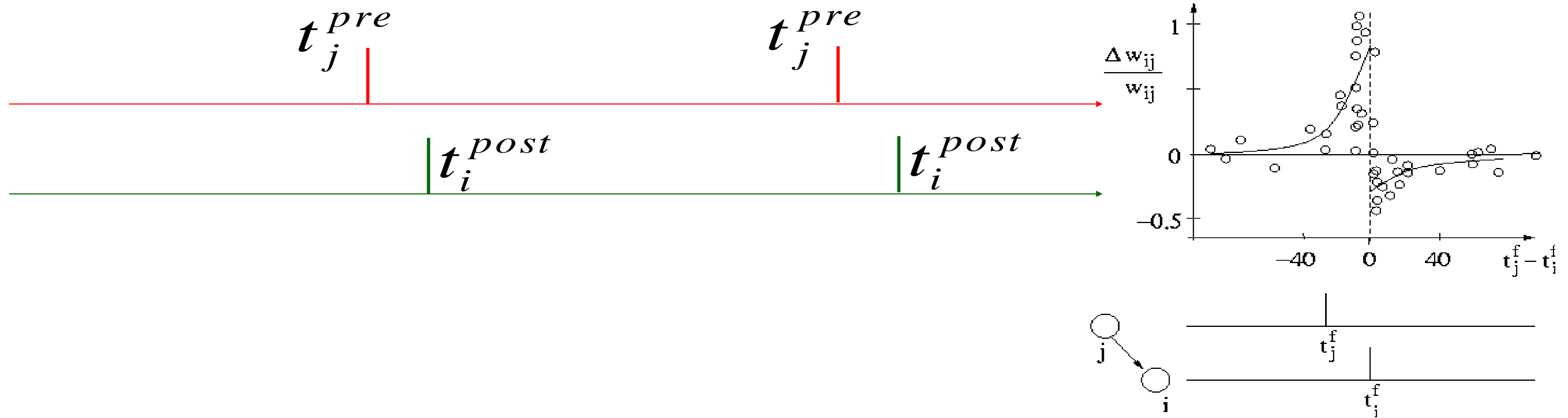
Reading for plasticity:
NEURONAL DYNAMICS
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1. Synaptic plasticity: aims
2. Classification of plasticity
3. Model of short-term plasticity
4. Models of long-term plasticity
5. Spike-Timing Models of plasticity
 - STDP as Hebbian learning
 - Model of STDP: synaptic traces
6. From spiking models to rate models
7. Triplet STDP model
 - Relation to experiments
 - Relation to BCM rule
8. Online learning of memories

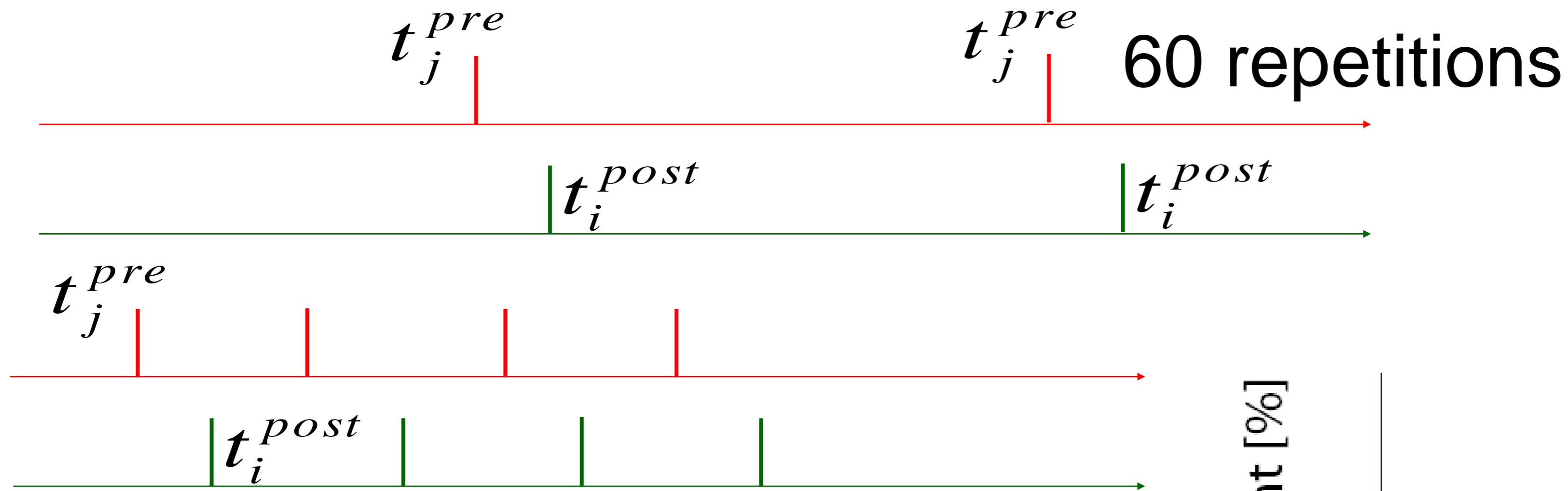
7. Why do we need a Triplet STDP model?



