Artificial Neural Networks: Lecture 4

Wulfram Gerstner

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Regularization and Tricks of the Trade in deep networks

Objectives for today:

- Bagging
- Dropout
- What are good units for hidden layers?
- Rectified linear unit (RELU)
- Shifted exponential linear (ELU and SELU)
- BackProp: Initialization
- Linearity problem, vanishing gradient problem, bias problem
- Batch normalization

Reading for this lecture:

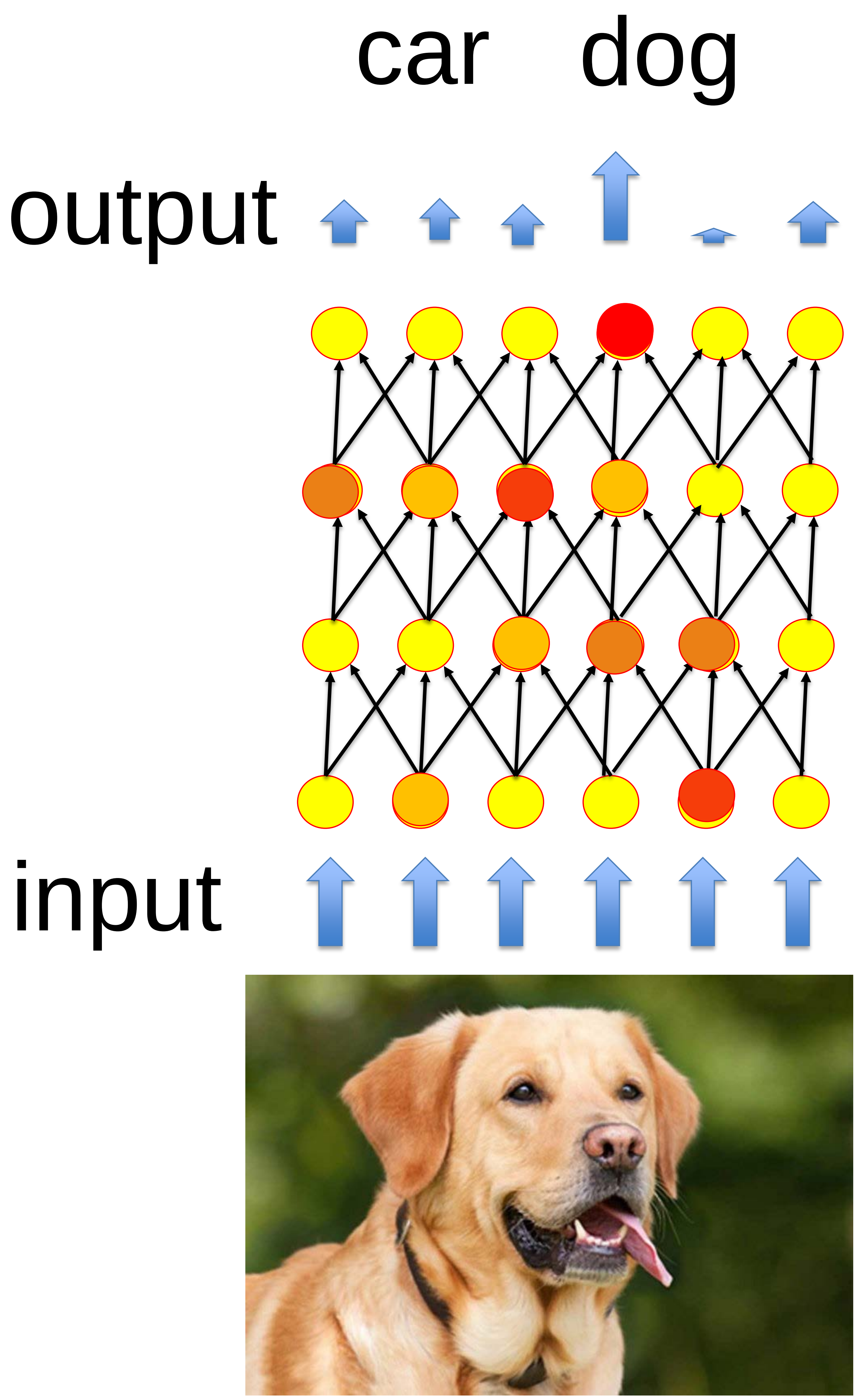
Goodfellow et al.,**2016** *Deep Learning*

- Ch 7.4, 7.8, **7.11 and 7.12,**
- **Ch. 8.4**

Further Reading for this Lecture:

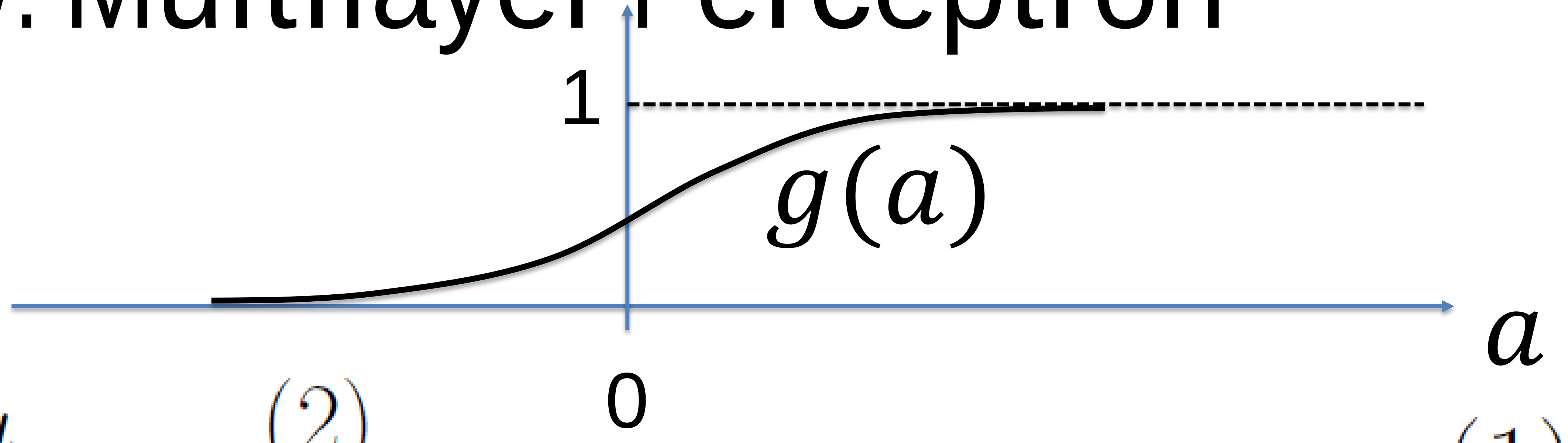
Paper: Klaumbauer, …, Hochreiter (2017)
Self-normalizaing neural networks
https://arxiv.org/pdf/1706.02515.pdf

review: Artificial Neural Networks for classification

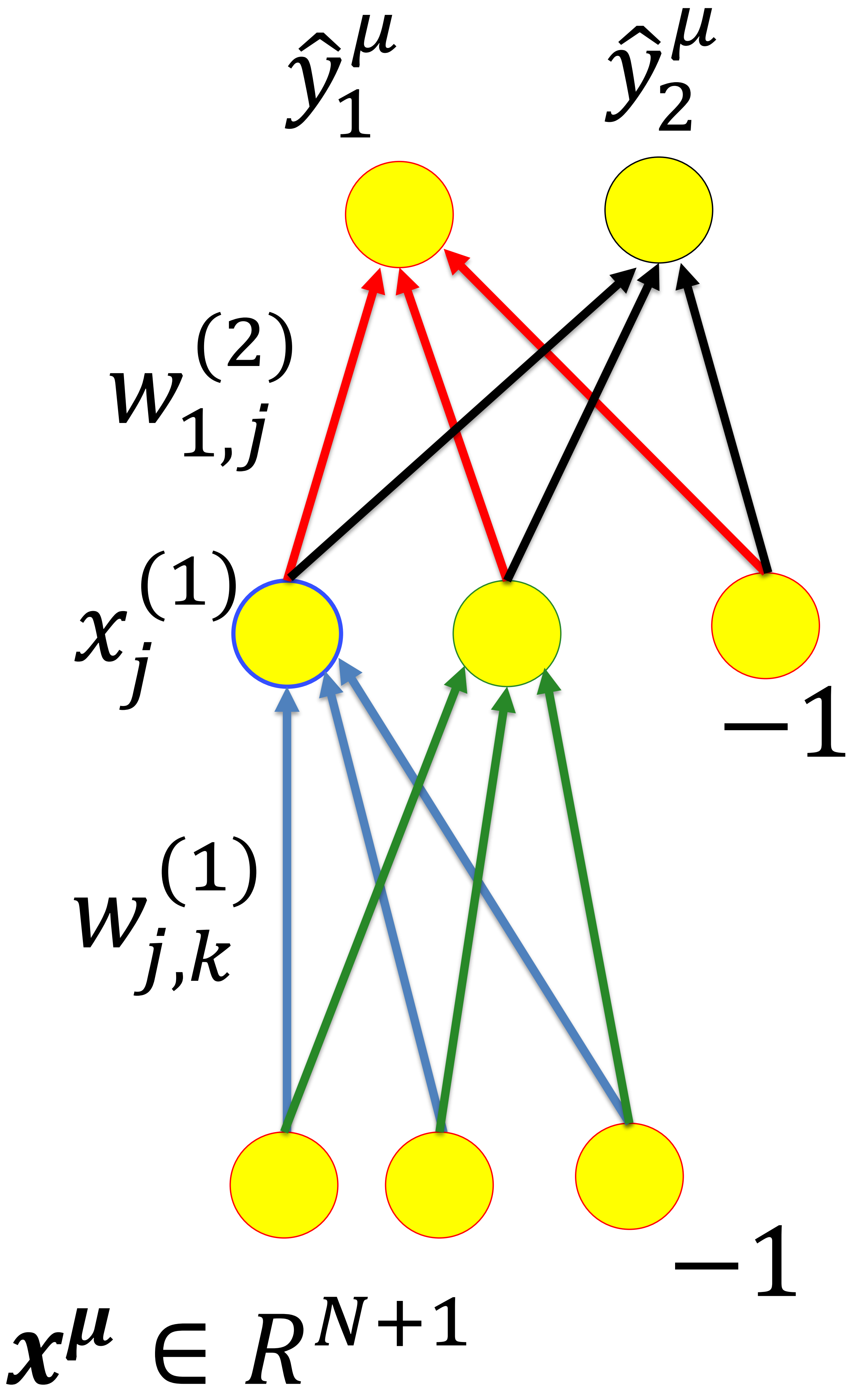


Aim of learning:
Adjust connections such that output is correct (for each input image, **even new ones**)

