Elementary Cryptography Welcome to the Quantum Era!

Serge Vaudenay





- Data Confidentiality
- Public-Key Cryptography
- Data Authentication
- 4 All Together Now
- Quantum Era

Cryptography = Science of Information and Communication Security















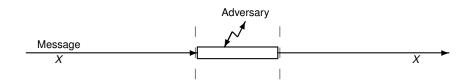




Cryptographic Problems in Ancient Time

privacy (by encryption)

The Fundamental Trilogy



- Confidentiality (C): defeat malicious access to X
- Authentication (A): defeat malicious forgery of X
- Integrity (I): defeat malicious modification of X

A Few Cryptographic Problems

- privacy (by encryption)
- detection malicious modification of information
- data authentication
- access control
- timestamping
- fair exchange
- digital rights management
- more privacy (anonymity, unlinkability, deniability, ...)

Applications

- bank cards
- E-commerce
- mobile telephony
- e-passport
- mobile communication (Bluetooth, WiFi...)
- traceability, logistic & supply chains (RFID)
- pay-TV, DRM
- access control (car lock systems, metro...)
- payment (e-cash)
- electronic voting

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What Can Be Assumed Secret?

- to design a cryptographic system is a difficult task
- products (implementing cryptography) are massively deployed
- we cannot assume that adversaries ignore which cryptographic system is used
- · we can assume the secrecy of a key, though
- security by obscurity can only fail!
 (i.e. using secret algorithms and relying on their secrecy)

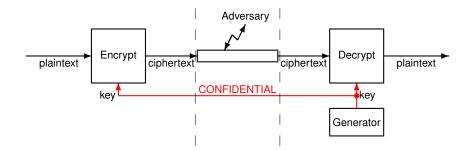
Kerckhoffs Principles (1883)

- 1 Le système doit être matériellement, sinon mathématiquement, indéchiffrable;
- Il faut qu'il n'exige pas le secret, et qu'il puisse sans inconvénient tomber entre les mains de l'ennemi;
- 3 La clef doit pouvoir en être communiquée et retenue sans le secours de notes écrites, et être changée ou modifiée au gré des correspondants;
- Il faut qu'il soit applicable à la correspondance télégraphique;
- [5] Il faut qu'il soit portatif et que son maniement ou son fonctionnement n'exige pas le concours de plusieurs personnes;
- Enfin, il est nécessaire, vu les circonstances qui en commandent l'application, que le système soit d'un usage facile, ne demandant ni tension d'esprit, ni la connaissance d'une longue série de règles à observer.



- meaning: security analysis must assume that the adversary knows the algorithms
- common misunderstanding: algorithms must be public

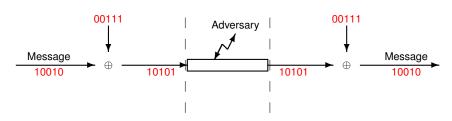
Symmetric Encryption



Vernam Cipher



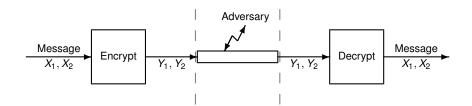




Using the Same Key Twice is Bad

$$Y_1 = X_1 \oplus K$$

 $Y_2 = X_2 \oplus K$



$$Y_1 \oplus Y_2 = (X_1 \oplus K) \oplus (X_2 \oplus K) = (X_1 \oplus X_2) \oplus (K \oplus K) = X_1 \oplus X_2$$

leakage of the $X_1 \oplus X_2$ value

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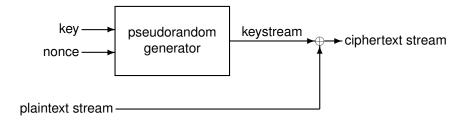
Information Theory

Claude Shannon

[Claude Shannon]

- formalized the notion of perfect secrecy
- the Vernam cipher (when correctly used) is perfectly secure
- perfect security implies that the key space is at least as large as the message space

Using a Pseudorandom Key: Stream Cipher



nonce = **n**umber which can be used **once** (necessary to avoid re-using a keystream)

Kinds of Symmetric Encryption Schemes

- stream cipher (length-preserving, needs a nonce)
- block cipher (encrypts only 128-bit blocks)
- block cipher in a mode of operation (some length-preserving, some with nonces)