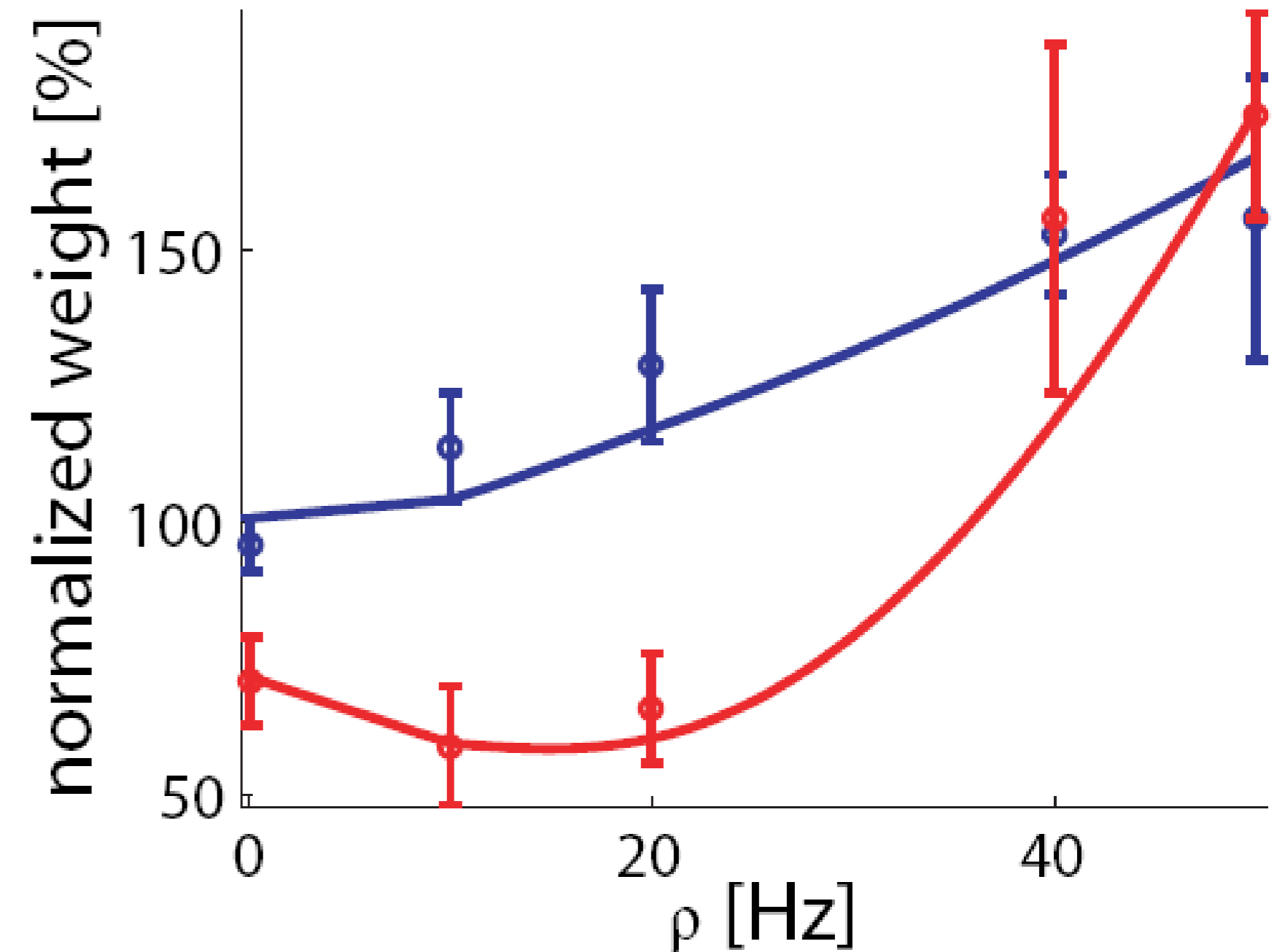
STDP window
is only part of story

Pair-based STDP model
is not sufficient

7. frequency dependenc of STDP

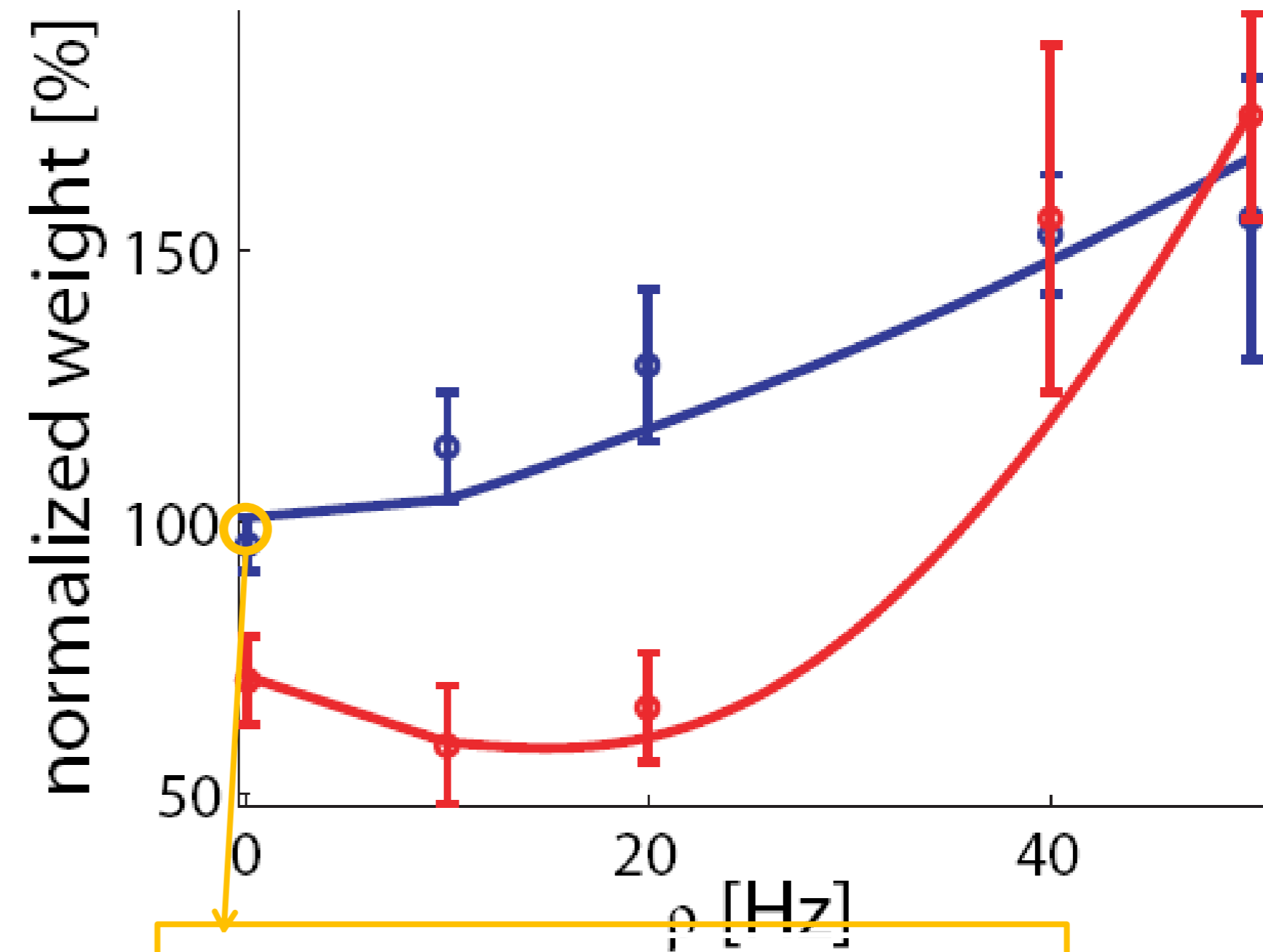


Sjostrom et al. 2001
See also:
Markram et al. 1997,
Senn et al. 2001,



7. frequency dependence of STDP

- Repetition frequency important
- No LTP at 0.1Hz

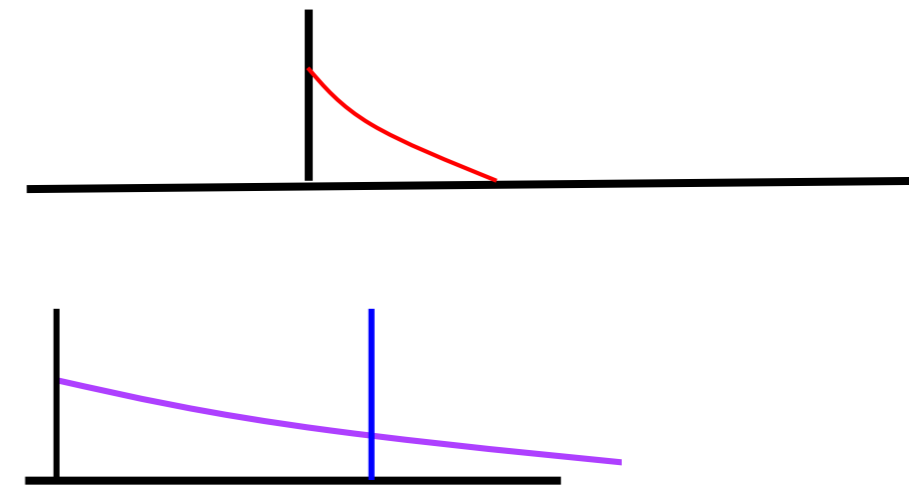


No plasticity
At low frequency

7. Triplet STDP model

Triplet
LTP

$$\frac{d}{dt} w_j^+ = +A^+ z_j^+ \underline{z_i^{slow}} \delta(t - t_i^{post})$$



pre

post

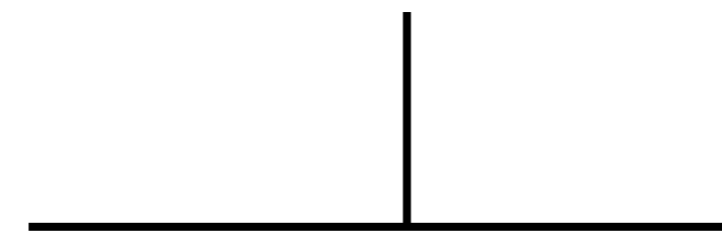
post

Triplet

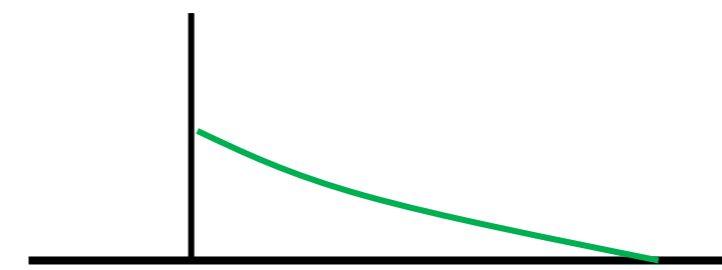
7. Triplet STDP model

Pfister and Gerstner, 2006

$$-B \underline{z_i^-} \delta(t - t_j^{pre})$$



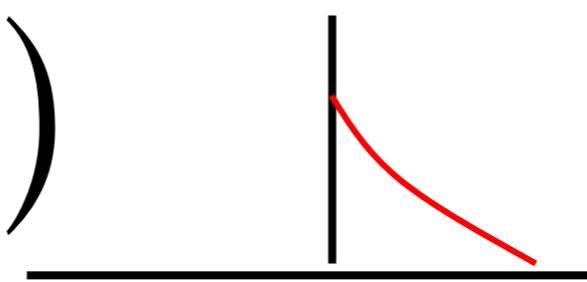
Pre: spike



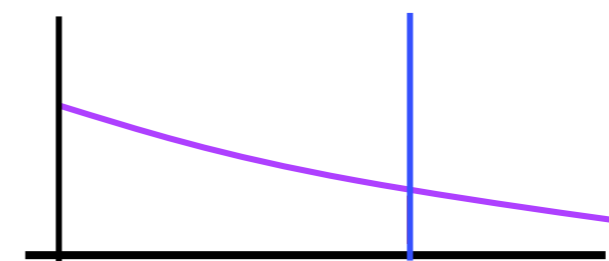
Post: spike-trace

$$\frac{d}{dt} W_j =$$

$$+A^+ z_j^+ z_i^{slow} \delta(t - t_i^{post})$$



Pre: spike-trace

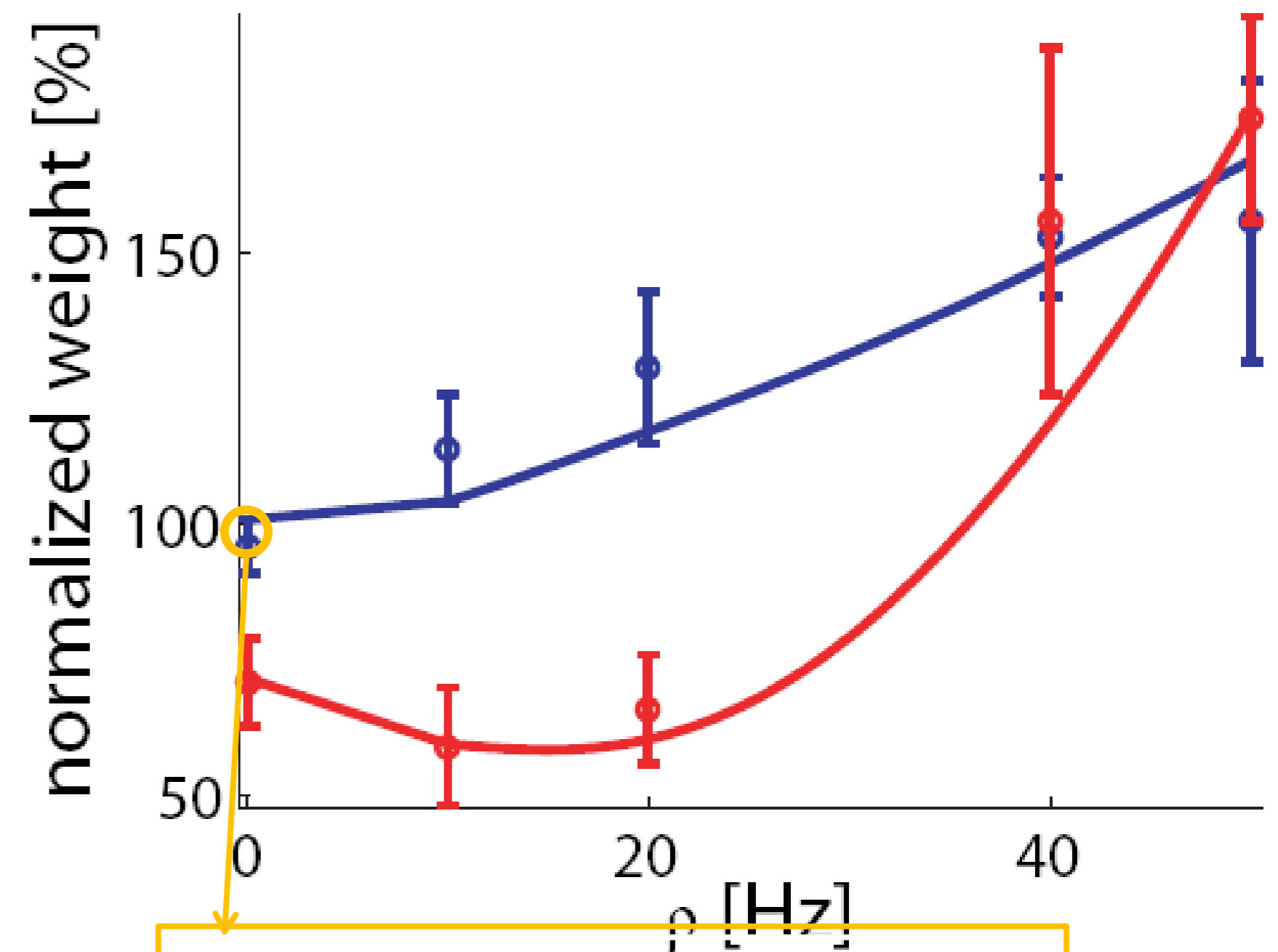


Post: spike-now
spike-trace

7. Triplet STDP model

Pfister and Gerstner, 2006

Similar triplet mechanism in
Senn et al. 2001,
Rubin et al. 2005,
Clopath et al. 2010

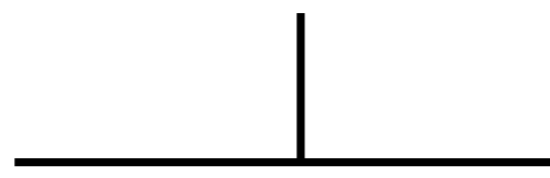


No plasticity
At low frequency

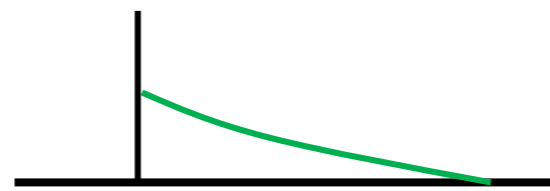
7. Triplet STDP model \rightarrow BCM model

$$\frac{d}{dt} W_j =$$

$$-B \underline{z_i^-} \delta(t - t_j^{pre})$$

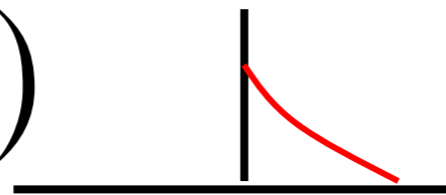


Pre: spike

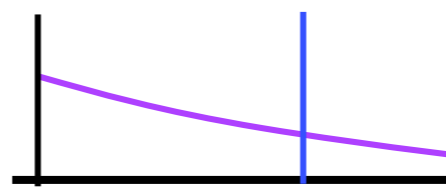


Post: spike-trace

$$+A^+ z_j^+ z_i^{slow} \delta(t - t_i^{post})$$



Pre: spike-trace



Post: spike-now
slow spike-trace

Assume Poisson firing

$$\frac{d}{dt} W_j =$$

7. Summary: Triplet STDP → BCM model

Triplet STDP model

- parameters can be extracted from experimental data
- for Poisson spikes closely related to rate-based BCM
- but captures additional spike-timing effects
- simple pair-based STDP model is not sufficient, because STDP depends also on repetition frequency (and not only on relative timing of pairs of spikes).