Review: Multilayer Perceptron



$$
\begin{aligned}
\hat{y}_i^{\mu} &= x_i^{(2)} && (1)\\
&= g^{(2)}[a_i^{(2)}] && (2)\\
&= g^{(2)}[\sum_j w_{ij}^{(2)} x_j^{(1)}] && (3)\\
&= g^{(2)}[\sum_j w_{ij}^{(2)} g^{(1)}(a_j^{(1)})] && (4)\\
&= g^{(2)}[\sum_j w_{ij}^{(2)} g^{(1)}(\sum_k w_{jk}^{(1)} x_k^{\mu})] && (5)
\end{aligned}
$$



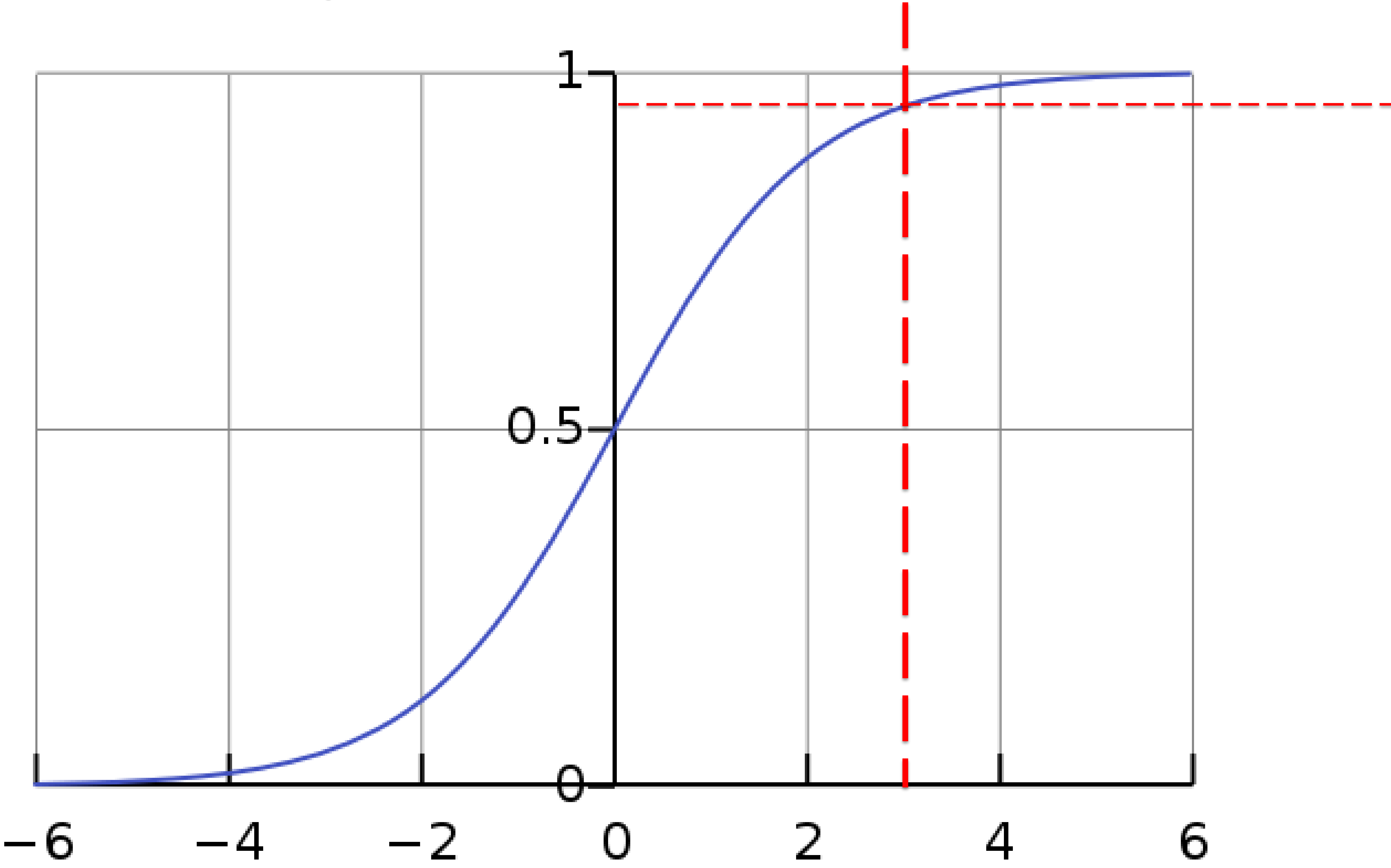
Review. sigmoidal output = **logistic function**

$$g(a) = \frac{1}{1+e^{-a}}$$

Rule of thumb:

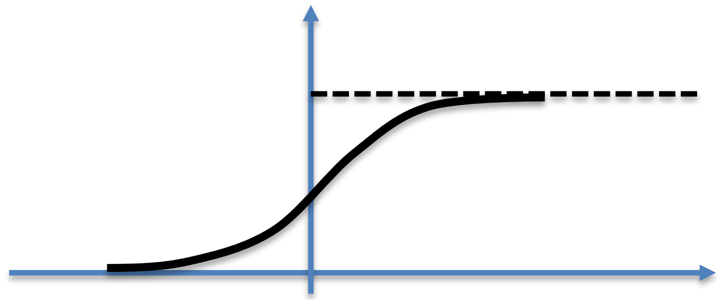
for $a = 3$: $g(3) = 0.95$

for $a = -3$: $g(-3) = 0.05$



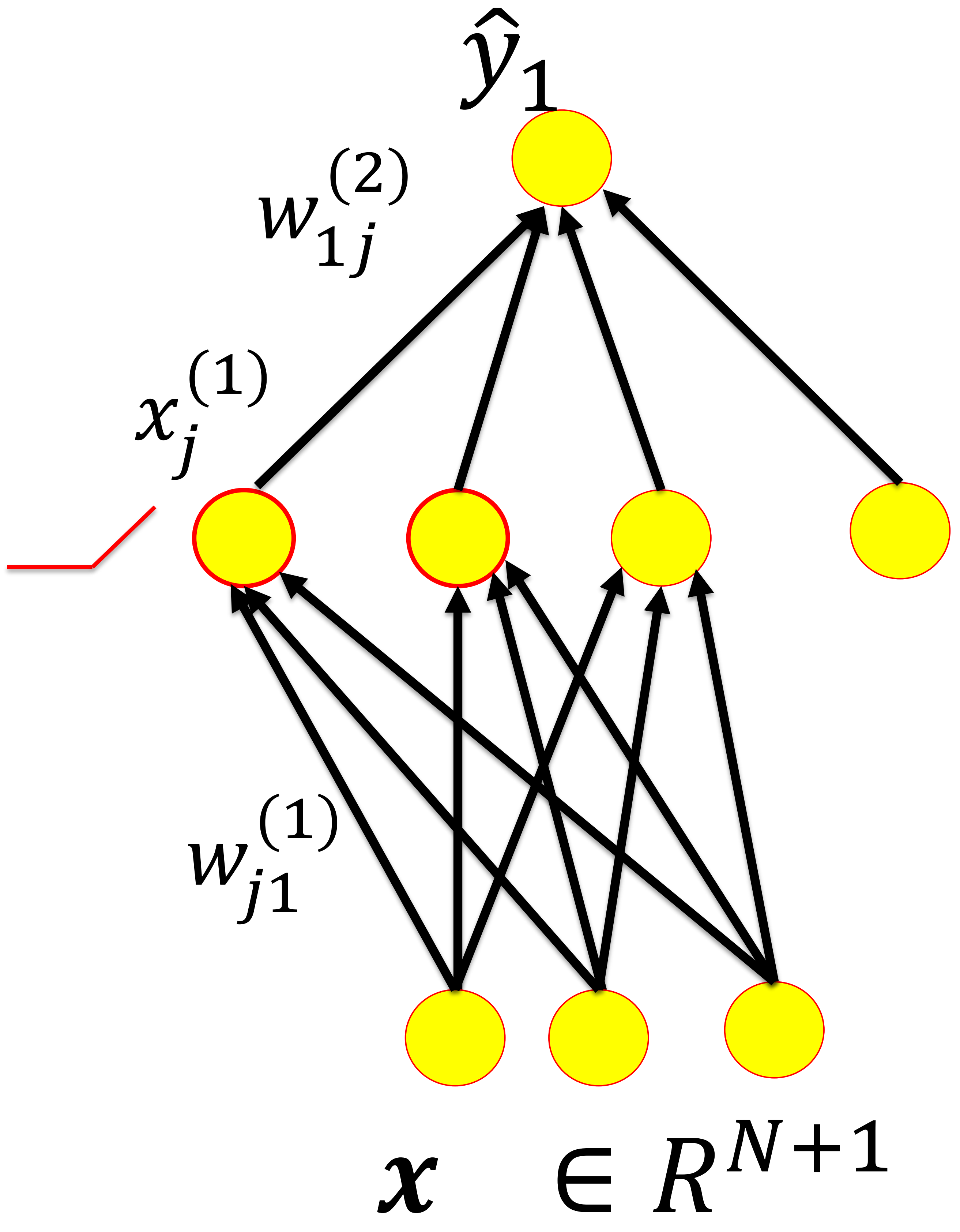
https://en.wikipedia.org/wiki/Logistic_function