Inventory of Symmetric Encryption Schemes

wildlife: ARMADILLO BEAR BLOWFISH DRAGON FOX FROG LION MOSQUITO RABBIT

SERPENT SHACAL SHARK TWOFISH

flora: CAMELLIA LILY SEED

pantheon: ANUBIS MARS KHAFRE KHUFU LUCIFER MICKEY SHANNON TURING

Qastronomic: COCONUT GRANDCRU KFC MILENAGE PEANUT WALNUT

elements: CRYPTON ICE ICEBERG RAINBOW SNOW

eccentric: ABC ACHTERBAHN AKELARRE CAST DEAL DECIM EDON FEAL FUBUKI GOST HELIX

HIEROCRYPT IDEA KASUMI KATAN KHAZAD KTANTAN LEX LEVIATHAN LOKI MACGUFFIN MADRYGA MAGENTA MIR MISTY NIMBUS NOEKEON NUSH PHELIX PRESENT PY QUAD REDOC RIJNDAEL SAFER SALSA SCREAM SFINKS SKIPJACK

SMS4 SQUARE SOBER SOSEMANUK XTEA 3-WAY YAMB

uninspired: A5 **AES** BMGL C2 CJCSG CMEA CS-CIPHER DES DFC E0 E2 FCSR HPC MMB Q RC2

RC4 RC5 RC6 SC TSC WG



A 128-Bit Key

c0930349 d3f27ba5 a93130de 2e4e1f21

number of combinations:

Exhaustive Search on 128 Bits?

Order of Magnitude of 2¹²⁸

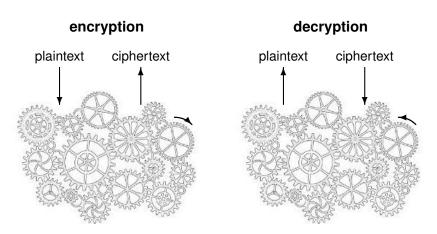
some big numbers:

- human population: 2³³
- number of cells in a human body: 2⁴⁷
- age of the universe: 2⁵⁹s
- number of atoms in 12g of carbon: 2⁷⁹
- diameter of the universe: 2⁹⁰m (2¹²³Å)
- mass of Earth: 2^{93} g ($\approx 2^{114}$ amoebas)
- number of atoms in the universe: 2²⁶⁶

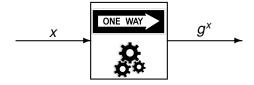
in 2007, a standard PC could test 1 000 000 keys per second to test 2¹²⁸ within 15 Billion years, we need 720 000 Billion of 2007-PCs!

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Reversibility in Symmetric Encryption

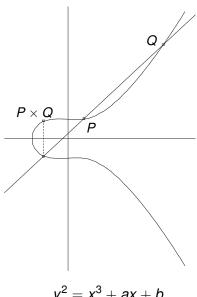


Hard-To-Invert Computation



in some algebraic structures, log is an intractable operation

Multiplication in an Elliptic Curve



$$y^2 = x^3 + ax + b$$

Multiplication Easy ⇒ Exponentiation Easy

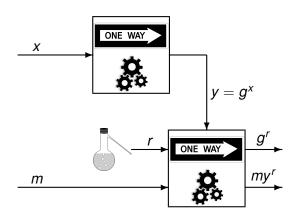
Theorem

We can compute g^x with 2n multiplications or less, for $x < 2^{n+1}$.

example: g^{54} in 8 multiplications

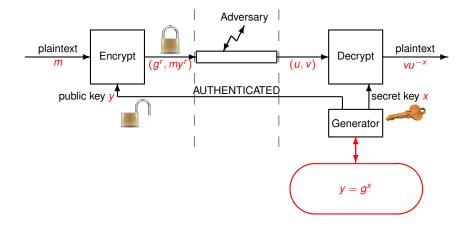
	54	squaring	multiplying
1		g	
2	= 2	$g imes g = g^2$	g^2
4	+4	$(g^2)\times(g^2)=g^4$	$ imes g^4$
8		$(g^4) \times (g^4) = g^8$	
16	+16	$(g^8) \times (g^8) = g^{16}$	$ imes g^{16}$
32	+32	$(g^{16}) \times (g^{16}) = g^{32}$	$\times g^{32}$
			$= g^{54}$

Encryption with a Public Key

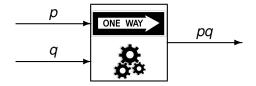


trick: x is a trapdoor allowing to decrypt! $m = (my^r)/(g^r)^x$ because $(g^r)^x = (g^x)^r$

A Public-Key Cryptosystem (ElGamal)



A Familiar Hard-To-Invert Computation



Factoring Record

complexity:
$$e^{\mathcal{O}\left((\ln n)^{\frac{1}{3}}(\ln \ln n)^{\frac{2}{3}}\right)}$$