Biological Modeling of Neural Networks

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Week 13

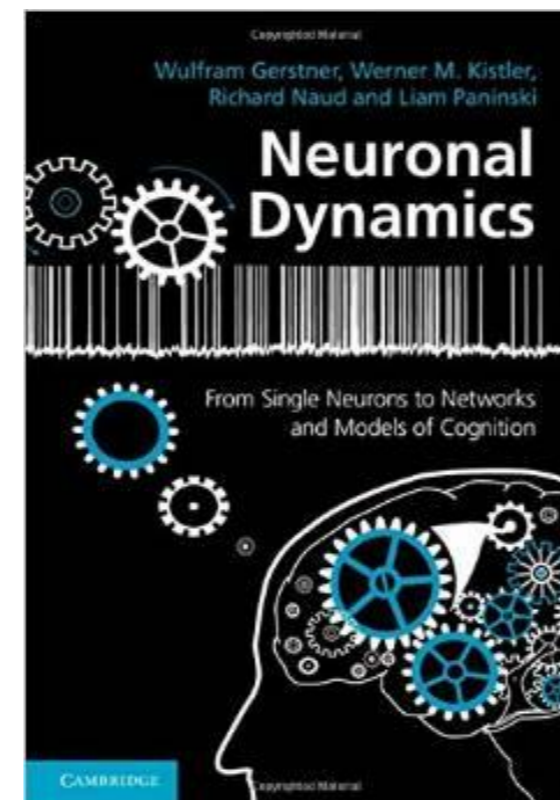
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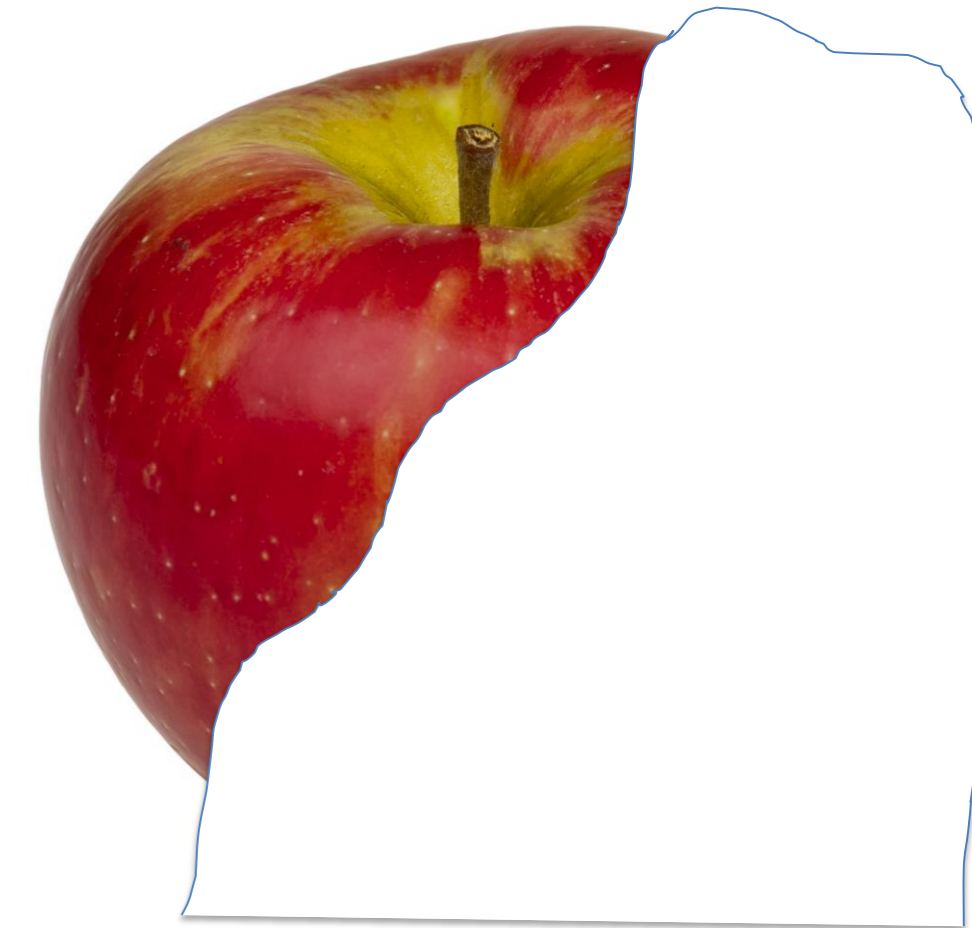
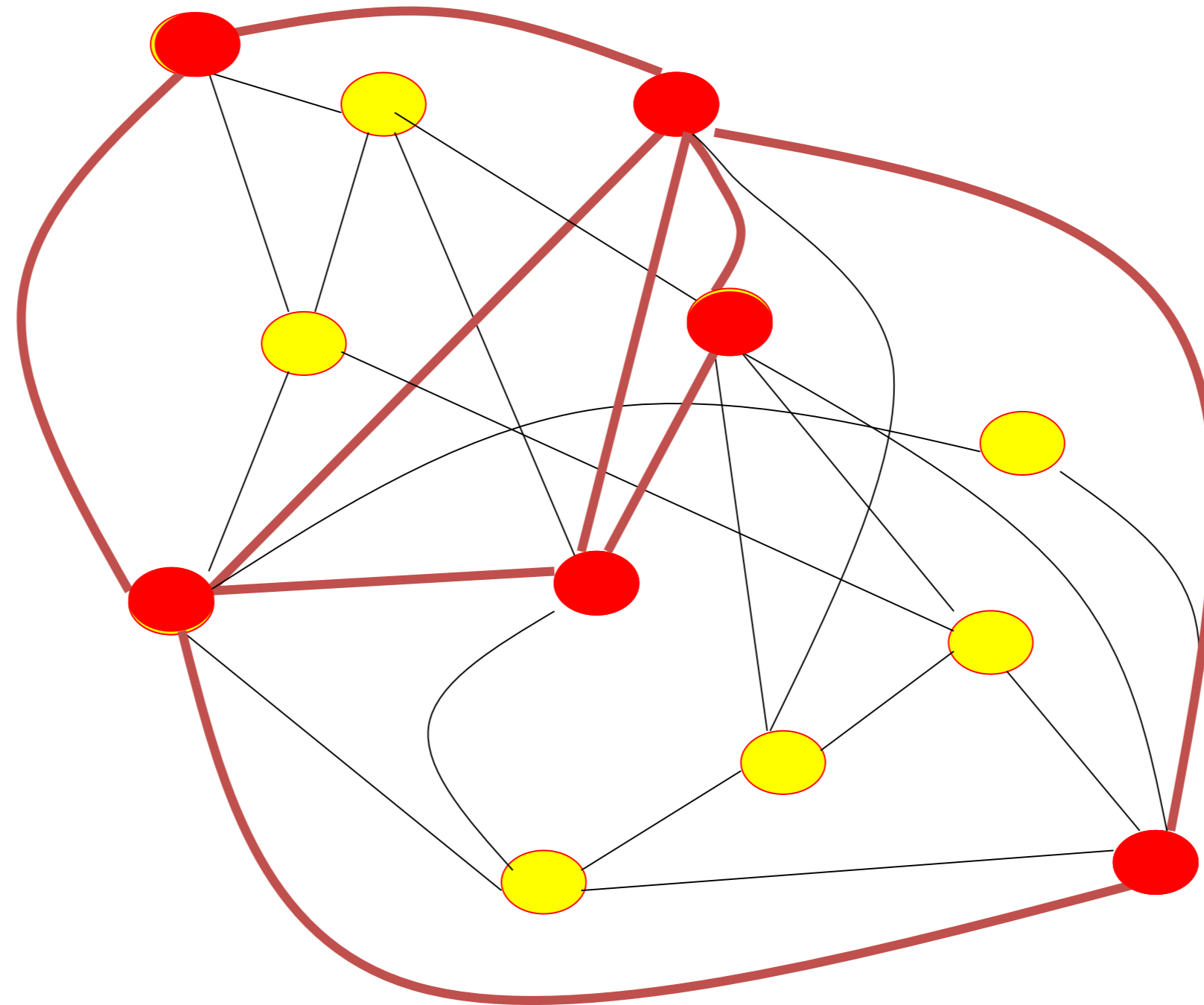
6. From STDP to rate models

7. Triplet STDP model

8. Online learning of memories

8. Review: Hebbian assemblies

Recall:
Partial info

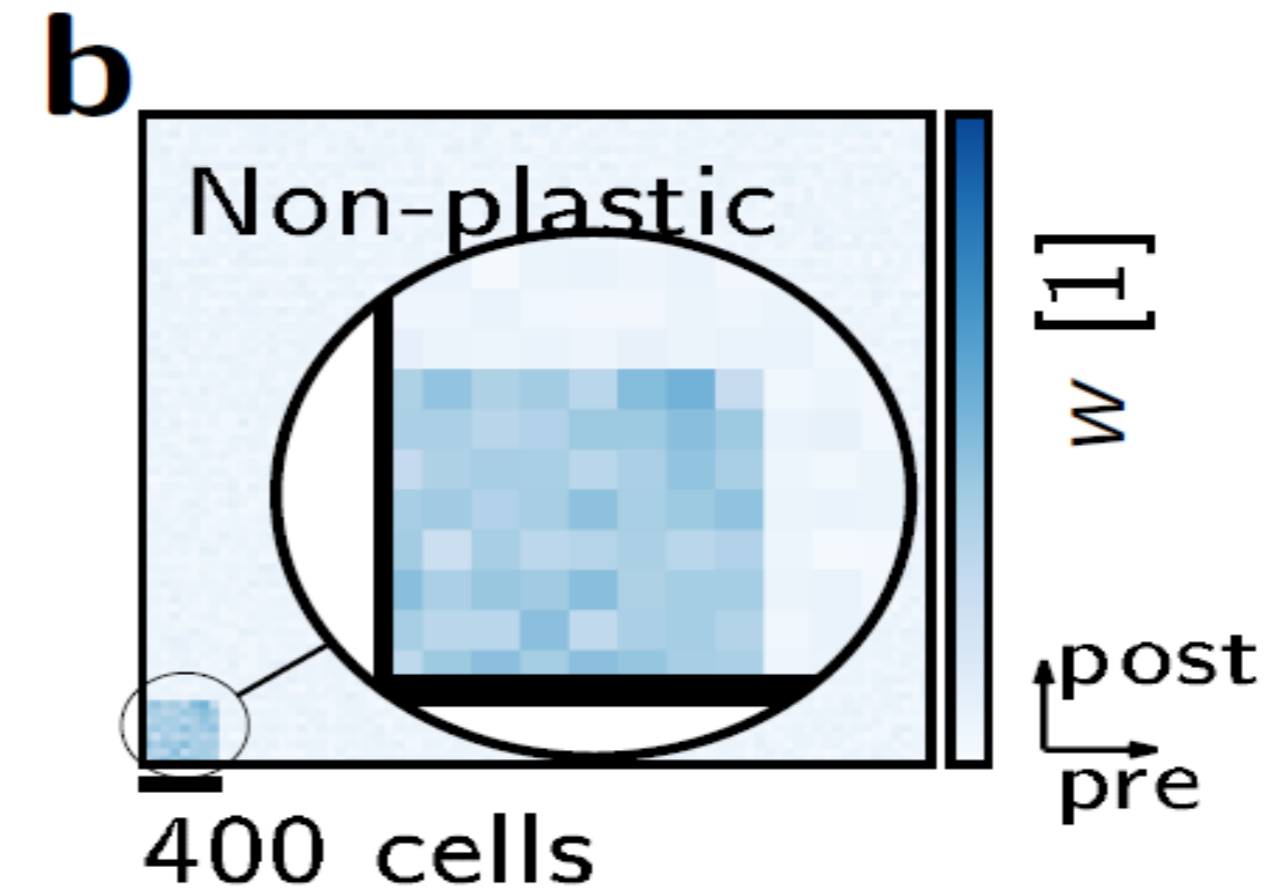
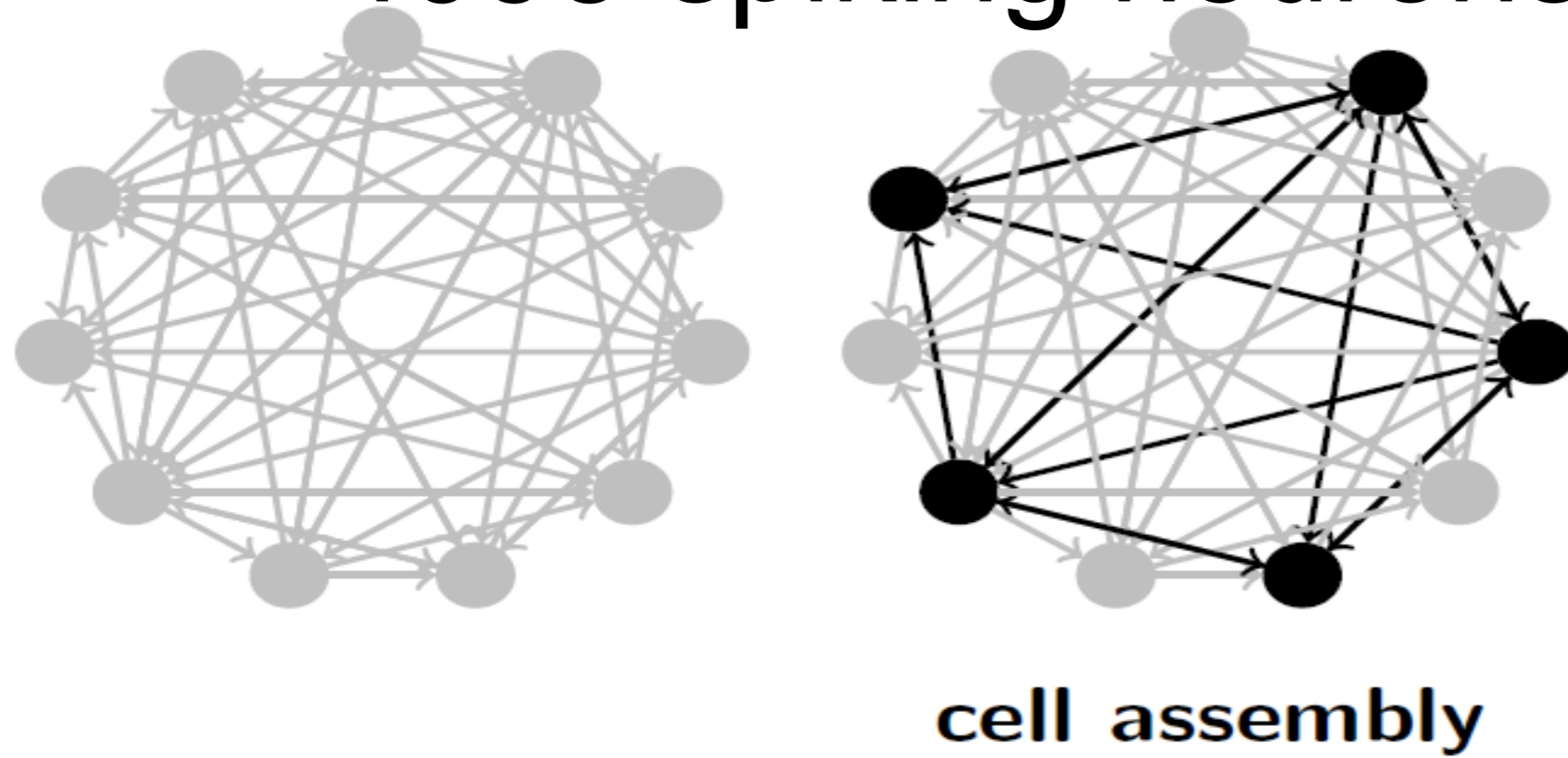