Review: Modern Neural Networks



output layer

use sigmoidal unit (single-class)
or softmax (exclusive mutlti-class)



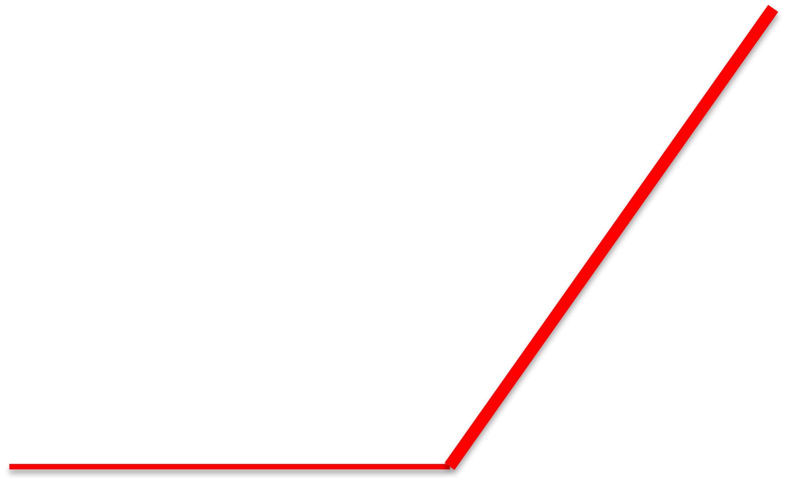
hidden layer

Why?

use rectified linear unit in $N+1$ dim.