RSA200

X

- = 27997833911221327870829467638722601621070446786955 42853756000992932612840010760934567105295536085606 18223519109513657886371059544820065767750985805576 13579098734950144178863178946295187237869221823983

79258699544783330333470858414800596877379758573642 19960734330341455767872818152135381409304740185467

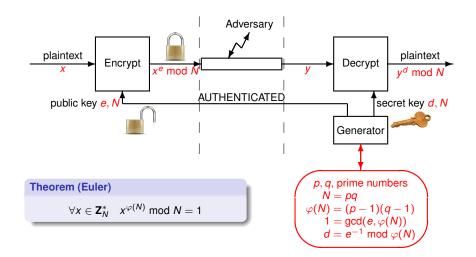
factored in 2005 with the equivalent of 55 years on a PC 2.2GHz

Rivest-Shamir-Adleman (RSA) (1978)

[Shamir, Rivest, Adleman]

- concrete trapdoor permutation
 (invertible transformation which is easy to compute in one direction but hard in the other, but with the knowledge of trapdoor information)
- public-key cryptosystem
- signature scheme

(Textbook) RSA



Public-Key Cryptosystems

RSA Rabin based on factoring Paillier ElGamal based on dis-ECC crete logarithm HECC NTRU lattice-based "post-quantum" McEliece TCHo

Symmetric vs Public-Key Cryptography

symmetric

public-key

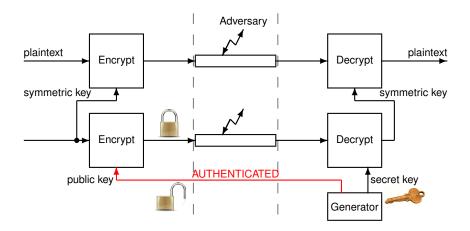
- link-based
- fast
- cheap
- robust

- user-based
- for short messages
- expensive
- sensitive

hybrid

- public-key crypto is used to establish a short-term symmetric key
- symmetric crypto is used to process the data

Hybrid Encryption



Diffie-Hellman

"New Directions in Cryptography" (1976)

[Merkle, Hellman, Diffie]

- invention of public-key cryptography
- notion of "trapdoor permutation"
- building a public-key cryptosystem from it
- building a digital signature scheme from it
- key agreement protocol

Diffie-Hellman Protocol

notion from Algebra (could be an elliptic curve) with a group generated by some *q*

Alice Bob

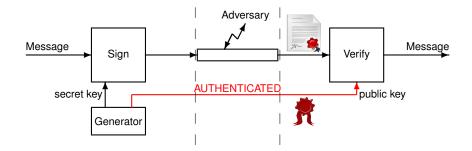
security requirement: given (g, g^x, g^y) , it must be hard to compute g^{xy} (**Computational Diffie-Hellman Problem**)

Key Agreement

- key agreement resist passive attacks vulnerable against man-in-the-middle attacks
- authenticated key agreement resist active attacks needs some prior authenticated information (e.g. public key, secret, password)

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Digital Signature: Encryption Upside Down!



Signature Schemes

RSA
Rabin
ElGamal
Schnorr
DSA
ECDSA
NTRU
lattice-based
based on discrete logarithm
"post-quantum"

Signature: From Paper to Bits

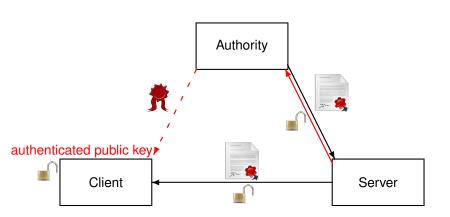
paper signature

- hard to copy
- same signature
- verified with a model
- needs human effort
- photocopies are non-binding

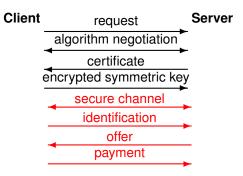
digital signature

- easy to copy
- message-dependent
- verified with a public key
- machine computable
- copies are digital evidence

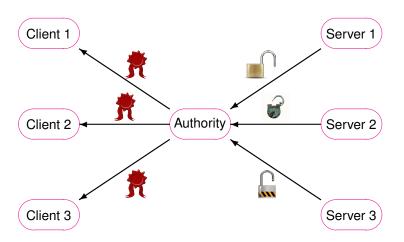
Public-Key Infrastructure (PKI)



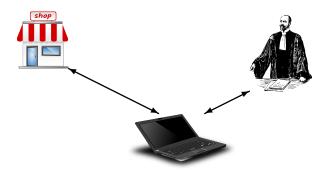
Transaction with "https"