item recalled

8. Preconfigured memory: bistable network

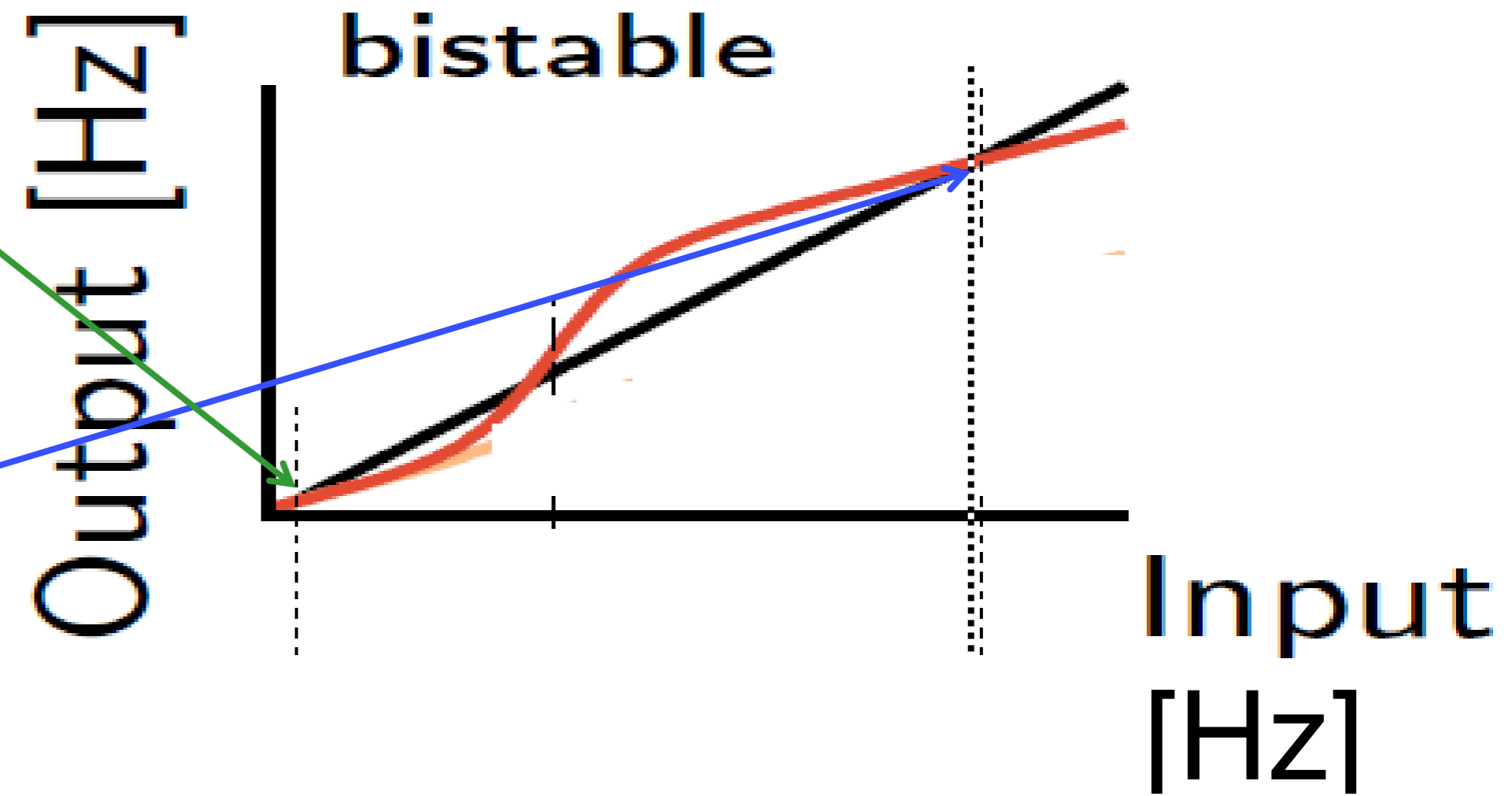
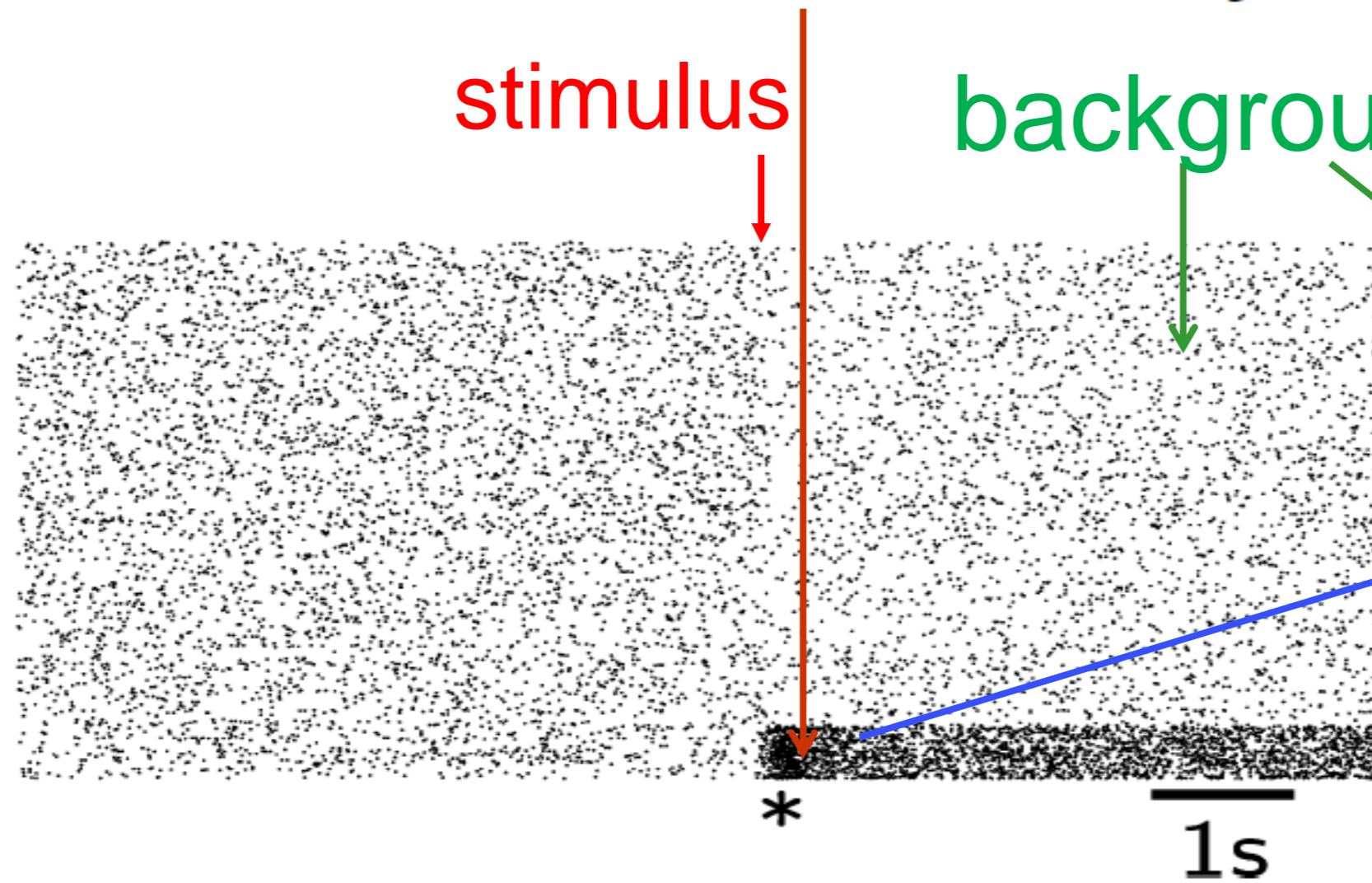
e.g., groups of Hopfield, Amit, Brunel, Fusi, Sompolinsky, Tsodyks,