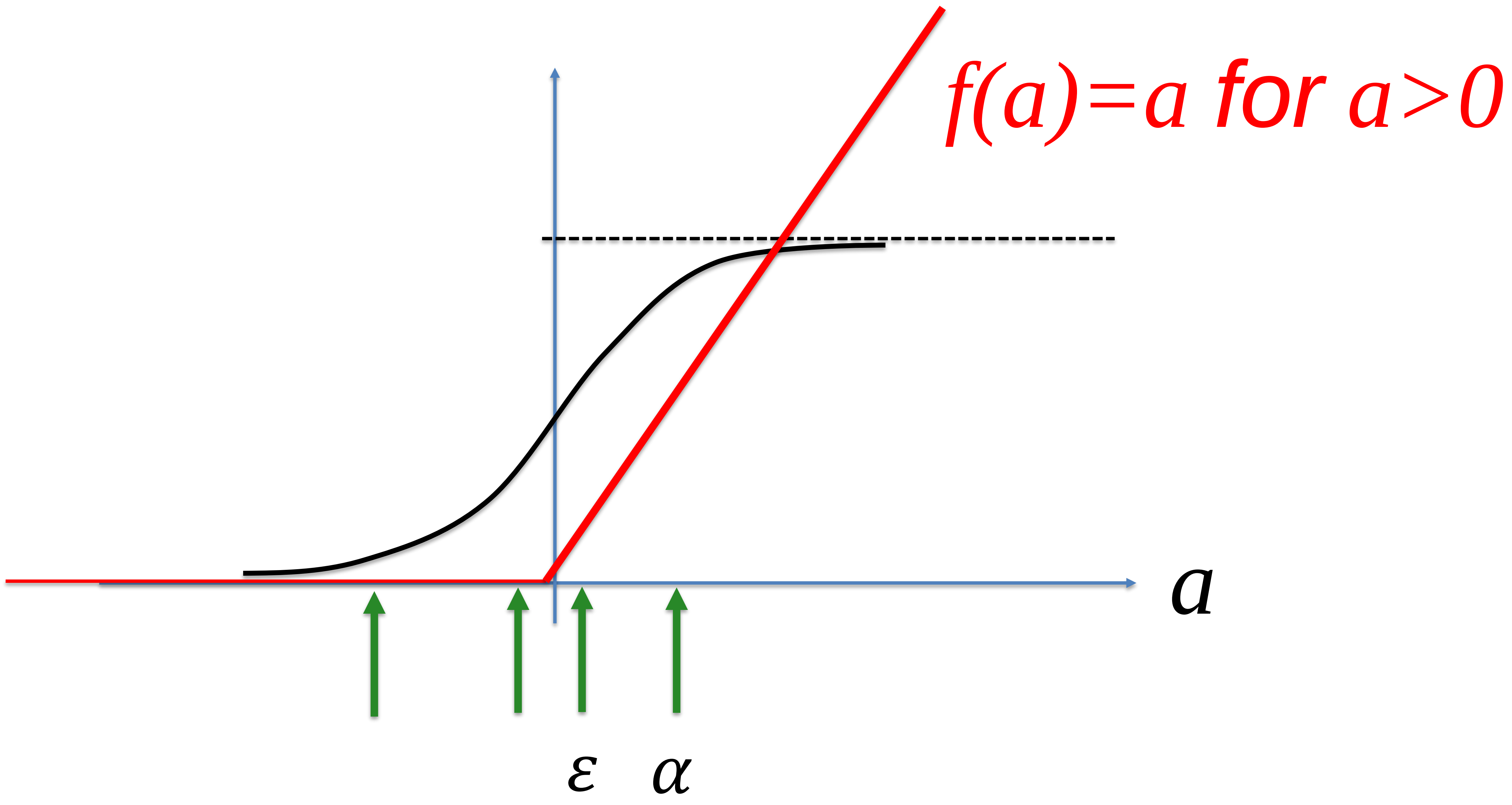
$f(x)=x$ for $x>0$

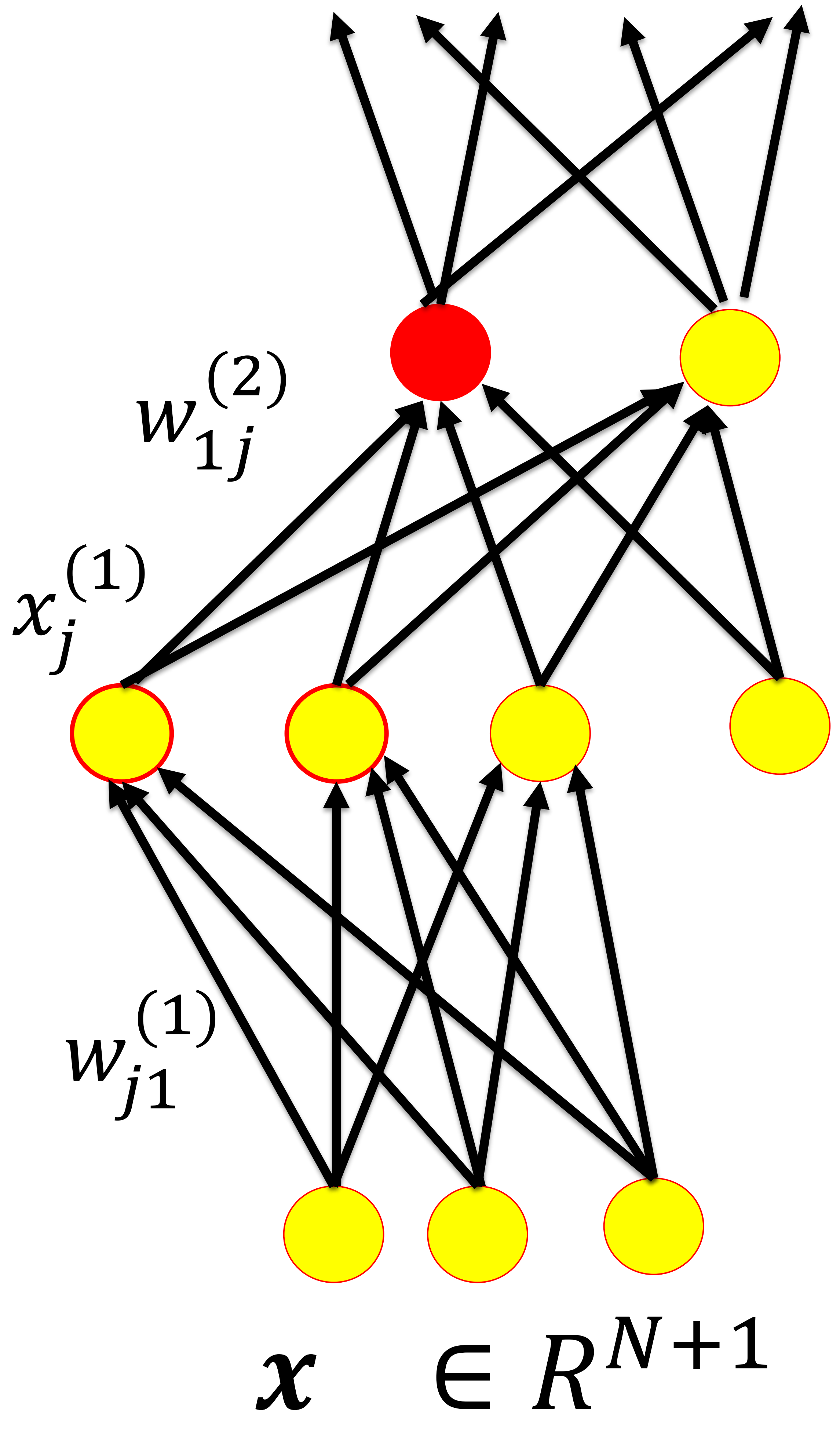
$f(x)=0$ for $x<0$ or $x=0$



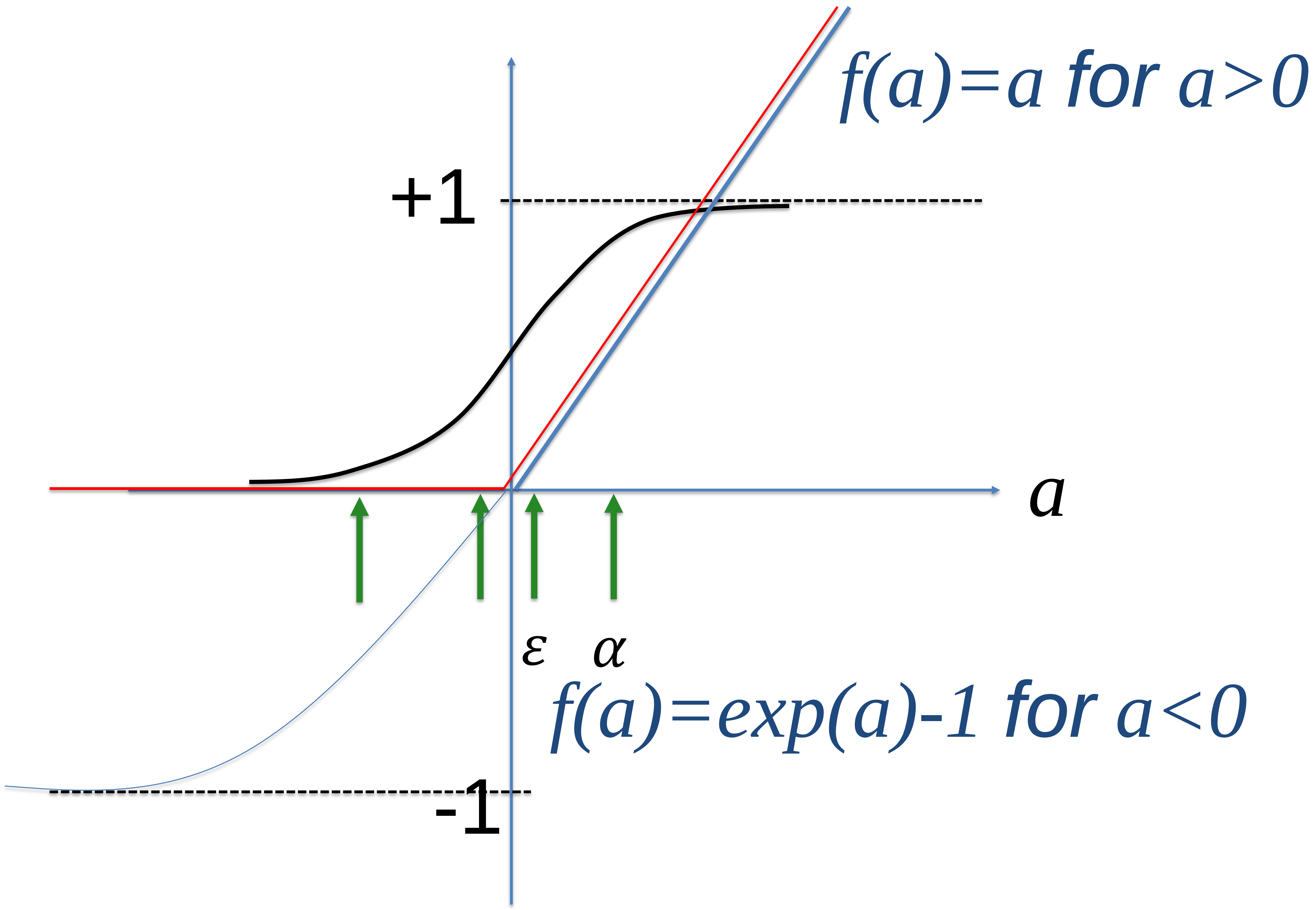
Better choices?