Critical Channels



Idealized Security



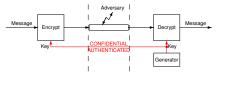
Security in Practice: Spot the Error



4 Main Cryptographic Primitives

confidential transmission

authenticated transmission





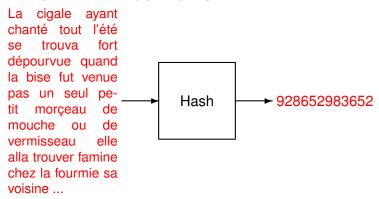




Cryptographic Primitives

- symmetric encryption
- message authentication code
- hash function
- key agreement protocol
- public-key cryptosystem
- digital signature

A 6th Important Cryptographic Primitive



- can hash a string of arbitrary length
- produce digests (hashes) of standard length (e.g. 224 bits)
- sometimes called "fingerprint"
- use: sign a hash instead of a randomly formatted message

Meaning of Breaking

for encryption

show that we can recover the decryption key (more generally): show that we can decrypt a target ciphertext when we have access to a decryption oracle, but without submitting the target to the oracle

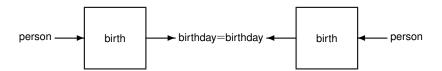
for signature

show that we can recover the signing key (more generally): show that we can forge the signature for a target message when we have access to a signing oracle, but without submitting the target to the oracle

for hashing

show that we can produce two documents with the same hash (same fingerprint)

Collision Search



Birthday Paradox

Theorem

If we pick independent random numbers in $\{1,2,\ldots,N\}$ with uniform distribution, n times, we get at least one number twice with probability $p\approx 1-e^{-\frac{n^2}{2N}}$ for $n\ll N$.

For N = 365:

n	_	_	-	_			-
probability	12%	25%	41%	57%	71%	81%	89%

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Common Algorithms

be careful with the key lengths

symmetric encryption: AES

hash function: SHA3

MAC/PRF: HMAC-SHA2

authentication encryption: AES-CCM, AES-GCM

be careful with the nonce

key agreement: DH, ECDH

cryptosystem: RSA, elliptic-curve cryptography

signature: RSA, DSA, ECDSA

be careful with the randomness

Key Length

- symmetric encryption/MAC: bit-security
- RSA: check tables
- hash with collision resistance: digest of twice bit-security
- hash without collision resistance: digest of bit-security
- discrete logarithm/DH in a group: twice bit-security caveat: if subgroup of Z_p*, p must be of size like for RSA

				+		•	→
method	year	sym.	RSA	DL		EC	hash
Lenstra-Verheul	2015	82	1613	145	1613	154	163
Lenstra updated	2015	78	1245	156	1245	156	156
ECRYPT II	2011–15	80	1248	160	1248	160	160
NIST	2011–30	112	2048	224	2048	224	224
FNISA	2010-20	100	2048	200	2048	200	200
BSI	2011–15	_	1976	224	2048	224	224
					A		

(http://www.keylength.com by Quisquater)

Secure Communication

all together now!

- be careful! security does not add up... too many ways to make mistakes
- all-in-one primitives exist (authenticated encryption)
- various setup assumptions
 - secure hardware (badge, SIM card, smart card, TPM)
 - trusted third party (key server, authority)
 - PKI (with one/several certificate authorities)
 - password/preshared secret
 - out-of-band channel (SAS)
- more modern security notions (e.g. for instant messaging)

Case Studies

if we had time...

- TLS
- wifi
- mobile telephony
- signal
- bluetooth
- blockchains
- NFC payment
- MRTD

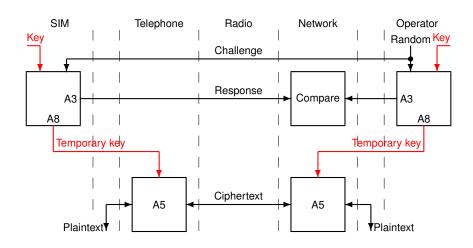
Mobile Telephony

- principle 1: authentication of mobile system
- principle 2: privacy protection in the wireless link

GSM architecture:

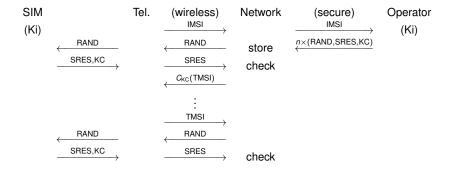
- challenge-response protocol based on Ki
- short-term encryption key (derived from Ki)
- identity IMSI replaced by a pseudonym TMSI as soon as possible
- Ki never leaves the security module (SIM card) or home security database (HLR)

