RBPA 2018:

Solutions

WEEK 12

Problem 1 :

The attenuation law is given by:

$$I(x) = I_0\, e^{-\mu x}.$$

This function is most sensitive to changes in μ where its derivative:

$$\frac{\partial}{\partial \mu} I(x) = I_0(-x)e^{-\mu x}$$ is steepest or has the highest value.

The value of x where this derivative has its maximum can be found according to standard analysis by calculating the first derivative and setting it equal zero:

$$\frac{\partial}{\partial x}\frac{\partial}{\partial \mu} I(x) = \frac{\partial}{\partial x} I_0(-x)e^{-\mu x}$$

$$= -I_0 e^{-\mu x} + I_0\, x \mu\, e^{-\mu x}$$

This expression equals zero if:

$$\cancel{I_0}\,\cancel{e^{-\mu x}} = \cancel{I_0}\, x\mu\, \cancel{e^{-\mu x}}$$

this is equivalent to

$$\boxed{\mu x = 1}\quad .:.$$

Problem 2 :

The mass of ^{241}Am is simply obtained by dividing its activity by the specific activity S of ^{241}Am. The specific activity of a radionuclide is the activity per gram mass of the radionuclide. (2)

As 1g of a nuclide ^{A}X contains $\frac{6.022 \times 10^{23}}{A}$ atoms and as

$A(t) = \lambda \cdot N(t)$, the specific activity can be calculated from the half-life by:

$$S = \lambda \cdot N(t) / 1g$$

$$= \frac{\ln 2}{T_{1/2}} \times \frac{6.022 \times 10^{23}}{A} \Big/ 1g$$

$$= \frac{1}{A} \times \frac{1}{T_{1/2}} \times 4.175 \times 10^{23} \Big/ 1g$$

with the half-life $T_{1/2} = 432.2a$ of ^{241}Am,

we obtain

$S \approx 1.3 \times 10^{11} \frac{Bq}{g}$, and the mass $\underline{\underline{m}}$ of ^{241}Am turns out to be $m = \frac{A}{S} = \frac{30 kBq}{S}$

$\Rightarrow$ $\boxed{m \approx 0.234 \mu g}$.:.

(3)

Problem 3:

- The number of tracer particles that are injected as an instantaneous pulse @ point A (see figure on slide 6 of lecture week[redacted]/ [redacted]) and @ $t=0$ is denoted N_T

- And we assume that $dN(t)$ is the number of tracer particles that pass through the point B (the exit of the system, where the detector is located) within the time interval from t to $t+dt$ after injection.

- The time t corresponds to the transit time of the particles between A and B.

- As not all particles will have identical transit times, the function

$$\frac{dN(t)}{dt}$$ will vary as

a function of t and describes the distribution of the transit times.

- If we divide (or normalize) $\frac{dN(t)}{dt}$ by N_T

we obtain the quantity

$$E(t) = \frac{dN(t)/dt}{N_T} ,$$

and $E(t)\,dt$ then corresponds to the probability that a tracer particle will have a residence time between t and $t+dt$.

• As $\underline{\underline{t}}$ is also the residence time that the $dN(t)$ tracer particles have spent in the system, the response $E(t)$ @ point B forms a probability distribution known as the residence time distribution (RTD). The radiotracer concentration $C(t)$ measured by the detector is related to $dN(t)$ by:

$$\boxed{dN(t) = C(t)dV = C(t)\,Q\,dt} \qquad (1)$$

Here dV is the volume of liquid flowing past B in the time interval between $\underline{\underline{t}}$ and $\underline{\underline{t+dt}}$, and Q is the flow rate (<u>assumed to be constant</u>). The fraction of the tracer particles passing B between $\underline{\underline{t}}$ and $\underline{\underline{t+dt}}$ can be written as $\underline{E(t)\,dt}$ or $\underline{\frac{dN(t)}{N_T}}$ (see above earlier).

Now, inserting Eq. (1) for $dN(t)$, we get:

$$E(t)\,dt = \frac{dN(t)}{N_T} = C(t)\,Q\,dt/N_T \qquad (2)$$

or, after "dividing by dt":

$$\boxed{RTD = E(t) = C(t)\,Q/N_T = \text{const.} \times C(t)} \qquad (3)$$

<u>In summary</u>: the detector response @ B to an instantaneous injection @ A is the RTD of the tracer particles in the system, provided that:

• the tracer is injected as an instantaneous pulse, and
• the flow rate Q through the system is constant.

Problem 4 :

As we neglect nuclear decays of the (5)
tracer in the system, all N_T tracer particles injected must pass by the detector @ point B (i.e., mass balance is given, no particle is lost):

$$\boxed{N_T = \int_0^\infty dN(t) = Q\int_0^\infty c(t)\,dt} \quad (4)$$

where we have used Eq. (1); see Problem 3.
Also, no activity ($A = \lambda \cdot N$) is lost, and we may rewrite Eq. (4) as:

$$\boxed{A_T = \int_0^\infty dA(t) = Q\int_0^\infty C_A(t)\,dt} \quad (5)$$

with the activity concentration $C_A(t) = \lambda C(t)$
(in units of Bq/l)

In the total sample method the sample is taken @ a constant rate q (in units of liter/second) for a time ΔT which includes the whole pulse. Again, due to mass (or better activity) balance, we have:

$$\int_0^\infty C_A(t)\,dt = c\,\Delta T = \frac{a}{q} \quad (6)$$

where c (in units of Bq/liter) is the average concentration of tracer in the total sample and a (in Bq) is the corresponding activity.

Substituting Eq. (6) in Eq (5) gives:

$$\boxed{Q = q\, A_T / a} \quad \therefore$$

In the total count method the registred countrate is proportional to the radioisotope concentration seen by the detector (immersed in the liquid or located externally). Therefore, the # of total counts $\underline{\underline{N}}$ recorded (after correction for background) is proportional to the integral $\int_0^\infty C_A(t)\,dt$.

If we denote the proportionality (or calibration) factor by F, we have:

$$N = F \int_0^\infty C_A(t)\,dt$$

Combining this result with Eq. (5) gives

$$\boxed{Q = A_T\, F / N} \quad \therefore$$

Problem 5 : In a rapidly stirred tank of volume V, an injected pulse of radiotracer will be instantaneously dispersed uniformly through the liquid in the tank.

Thus, the concentration will rapidly adopt the value:

$$\boxed{C_0 = A_T / V}$$, where

A_T is the activity of the injected tracer. Assuming a constant flow rate through the tank, the change $\underline{dC}$ in the tracer concentration between $\underline{t}$ and $\underline{t+dt}$ is given by the difference between the inflow (which is zero after the instantaneous pulse) and the outflow, which is equal to $\frac{C(t)\,Q\,dt}{V}$.

Thus, $$\boxed{dC = C(t) - C(t+dt) = -\frac{C(t)\,Q}{V}\,dt = -\frac{C(t)}{\tau}\,dt}$$

with the mean residence time $\underline{\tau = \frac{V}{Q}}$.

This differential equation can easily be integrated with the result:

$$\boxed{C(t) = C_0\, e^{-\frac{t}{\tau}}}$$.:.

Problem 6 :

As, mathematically, the average, or MRT, is given by:

$$MRT = \int_0^\infty t\,E(t)\,dt$$

<u>Recall Eq. 3 ~~of~~ the Problem 3!</u>

$$RTD = E(t) = \frac{C(t)\,Q}{N_T} = \text{const.} \times C(t)$$

we immediately obtain:

$$\boxed{MRT = \frac{Q}{N_T}\int_0^\infty t\,C(t)\,dt} \therefore$$