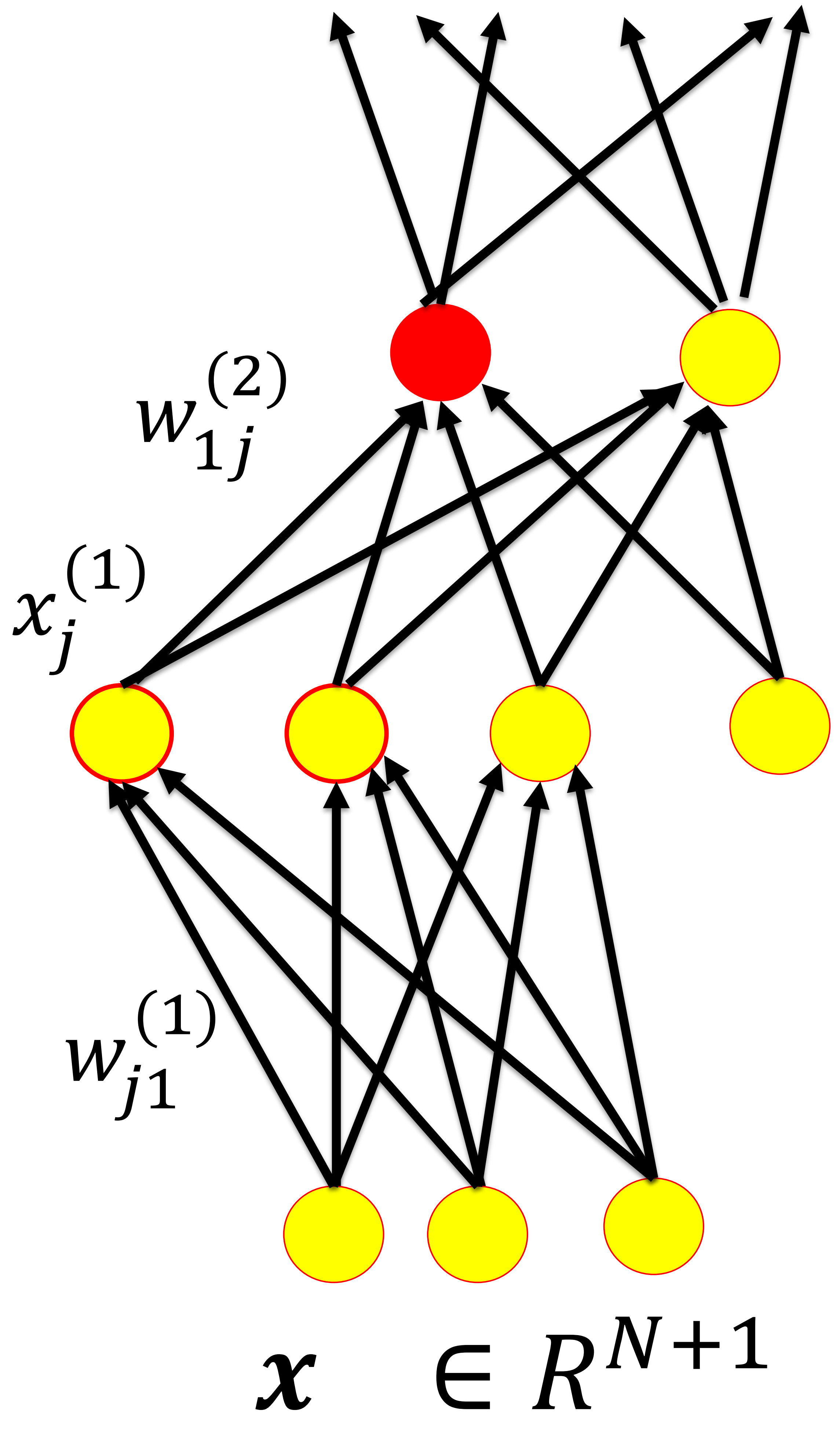
Rectified Linear (RELU) vs. Sigmoidal



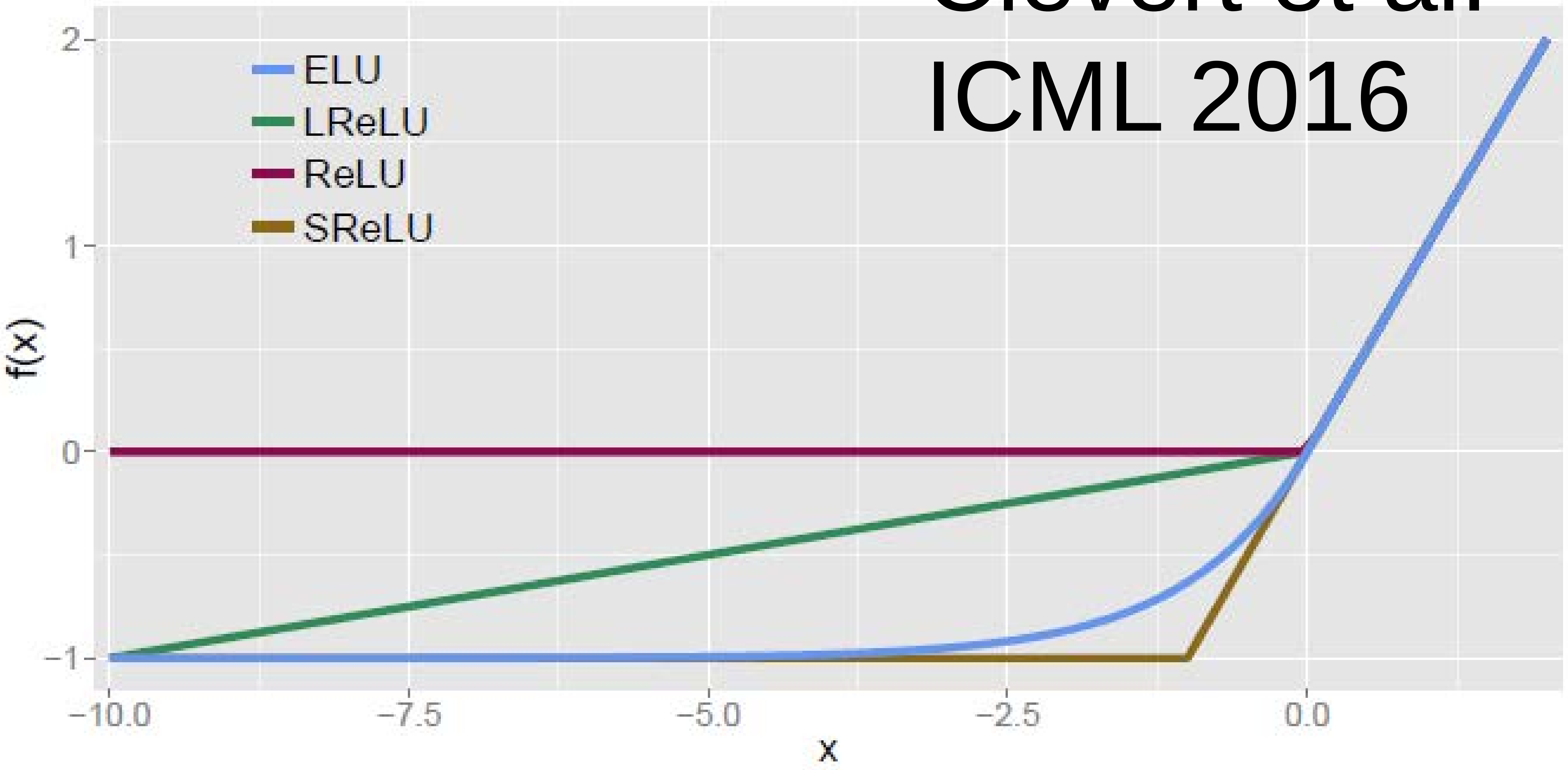


Exponential Linear vs. Sigmoidal

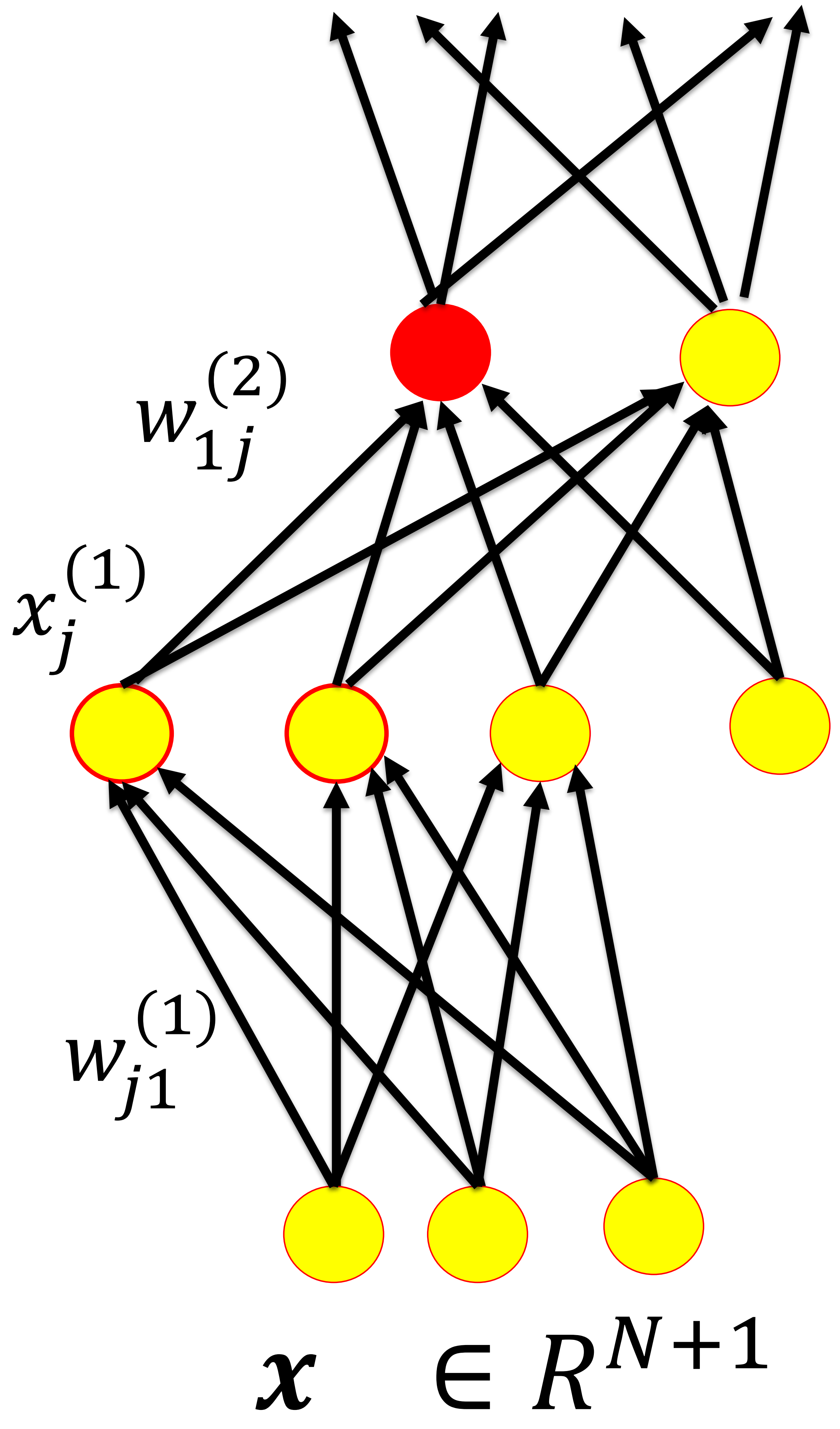




Exponential Linear (ELU) vs. Sigmoidal



Clevert et al.
ICML 2016



Shifted ReLU (SReLU)
Leaky ReLu (LReLU)

Question 1 for this week:

What are good models for hidden neurons?

… and why?

BackProp

0. Initialization of weights

1. Choose pattern $\mathbf{x}^\mu$

   input $x_k^{(0)} = x_k^\mu$

2. Forward propagation of signals $x_k^{(n-1)} \longrightarrow x_j^{(n)}$

$$x_j^{(n)} = g^{(n)}(a_j^{(n)}) = g^{(n)}(\sum w_{jk}^{(n)} x_k^{(n-1)}) \tag{1}$$

   output $\hat{y}_i^\mu = x_i^{(n_{\max})}$

3. Computation of errors in output

$$\delta_i^{(n_{\max})} = g'(a_i^{(n_{\max})})\ [t_i^\mu - \hat{y}^\mu] \tag{2}$$

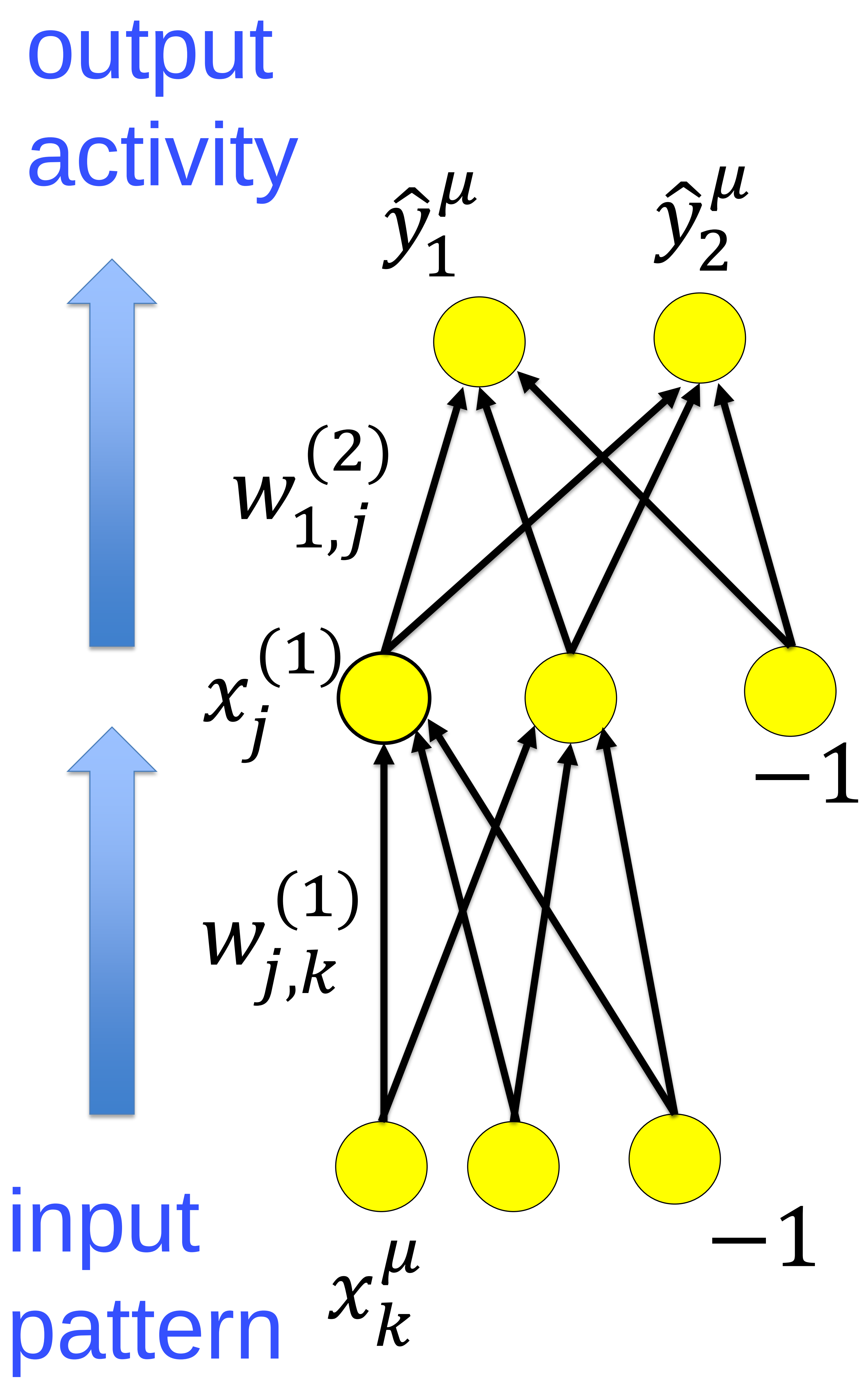
4. Backward propagation of errors $\delta_i^{(n)} \longrightarrow \delta_j^{(n-1)}$

$$\delta_j^{(n-1)} = g'^{(n-1)}(a^{(n-1)}) \sum_i w_{ij}\, \delta_i^{(n)} \tag{3}$$