GSM Protocol



GSM Authentication

$$A3/8(Ki, RAND) = (SRES, KC)$$



GSM Encryption

- several standard algorithms: A5/0, A5/1, A5/2, A5/3
- cipher imposed by network
- new KC for each session
- synchronized frame counter (used as a nonce)

Security of Privacy protections

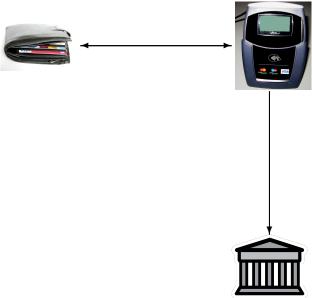
- blinding the identity is not effective at all:
 - challenges can be replayed to trace mobile telephones
 - fake network can force identification in clear (re-synchronization protocol)
- security of A5/0 (no encryption) void
- security of A5/2 weak
- security of A5/1 not high
- security of A5/3 high
- fake network can force to weak encryption (they all use the same key)
- replaying a challenge will force reusing a one-time key
- message integrity protection is ineffective

security: 😉

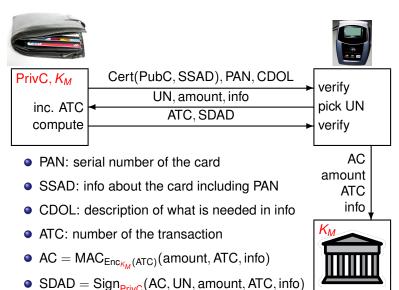
Improvements in 3G Mobile Telephony

- challenges are authenticated (fake network cannot forge them)
- integrity protection (MAC)
- protection against challenge-replay attacks
- uses block cipher KASUMI instead of stream cipher A5/1

NFC Payment



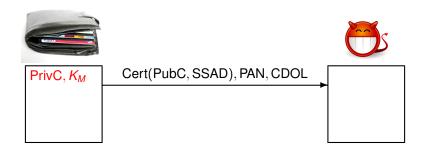
(Simplified) EMV PayPass Protocol



From Paper to Bits...

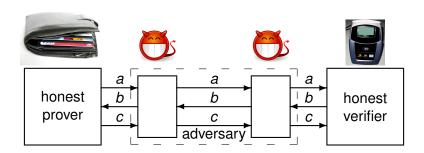
- holder is not aware a payment is happening
- holder is not aware of the payment amount
- no access control of the payment terminal (no PIN)
- payee is not authenticated (info could be anyone)
- privacy issue (SSAD leaks)

Skimming

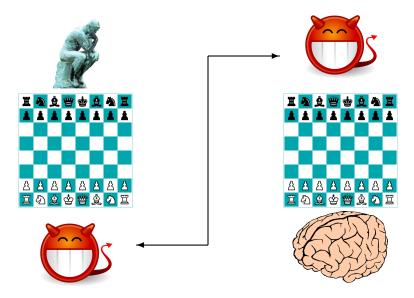


get name on card, credit card number, expiration date, etc

Relay Attacks



Playing against two Chess Grandmasters



Relay Attacks in Real

- opening cars and ignition (key with no button)
- RFID access to buildings or hotel room
- toll payment system
- NFC credit card (for payment with no PIN)
- access to public transport
- ...

Signal

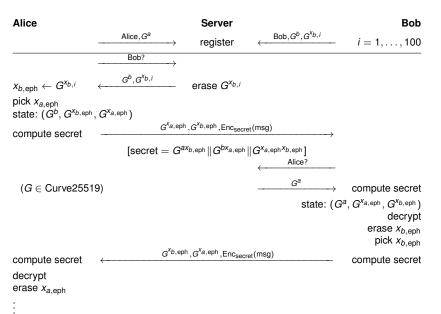


Signal

used in WhatsApp

- secure messaging (confidentiality, authenticity, integrity of messages)
- forward and future secrecy (confidentiality preserved even though secrets leak)
- deniability (no transferable proof of message authorship leaks)
- asynchronous (can be done offline)
- detect replay/reorder/deletion attacks
- allow decryption of out-of-order messages
- don't leak metadata

Initial Key Agreement



SV 2018

Ratchet

A ratchet is a mechanical device which can only move forward.