a 4096 spiking neurons



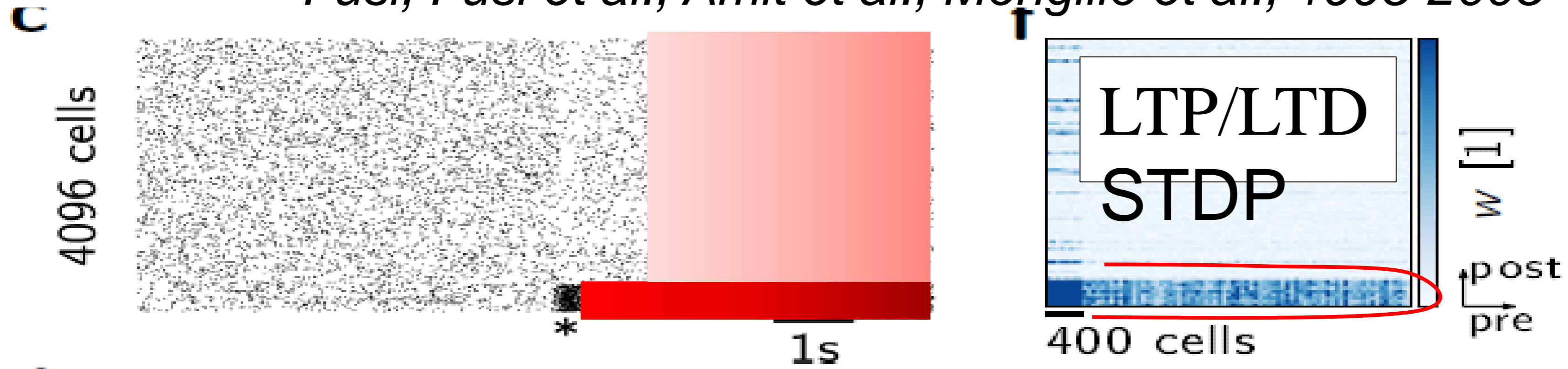
c

4096 cells

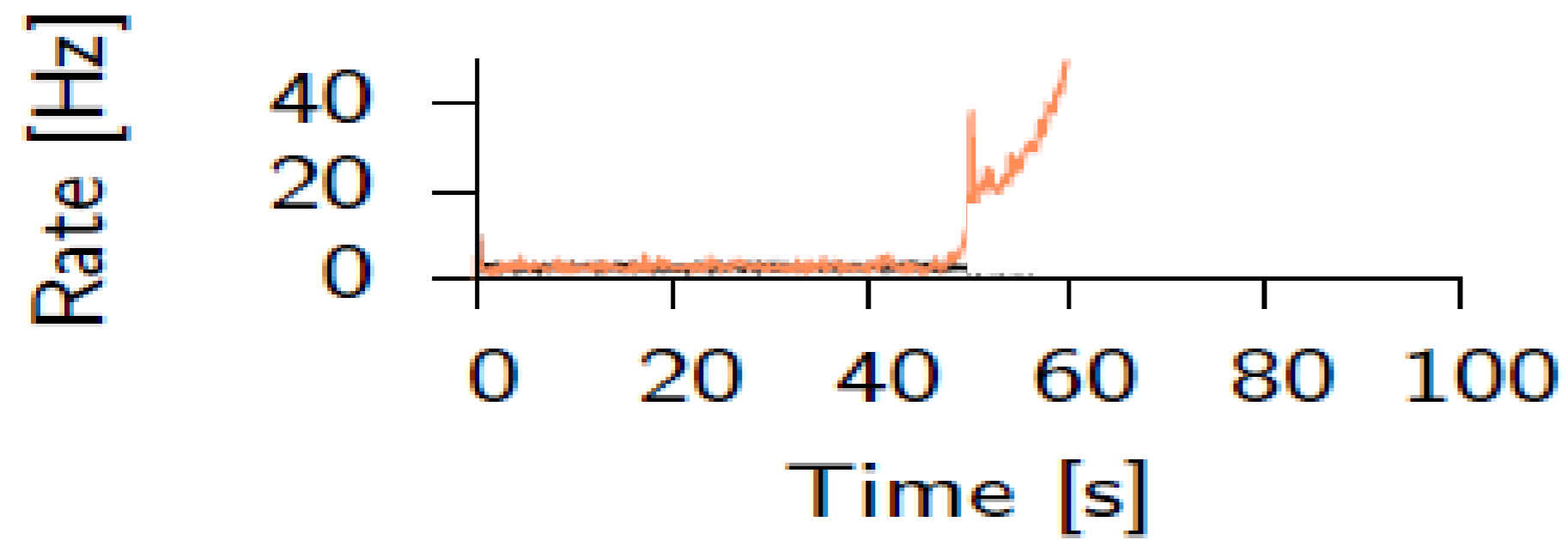


8. Learning the memory: very hard

Fusi, Fusi et al., Amit et al., Mongillo et al., 1995-2005



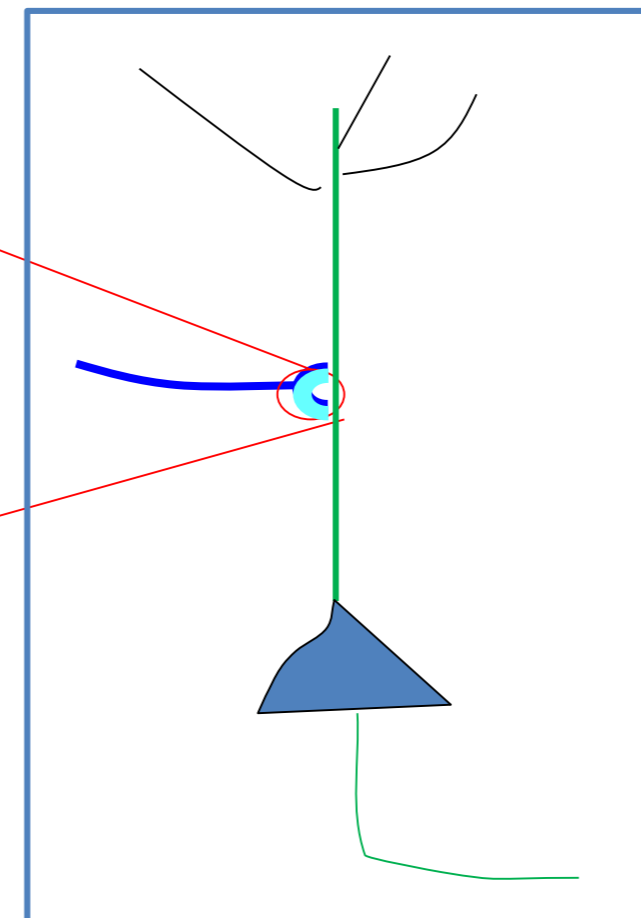
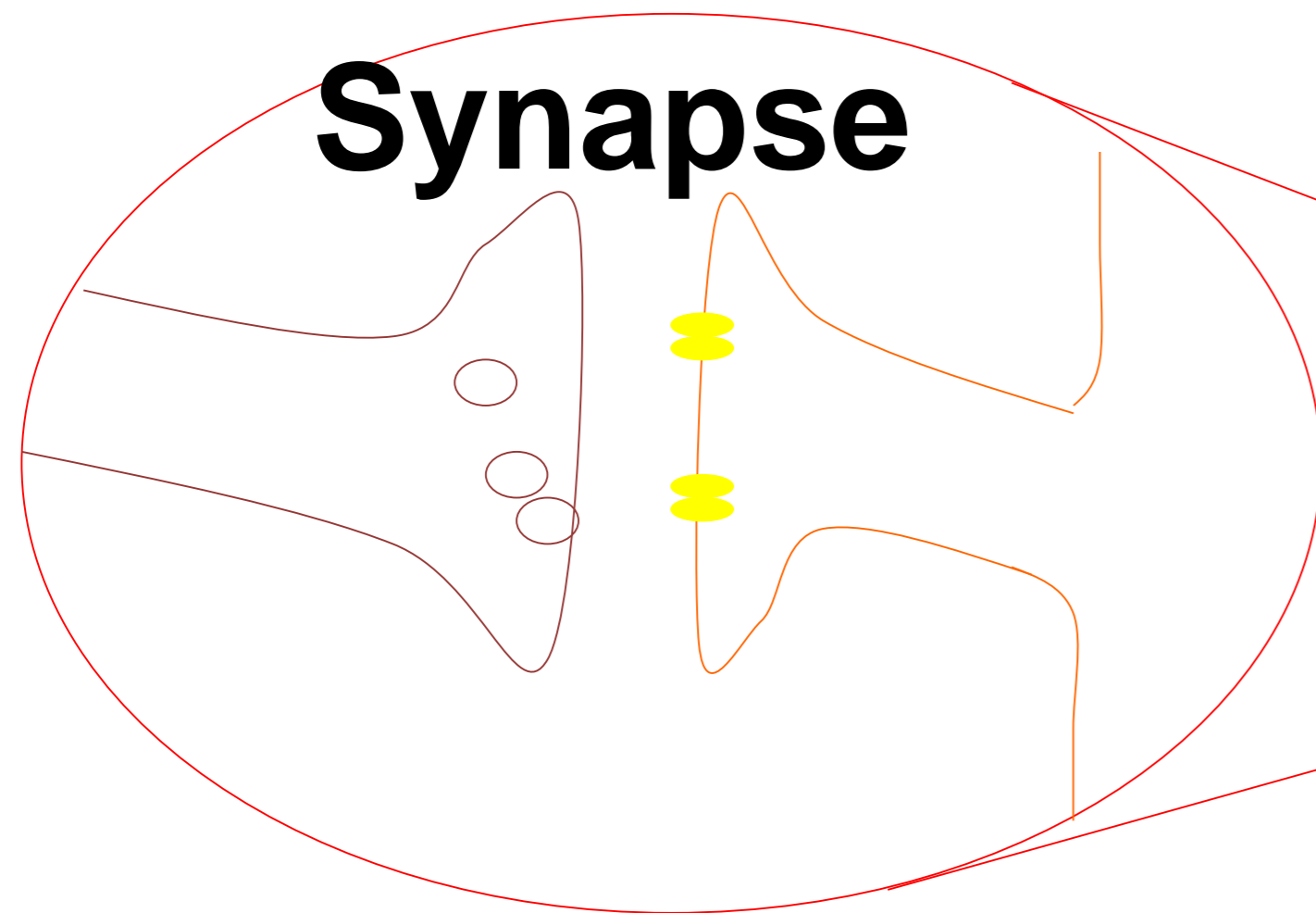
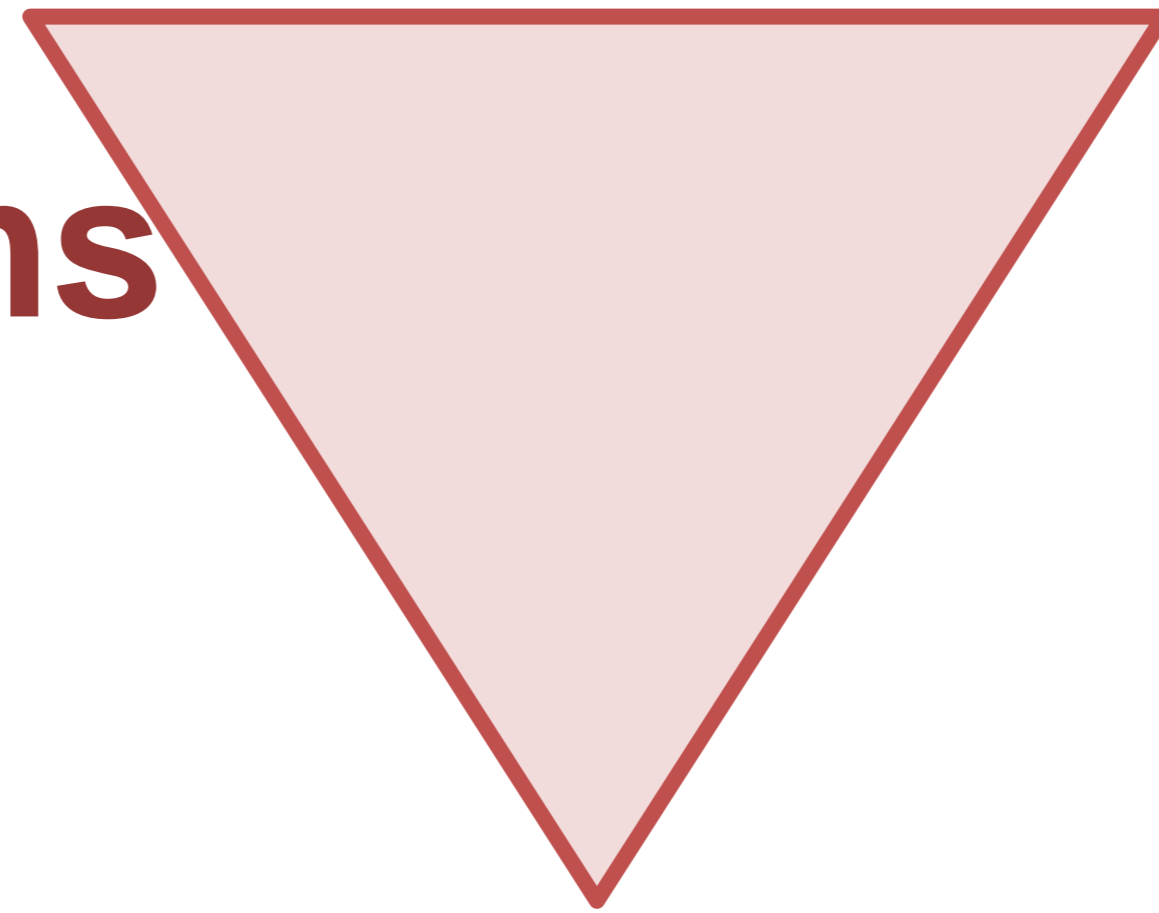
stimulus