5. Update weights (for each (i,j) and all layers (n))

$$\Delta w_{ij}^{(n)} = \eta\, \delta_i^{(n)}\, x_j^{(n-1)} \tag{4}$$

6. Return to step 1.



BackProp

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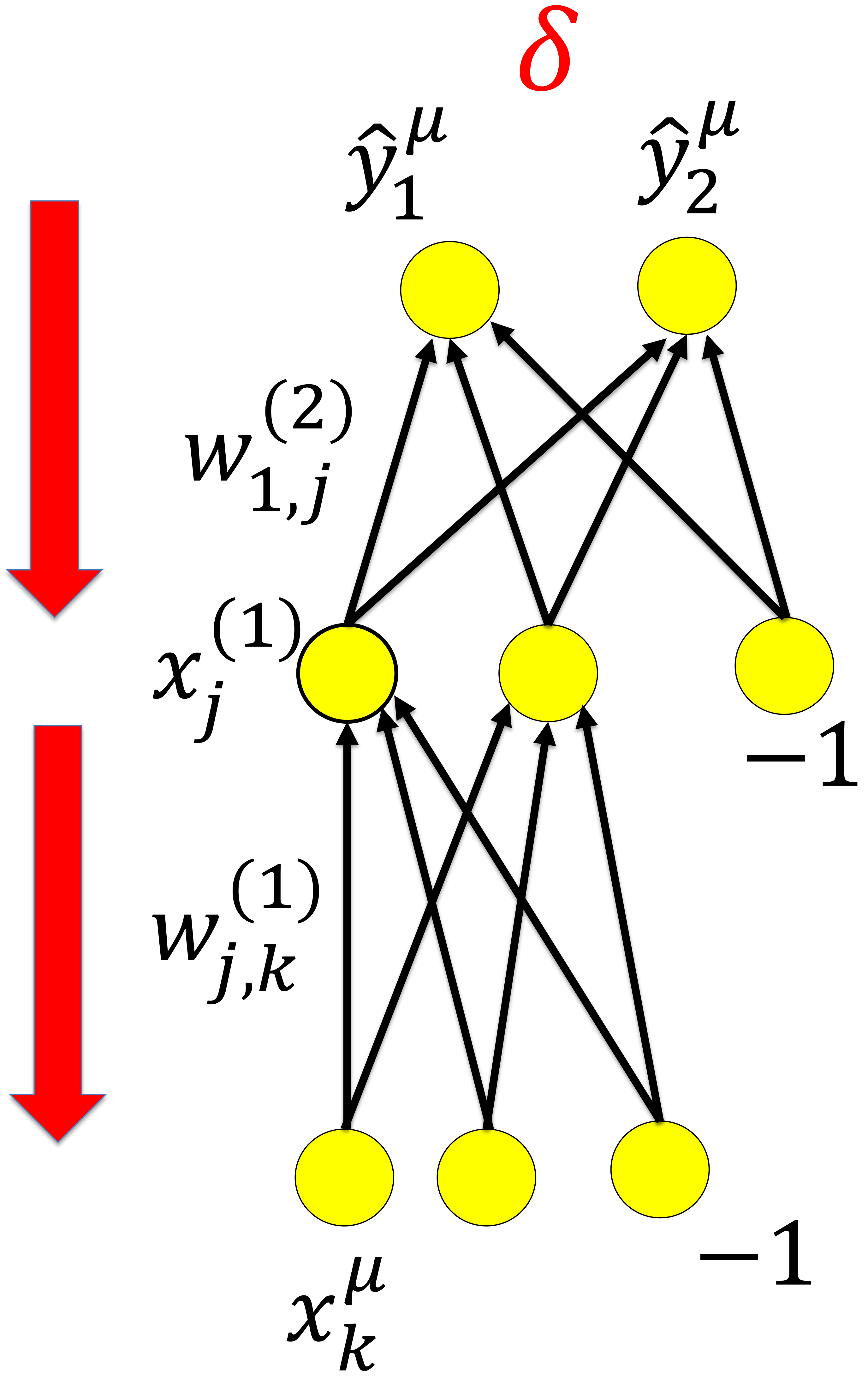
Calculate output error



BackProp

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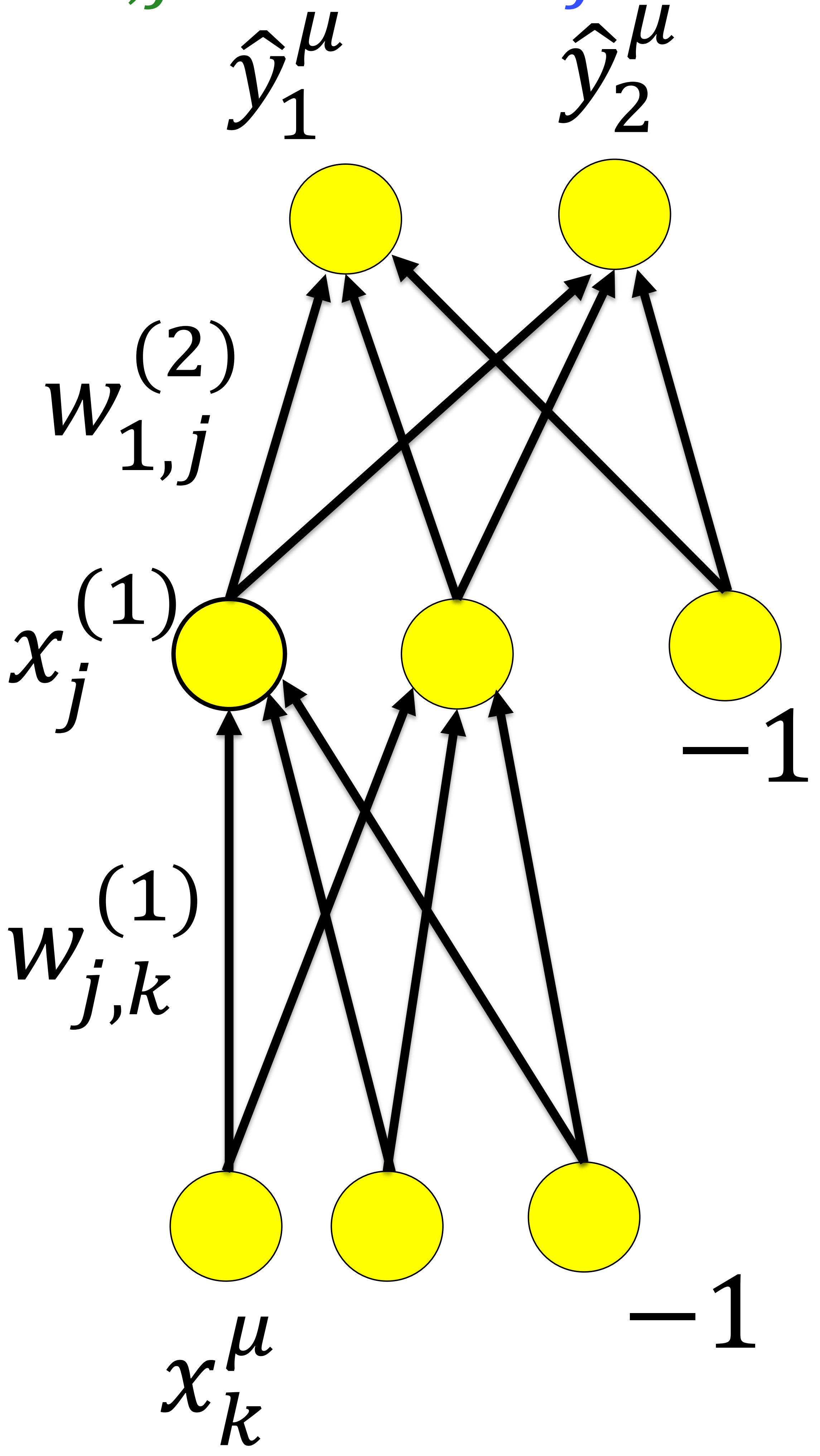
5. Update weights (for each (i,j) and all layers (n))