- forward secrecy: protects past sessions against future compromises of long-term secret keys
- future secrecy: protects future sessions against compromises of ephemeral secret keys

Double Ratchet in Signal

- 3DH: a ratchet for every time the direction of exchange changes
 - needs synchronization between the two participants
 - good forward and future secrecy
- a ratchet for a sequence of messages in the same direction
 - no real future secrecy
 - plausible deniability

MRTD



ICAO-MRTD Objectives

(MRTD=Machine Readable Travel Document)

more secure identification of visitors at border control

- → biometrics
- → contactless IC chip
- → digital signature + PKI

maintained by UN/ICAO (International Civil Aviation Organization)

MRTD History

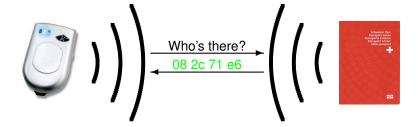
- 1968: ICAO starts working on MRTD
- 1980: first standard (Machine Readable Zone (MRZ))
- 1997: ICAO-New Tech. WG starts working on biometrics
- 2001 9/11: US want to speed up the process
- 2002 resolution: ICAO adopts facial recognition (+ optional fingerprint and iris recognition)
- 2003 resolution: ICAO adopts contactless IC media (instead of e.g. 2D barcode)
- 2004: version 1.1 of standard with ICC
- 2005: deployment of epassports in several countries
- 2006: extended access control in the EU
- now part of Doc9303

MRZ Example

PMCHEDUPONT<<JEAN<<<<<<<<<<<<<<<<<<<<<<<<<<<>X337803X<6CHE7208066M1308147<<<<<<<<<

- document type
- issuing country
- holder name
- doc. number + CRC
- nationality
- date of birth + CRC
- gender
- date of expiry + CRC
- options + CRC

ISO 14443 (RFID)



frequency: 13.56MHz

typical range: 2cm

reported range (with legal equipment): 12m



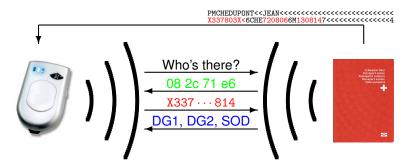
Advantages

Problems

- robust
- large storage capacity
- dynamic

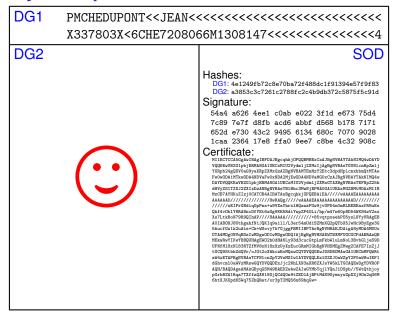
- leaks information
- answers to anyone

ICAO (MRTD): BAC and Passive Authentication



- DG1: official name, citizenship, X337 ··· 814, gender
- DG2: facial picture
- SOD: signature by authorities of the hash of DG's

Identity Example



MRTD

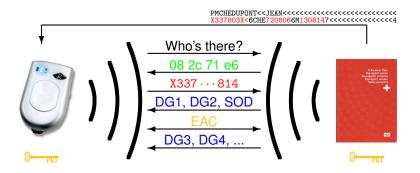
Advantages

- impossible to forge an identity
- protect against non-organized illegal immigration

Problems

- encourage identity theaft
- facial recognition is weakly reliable
- passeport cloning
- tracking people
- leakage of evidence
 - proof of official name
 - proof of wedding
 - proof of age
 - proof of gender
- anonymity loss

EAC: Access Control and Active Authentication



- EAC: chip authentication
- EAC: terminal authentication
- DG3...: fingerprint, other data



EAC

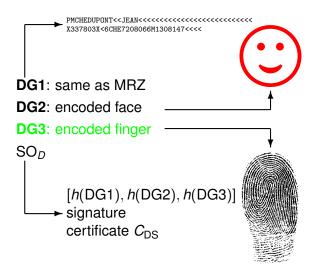
Advantages

- anti-cloning
- better access control
- better identification

Problems

- only where EAC is available
- still evidence leakages
- a new PKI

LDS Example

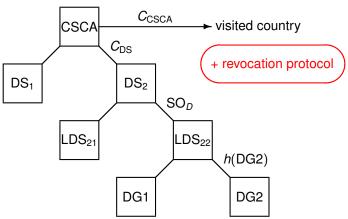


LDS (MRTD Memory) Structure

- K_{ENC}, K_{MAC}, KPr_{AA}
- COM: present data groups
- DG1: same as MRZ
- DG2: encoded face
- DG3: encoded finger(s)
- DG4: encoded eye(s)
- DG5: displayed portrait
- DG6: (reserved)
- DG7: displayed signature
- DG8: data feature(s)