8. Learning: the task of modeling

Learning Algorithms

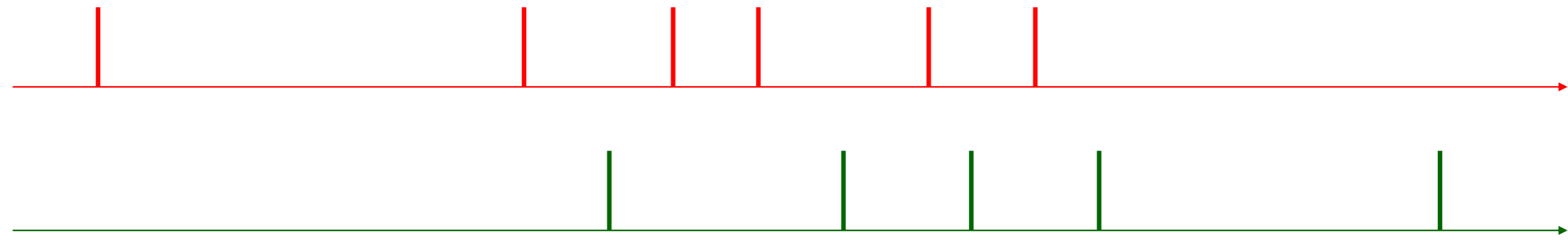
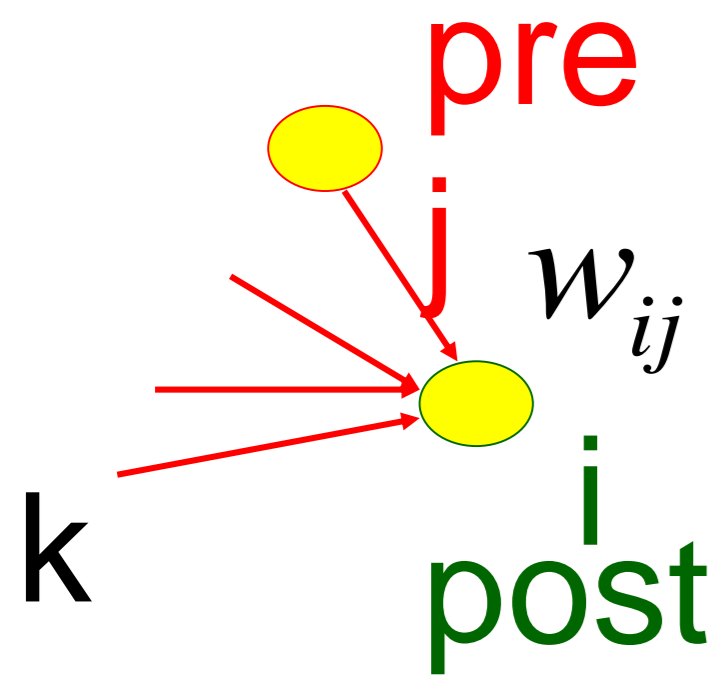
Functional or Behavioral Consequences



**Memory formation
Memory retention
Network stability
Plasticity data**

Synaptic Plasticity

8. Review: Rate models of Hebbian learning



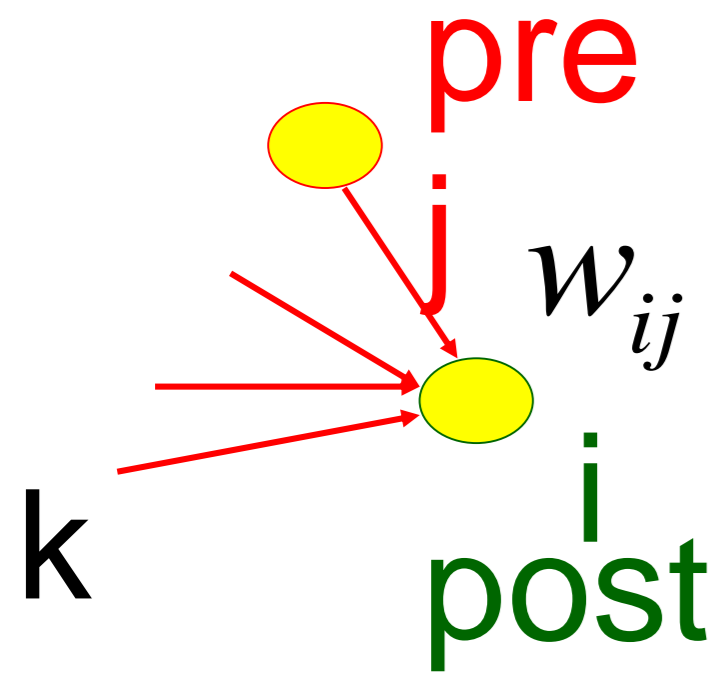
$$\frac{dw_{ij}}{dt} = F(w_{ij}; \overset{\text{rate}}{v_j^{pre}}, \overset{\text{rate}}{v_i^{post}})$$

- local rule
- simultaneously active

$$\frac{dw_{ij}}{dt} = a_0 + a_1^{pre} v_j^{pre} + a_1^{post} v_i^{post} + a_2^{corr} \underbrace{v_j^{pre} v_i^{post}}_{\text{pair}} +$$

depend on w_{ij}

8. Induction of Plasticity



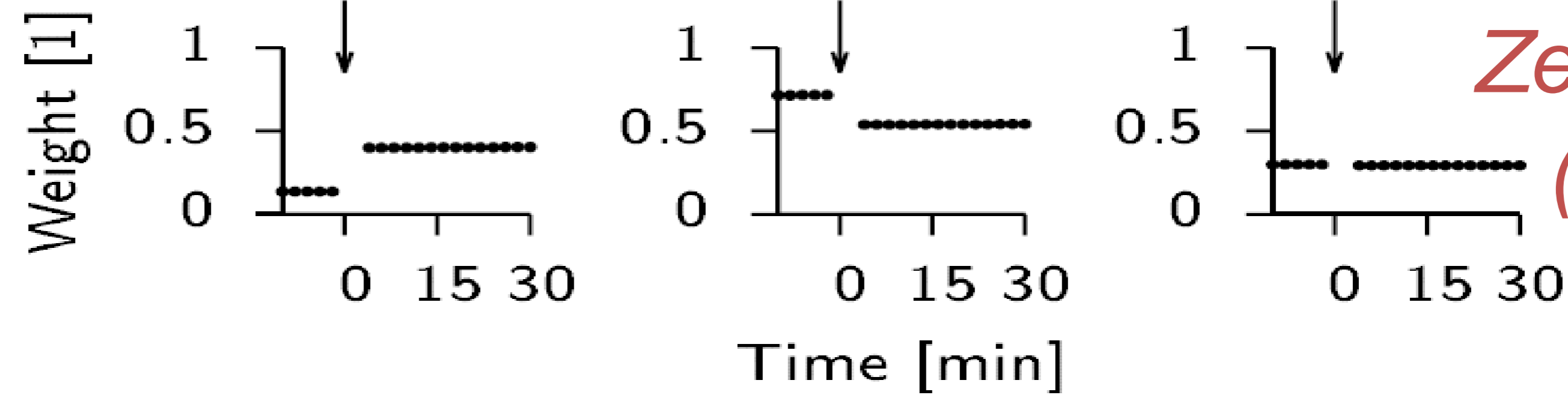
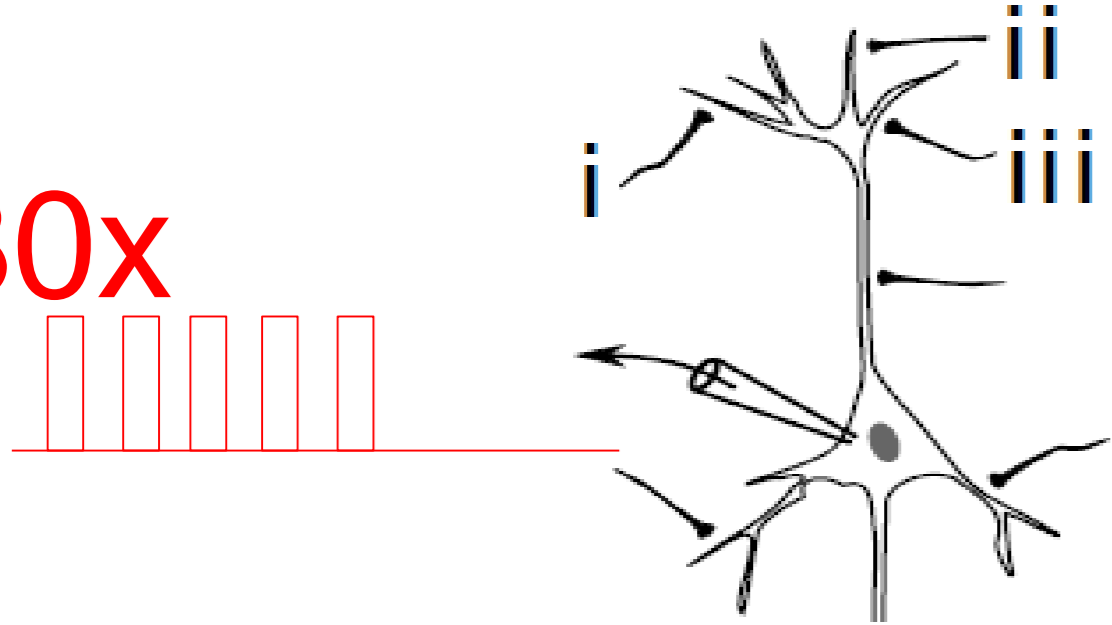
- **homosynaptic/Hebb** ('pre' and 'post')
- **heterosynaptic plasticity** (pure 'post'-term)
- **transmitter-induced** (pure 'pre'-term)

$$\frac{dw_{ij}}{dt} = a_0 + a_1^{pre} v_j^{pre} + a_1^{post} v_i^{post} + a_2^{corr} v_j^{pre} v_i^{post} +$$
$$+ a_3^{BCM} v_j^{pre} (v_i^{post})^2 + a_4^{post} (w_{ij}) [v_i^{post}]^4$$

8. Heterosynaptic Plasticity (exper. and model)

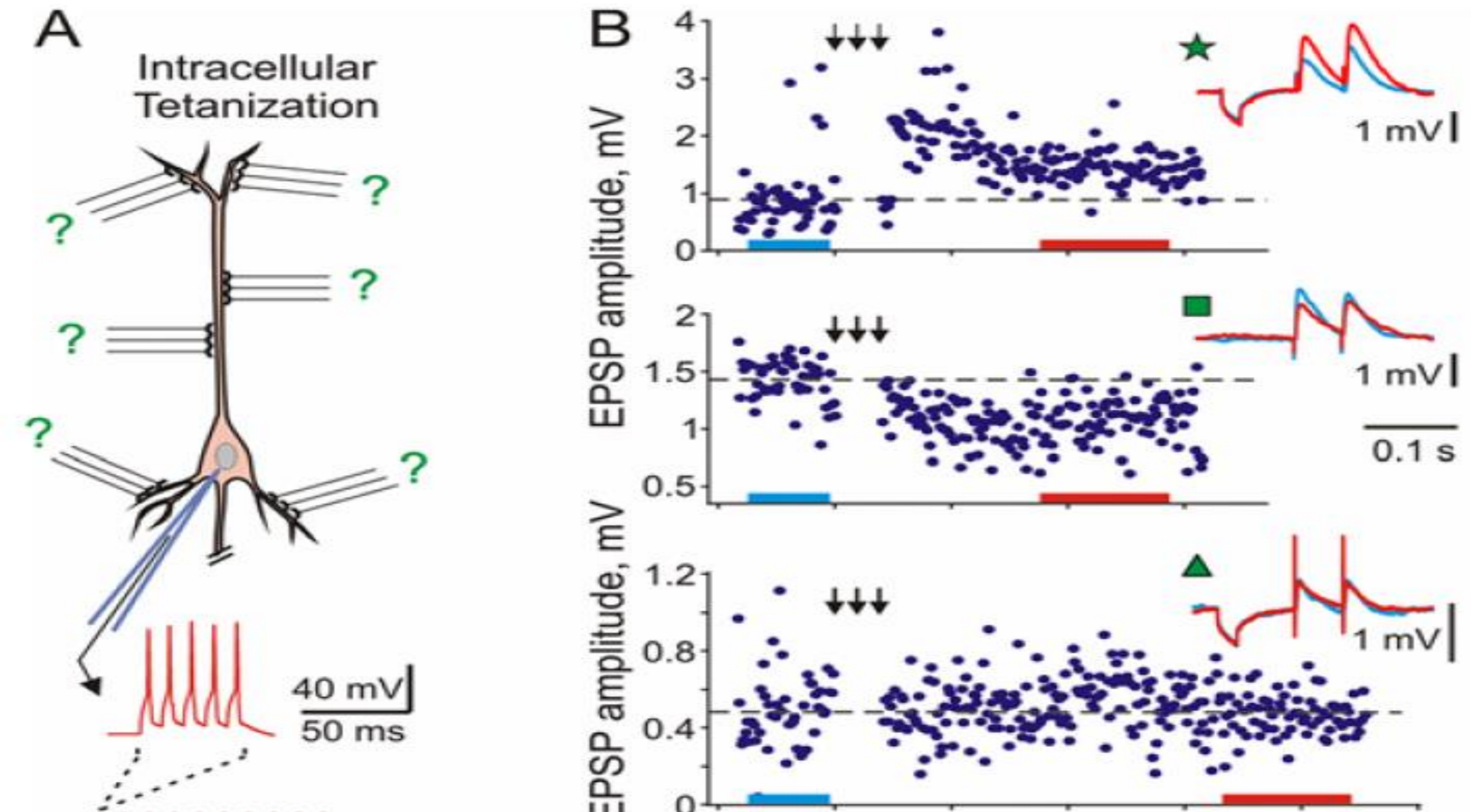
3 trains at 1/60 Hz, 10 bursts per train at 1 Hz, 5 spikes per burst at 100 Hz