$$\Delta w_{ij}^{(n)} = \eta\, \delta_i^{(n)}\, x_j^{(n-1)} \tag{4}$$

6. Return to step 1.

update **all** weights

$$\Delta w_{i,j}^{(n)} = \delta_i^{(n)} x_j^{(n-1)}$$

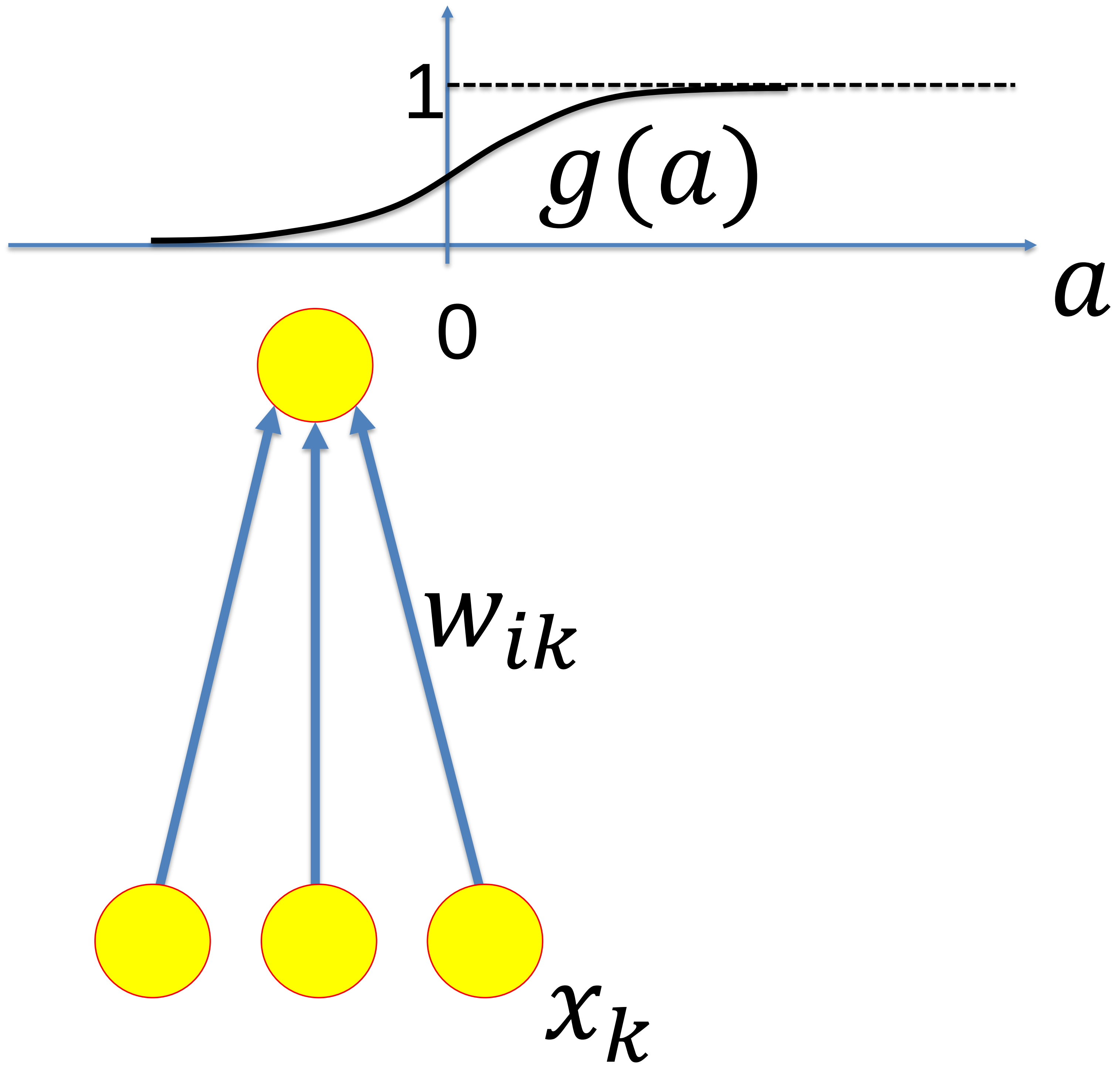


Question 2 for this week:

Why does the initatialization or normalization matter in backprop?

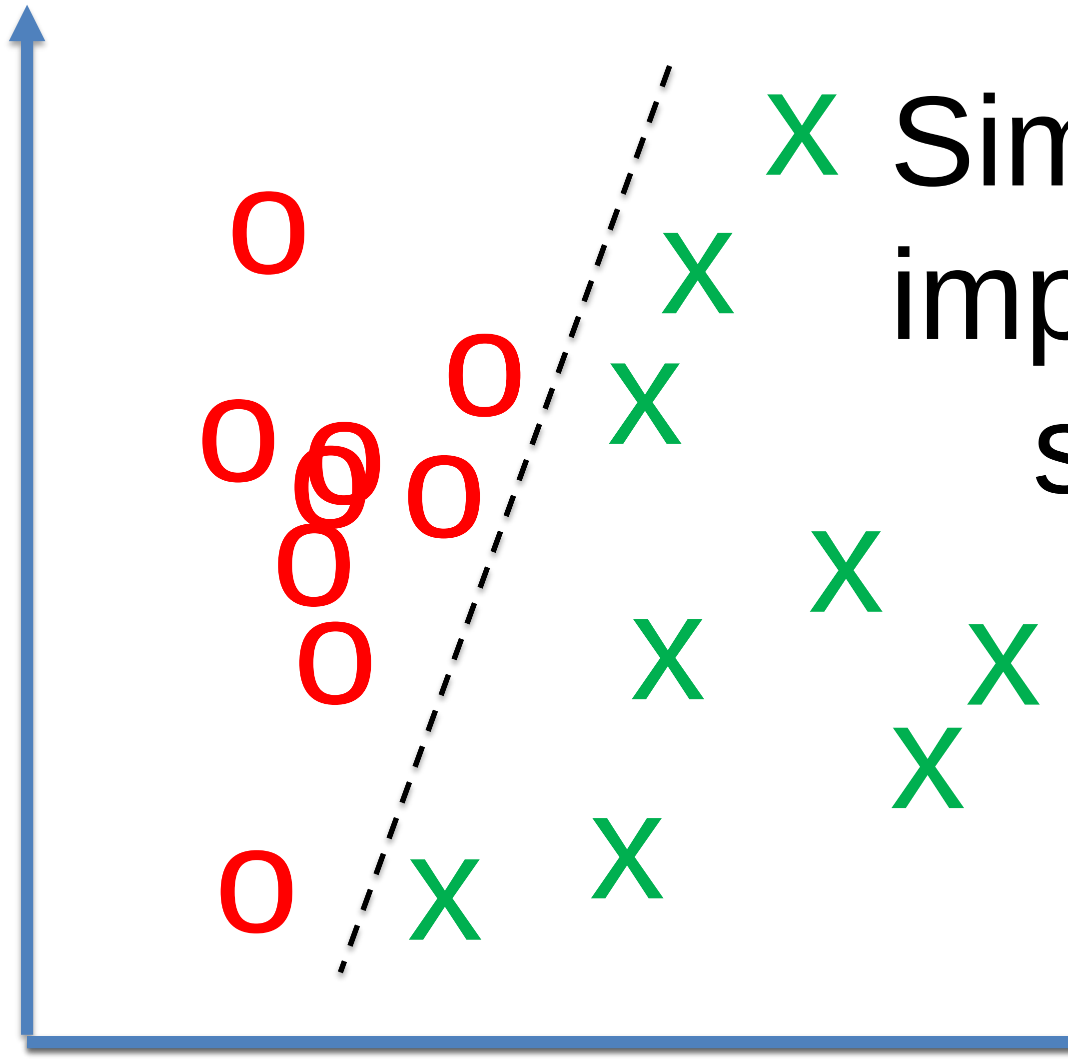
Review: Single-Layer networks/simple perceptron