- DG9: structure feature(s)
- DG10: substance feature(s)
- DG11: add. personal detail(s)
- DG12: add. document detail(s)
- DG13: optional detail(s)
- DG14: security options
- DG15: KPu_{AA}
- DG16: person(s) to notify
- SO_D

(Country-wise) PKI



- one CSCA (Country Signing Certificate Authority)
- several DS (Document Signer) per country
- SO_D: signature of LDS
- fingerprint of a DG

Passport: From Paper to Bits

paper passport

- invisible if not shown
- hard to copy
- photocopies are non-binding
- needs human check
- access control by the holder

MRTD

- detectable, recognizable
- easy to copy with no AA
- SOD is a digital evidence
- readable automatically
- needs specific access control

MRZ_info

PMFRADUPONT<<<<JEAN<<<<<<<<<<<<<<<<<<<<<<<<<>74HK8215<6CHE7304017M0705121<<<<<<<<<

- document type
- issuing country
- holder name
- doc. number + CRC
- nationality
- date of birth + CRC
- gender
- date of expiry + CRC
- options + CRC

Basic Access Control

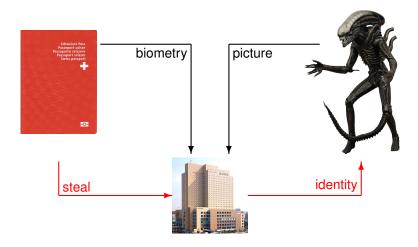
Authenticated Key Exchange Based on MRZ_info

IFD ICC (derive K_{ENC} and K_{MAC} from MRZ_info) **GET CHALLENGE** RND.ICC pick RND.IFD, K.IFD pick RND.ICC, K.ICC check RND.ICC $S \leftarrow \mathsf{RND}.\mathsf{IFD} \| \mathsf{RND}.\mathsf{ICC} \| \mathsf{K}.\mathsf{IFD}$ check RND.IFD $R \leftarrow \mathsf{RND}.\mathsf{ICC} \| \mathsf{RND}.\mathsf{IFD} \| \mathsf{K}.\mathsf{ICC}$ (derive KS_{ENC} and KS_{MAC} from $K_{seed} = K.ICC \oplus K.IFD$)

Security and Privacy Issues

- collision avoidance discrepancies
 - → deviating from standard induce leakages
- MRZ_info entropy
 - → online attack or offline decryption from skimming
- underestimated wireless range limits
 - → claimed to be possible at a distance of 25m
- identity theft (by stealing/cloning MRTD)
 - \rightarrow facial recognition is weak
- remote passport detection
 - \rightarrow nice to find passports to steal
- relay attacks
- denial of services
- ...

Identity Theft



a few 100 customers are enough

Extended Access Control (EAC)

- PACE > BAC
- Chip Authentication
- Terminal Authentication to access non-mandatory data
- more biometrics (finger) for more secure identification
- using state-of-the-art cryptography (public-key crypto, PAKE, elliptic curves)
- secure access control but requires a heavy PKI for readers
- in-process standard: protocols with different versions, variants, described in different documents, with different notations...

Terminal Authentication Issues

Terminal revocation issue:

- MRTDs are not online!
- MRTDs have no reliable clock

Information Leakage

- SO_D leaks the digest of protected DGs before passing EAC
- could be used to recover missing parts from exhaustively search
- could be used to get a proof if DG is known

Conclusion on MRTD

LDS: contains too much private information

passive authentication: leaks evidence for LDS

• BAC: does a poor job

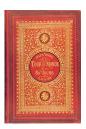
secure messaging: OK

AA: leaks digital evidences, subject to MITM

EAC: much better, but still leaks + revocation issue

RFID: leaks

• biometrics: leaks template



"Les passeports ne servent jamais qu'à gêner les honnêtes gens et à favoriser la fuite des coquins."