30x



Zenke et al. (2015)

Experiments



Chen et al. 2013,
Chistiakova et al. 2014
See also:
Lynch et al. 1977

8. Induction of Plasticity (rate-based)

- **nonlinear Hebb for potentiation**

$$+a_3^{BCM} v_j^{pre} (v_i^{post})^2$$

- **pre-post for depression**

$$-a_2^{LTD} v_j^{pre} v_i^{post}$$

Bienenstock et al., 1982
Pfister and Gerstner, 2006

- **heterosynaptic plasticity (pure 'post')**

$$-a_4^{het} (w_{ij} - z_{ij}) [v_i^{post}]^4$$

- **transmitter-induced (pure 'pre')**

$$+a_1^{pre} v_j^{pre}$$

8. Plasticity model in network

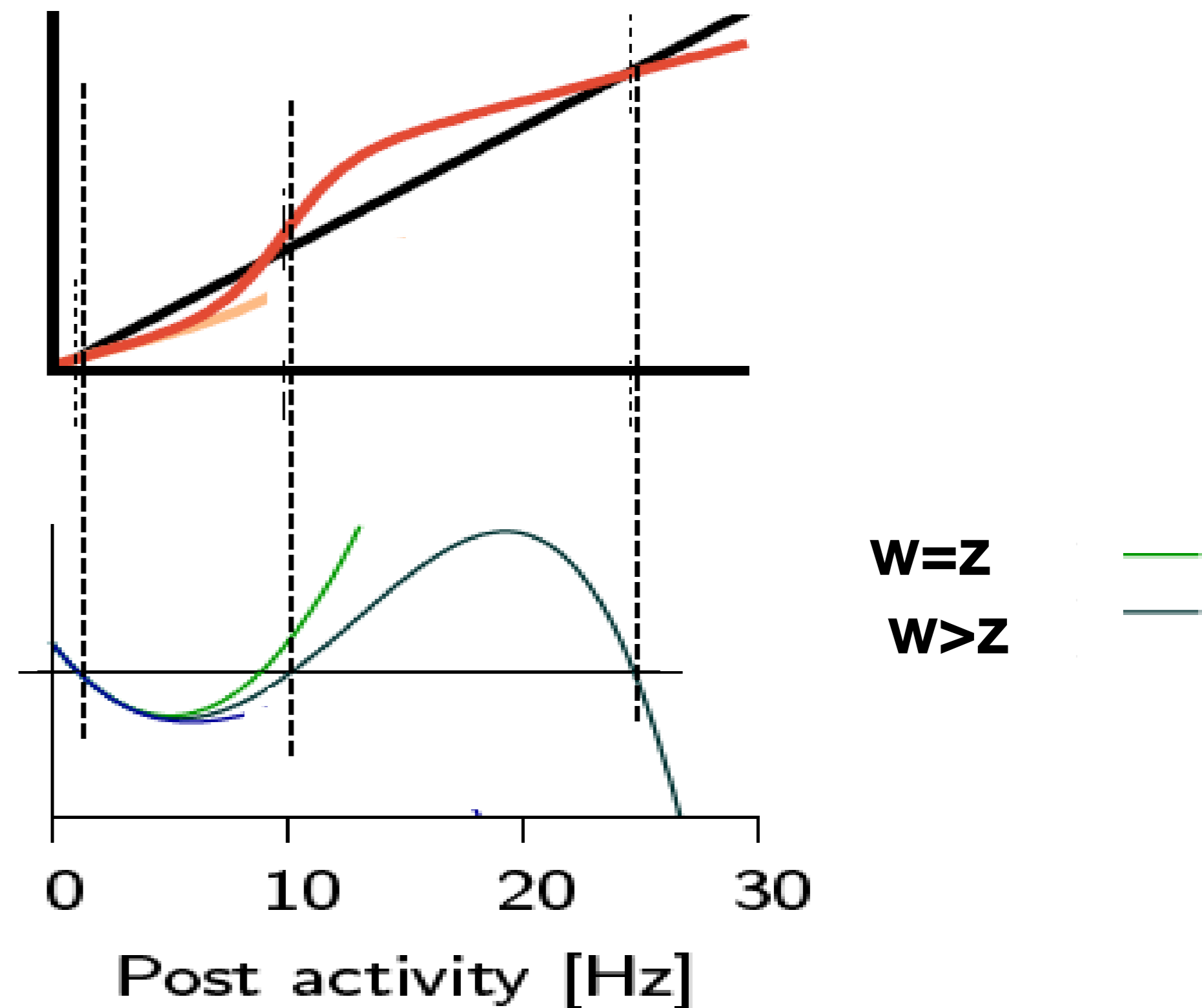
$$\frac{dw_{ij}}{dt} = a_1^{pre} v_j^{pre} - a_2^{LTD} v_j^{pre} v_i^{post} + a_3^{BCM} v_j^{pre} (v_i^{post})^2 - a_4^{het} (w_{ij} - z_{ij}) [v_i^{post}]^4$$

→ Self-stabilizing!