> Jules Verne, 1872 Le tour du monde en 80 jours

Other Useful Primitives

- zero-knowledge (for privacy)
- property-preserving encryption (for databases): searchable, order-revealing, format-preserving
- homomorphic things (for privacy)
- multiparty computation
- identity-based cryptography
- obfuscation

Intrinsic Threats to Cryptography

- Moore law natural increase of the computational power of computers
 → security gracefully decreases
- Shor algorithm
 quantum computer
 → security will collapse in an earthquake
- non-guaranteed hypotheses finally, factoring could be easy
 - \rightarrow security fall can occur at any time



Lack of Cryptodiversity



Erroneous Security Proofs

[illustration by Fred]

Bad Random Sources

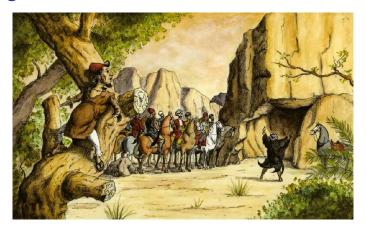


- the secret key of servers can be guessed
- the secret key of routers can be guessed
- play stations can be cracked
- bitcoins can be stolen
- bad algorithms may be (maliciously) imposed as standards

Lack of Composability Results

secure + secure
$$\stackrel{?}{=}$$
 secure

Leakage Due to Hardware



power consumption radio emanation cache fails response to stress vibration format validation

time of computation branch prediction

Trust in Security Infrastrutures

[Disney illustration]

Hot Issues

- good authenticated encryption
- side channel mitigation
- leakage resiliency
- randomness generation
- postquantum cryptography

- Data Confidentiality
- Public-Key Cryptography
- Data Authentication
- All Together Now
- Quantum Era

Principles of Quantum Mechanics

- randomness is inherent
- state of an isolated system: unit vector with complex coordinates
- transformations of an isolated system are reversible
- observing a system degrades the state of the system
- weird things:
 - superposition state
 - entangled state
 - no-cloning principle

Superposition State

- ullet elementary states defined with the $|\cdot\rangle$ notation
- example: the aliveness of Shrödinger's cat

$$X = |alive\rangle$$

 $Y = |dead\rangle$

a state could be a combination $\alpha X + \beta Y$

• observing would end up in state $\left\{ egin{array}{l} X \text{ with probability } |\alpha|^2 \\ Y \text{ with probability } |\beta|^2 \end{array} \right.$

Entangled State

 with two particules A and B which can be either "up" (1) or "down" (0), we have the orthogonal states

$$|00\rangle$$
 $|01\rangle$ $|10\rangle$ $|11\rangle$

we could have the state

$$\frac{1}{\sqrt{2}}\left(\ket{00}+\ket{11}\right)$$

observing one particule affects the other!

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Quantum Computing

- qbits of memory are complex vectors of dimension two: $\alpha |0\rangle + \beta |1\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$
- a quantum memory of n qbits is a combination $\sum_{b_1 \cdots b_n} \alpha_{b_1 \cdots b_n} | b_1 \cdots b_n \rangle$
- operations are unitary linear operations
- if we have a classical circuit to compute f, we have an equivalent quantum circuit to transform |x y > into |x y + f(x)>
- we can create $\frac{1}{\sqrt{2^{\text{length}(x)}}} \sum_{x} |x| f(x) \rangle$ (free parallelism)
- observing one qbit is a linear projection

Main Algorithms

Shor algorithm

- factoring in quasi-linear time (instead of sub-exponential)
 † RSA
- discrete logarithm in quasi-linear time
 † Diffie-Hellman, DSA, elliptic curves
- → need alternate cryptography: postquantum cryptography

Grover algorithm

- finds a needle in a haystack in square root time
- → need to double key lengths in symmetric cryptography

Quantum Cryptography

by transmitting photon in a quantum state, we can make a key agreement

- unconditionally secure
 (if the adversary can only see or modify the transmitted photons and any other classical communication)
- not secure against side-channel attacks
- the key can only be used with the Vernam cipher
- needs a quantum channel

Bennett-Brassard Protocol

Alice Bob

$$\begin{array}{ll} \operatorname{pick} x, e \in \{0,1\} & \operatorname{pick} d \in \{0,1\} \\ |A\rangle = H^e |x\rangle & \xrightarrow{|A\rangle} & y = \operatorname{measure}_{d=0?Z:X}(|A\rangle) \\ \operatorname{erase} x \ \operatorname{if} \ e \neq d & \xleftarrow{d} \\ & \xrightarrow{\operatorname{kept \ or \ erased}} & \operatorname{erase} y \ \operatorname{is \ Alice \ erased} \\ & (x = y) & \end{array}$$

$$\Pr[\mathsf{measure}_{d=0?Z:X}(H^e|x\rangle) = x] = \begin{cases} 0 \text{ if } d = e \\ \frac{1}{2} \text{ if } d \neq e \end{cases}$$

this must be followed by a verification of having received enough bits and of correctness

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Conclusion

- many techniques and algorithms to protect information
- almost always relying on some hardness assumption
- does not compose
- delicate to use
- quite multidisciplinary and lively!
- a science for

malicious behaviors and protection techniques



Pub