Heterosynaptic plasticity
must act on the same time scale

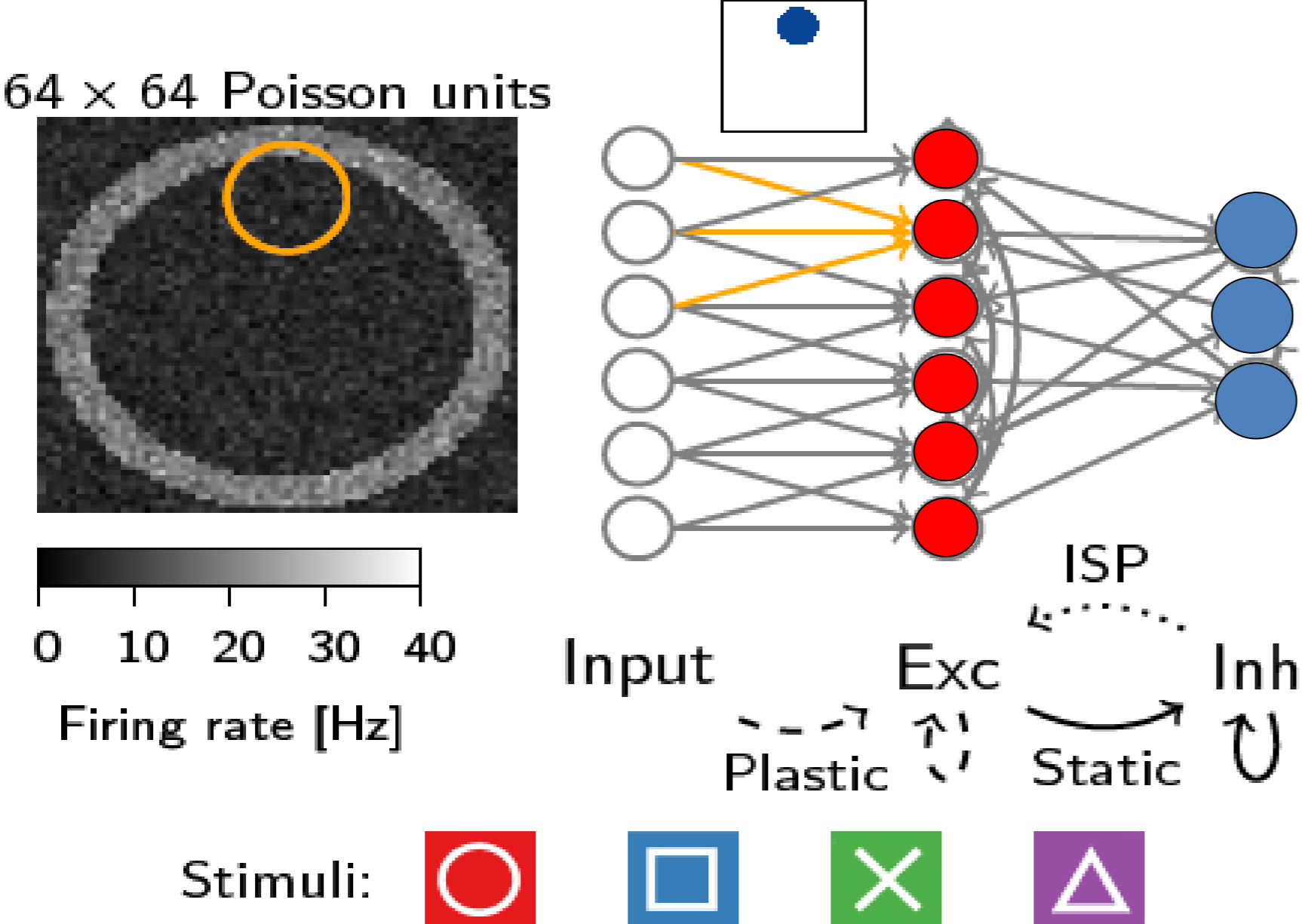
Zenke+Gerstner, PLOS Comp. B. 2013

Zenke et al., Nat. Comm., 2015



8. Plasticity in feedforward /recurrent connections

a

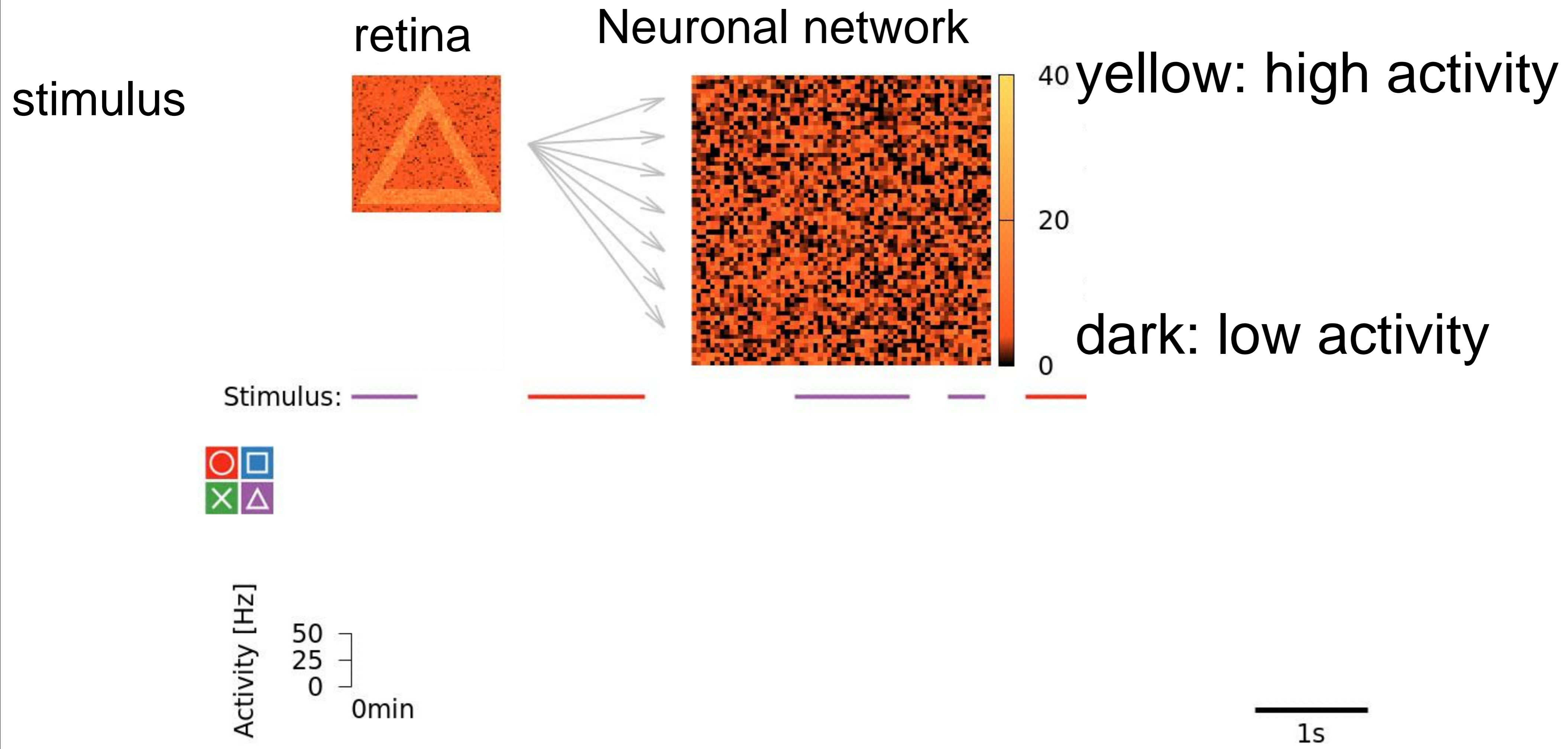


b



*Zenke et al.,
Nat. Comm.
(2015)*

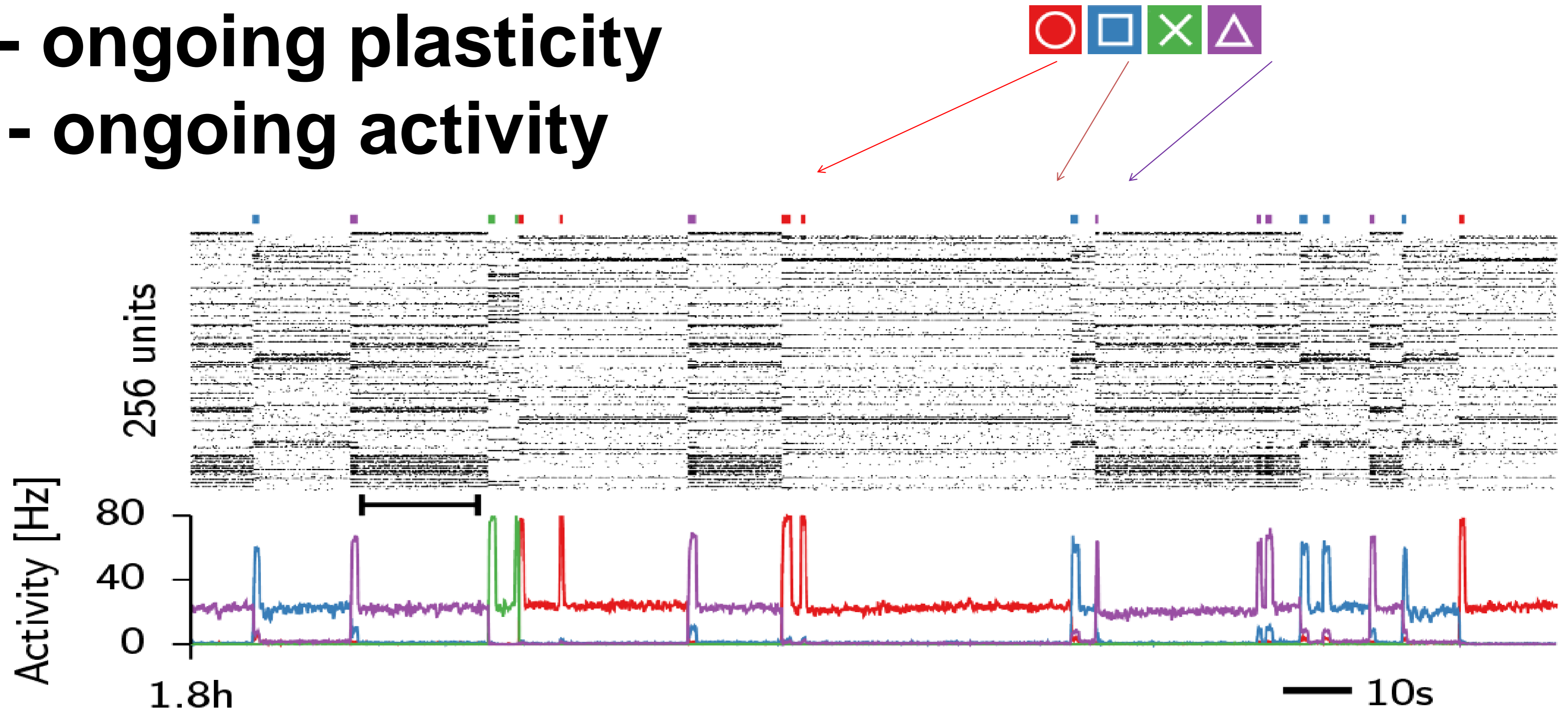
8. Theory and Simulation: first minute



8. Plasticity model in network: two hours later

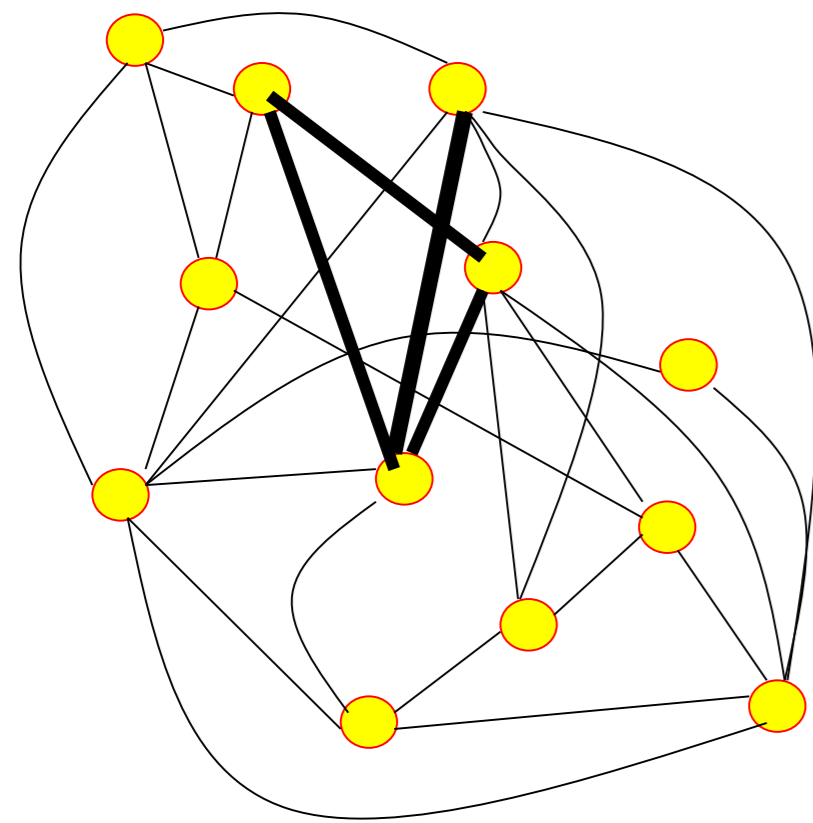
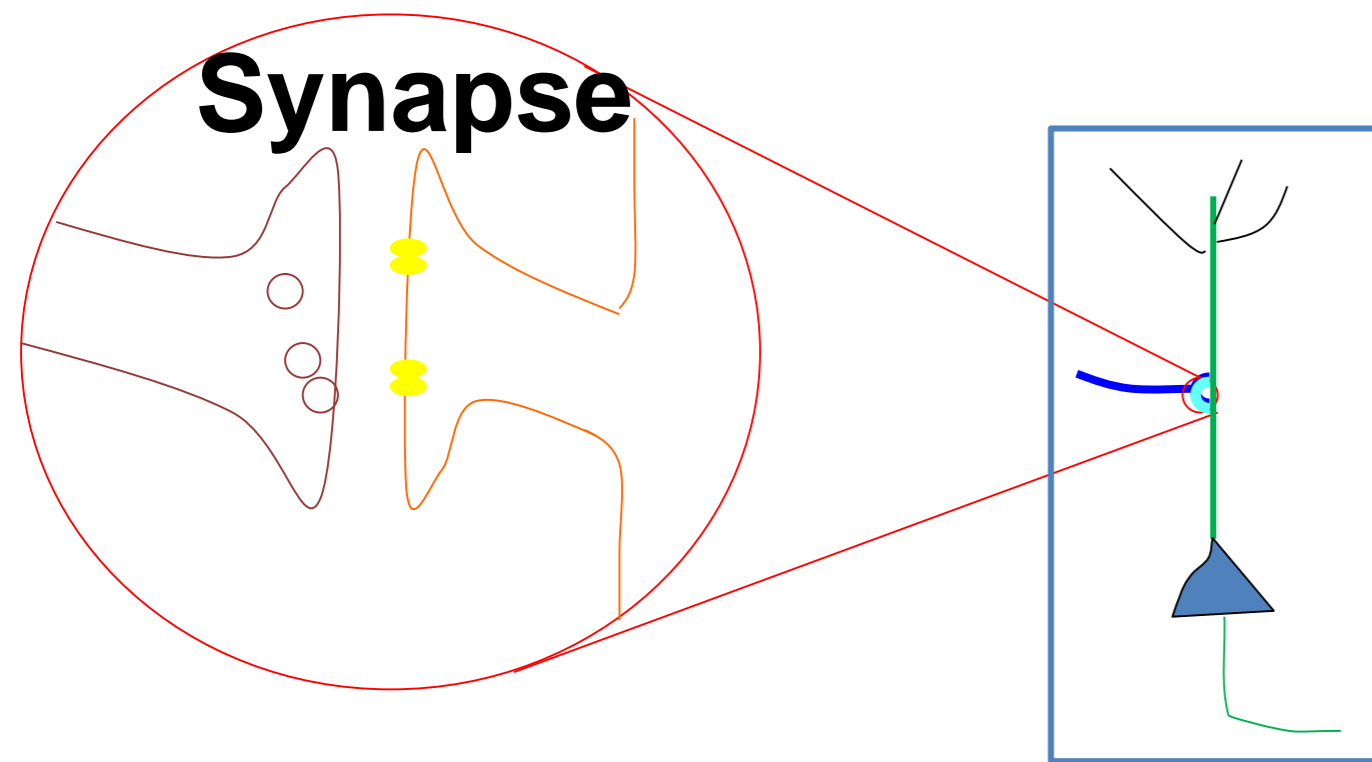
Stable memory recall despite

- ongoing plasticity
- ongoing activity



Zenke et al., Nat. Comm. (2015)

8. Synaptic plasticity, Learning and Memory



Should enable **Learning**

- adapt to the statistics of task and environments
(receptive fields, allocate space etc)
- memorize facts and episodes
- learn motor tasks

Should avoid:

- blow-up of activity **homeostasis**
- unnecessary use of energy

abstract models capture the essence
(but leave out many, many details)

8. Summary: Synaptic plasticity and Memory

Hebbian rules are a family of unsupervised learning rules which describe changes that only depend on presynaptic spike arrivals and the state (depolarization, firing rate, or bursts of spikes) of the postsynaptic neuron. If we make a Taylor expansion of local unsupervised rules, we find terms that depend on the correlations (Hebbian terms/homosynaptic terms) and terms that depend only on the state of the postsynaptic neuron (heterosynaptic terms). The heterosynaptic terms are useful to control the total firing activity of a neuron.

A clever combination of BCM-type homosynaptic terms (triplet STDP model) and of heterosynaptic terms (fourth-order in the postsynaptic firing rate) enables to build network models that learn to form stable attractors. These correspond to the attractors we have seen in the weeks on Hopfield model and memory; but now with true online learning: synaptic plasticity is always ongoing (and not switched off during retrieval).