$$\hat{y} \quad = 0.5[1 + tanh(\sum_k w_k\, x_k - \vartheta)]$$



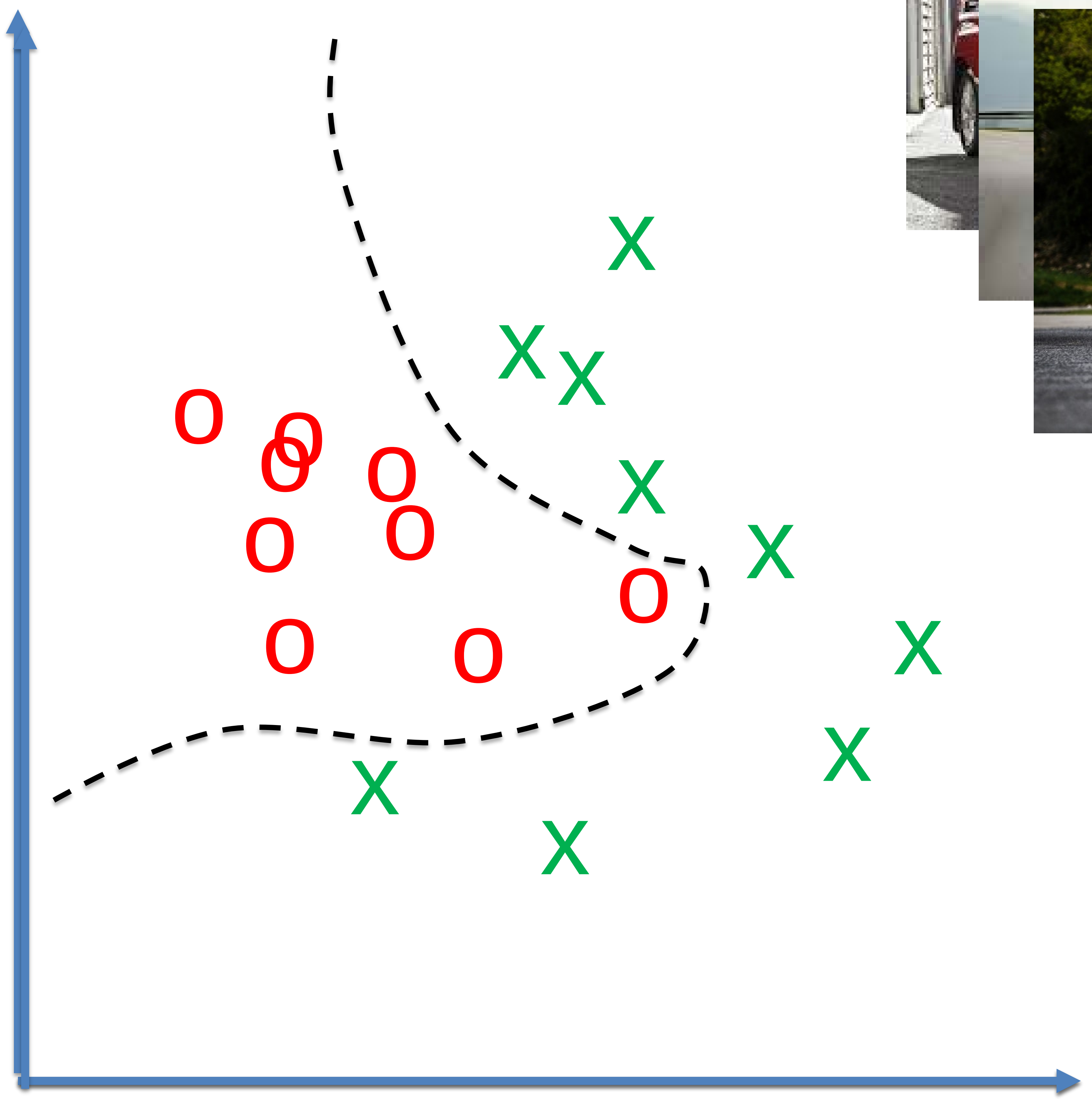
vector $\boldsymbol{x}$

$$d(\boldsymbol{x}) = \sum_k w_k\, x_k - \vartheta = 0$$



Simple perceptron imposes a linear separation

Review: Classification as a geometric problem

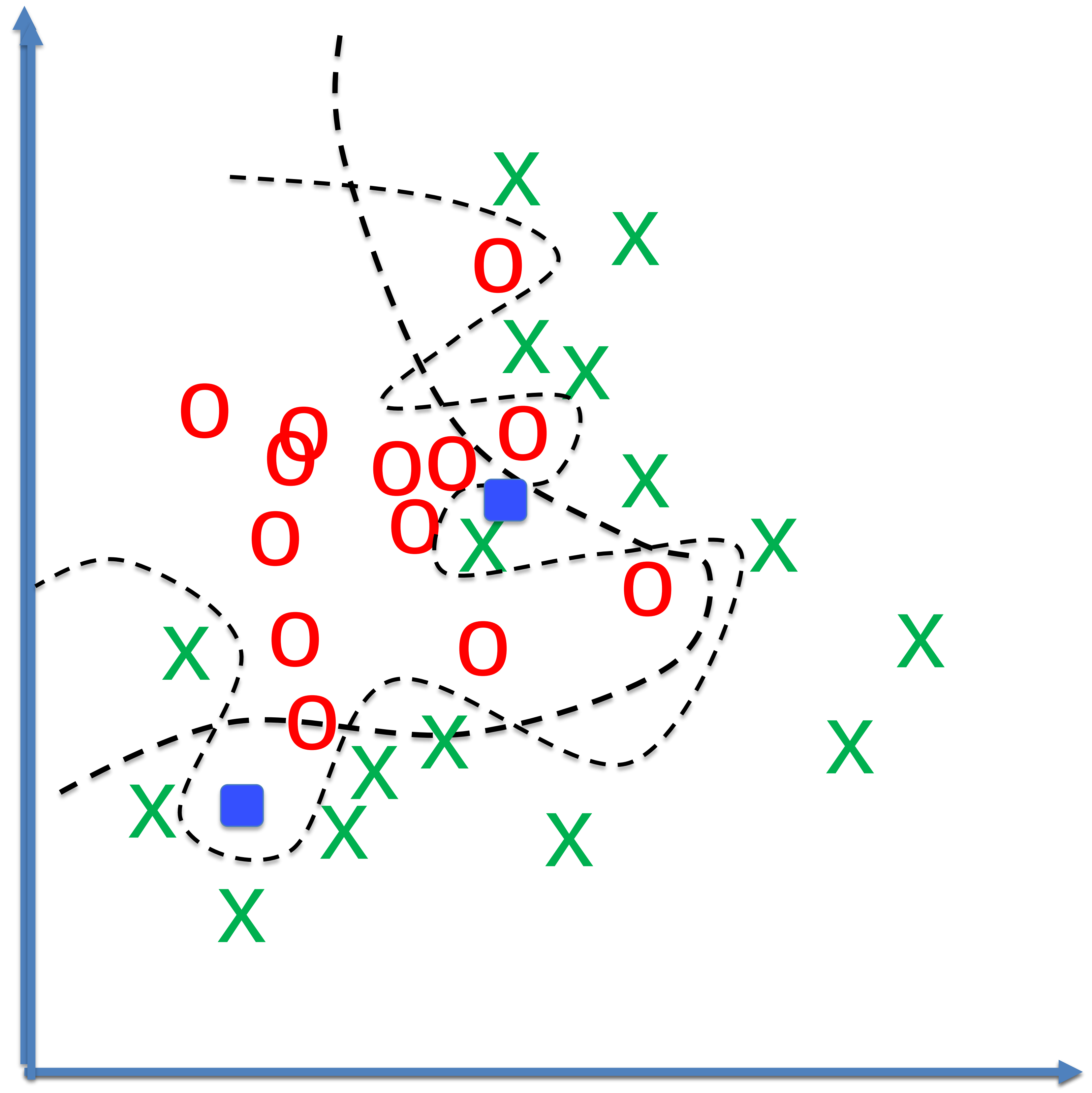


Review: The problem of overfitting

Big Multilayer perceptrons are flexible and can be trained by BackProp to minimize classification error

… but is flexibility always good?

Network has to work on future data:
test data base



Question 3 for this week:

What are good models for regularization?

… and why?

We start with this question!

Artificial Neural Networks: Lecture 4

Tricks of the Trade in deep networks

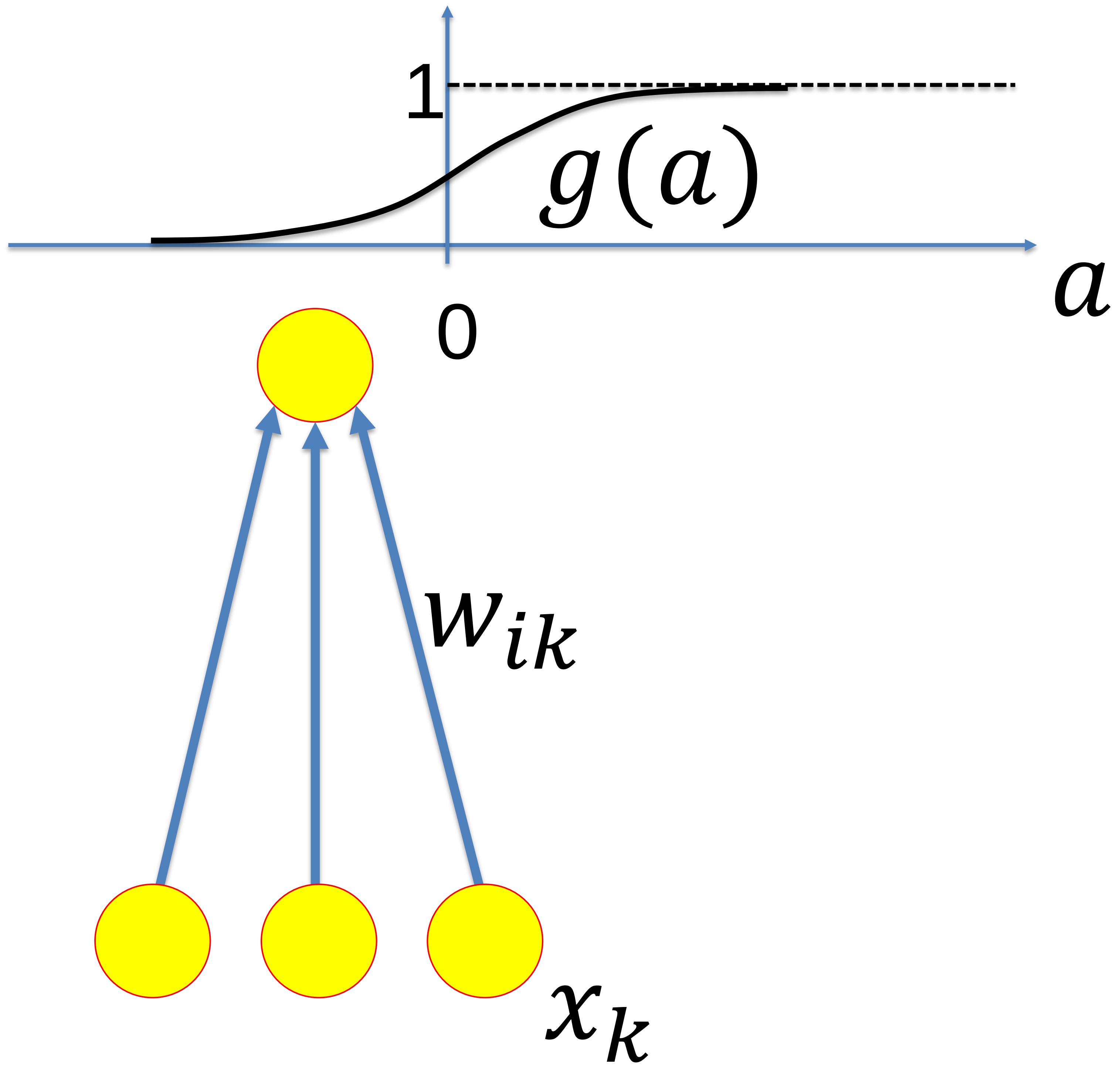
Wulfram Gerstner

EPFL, Lausanne, Switzerland

1. Bagging

1. Bagging Example: simple perceptron

$$\hat{y} \quad = 0.5[1 + tanh(\textstyle\sum_k w_k\, x_k - \vartheta)]$$



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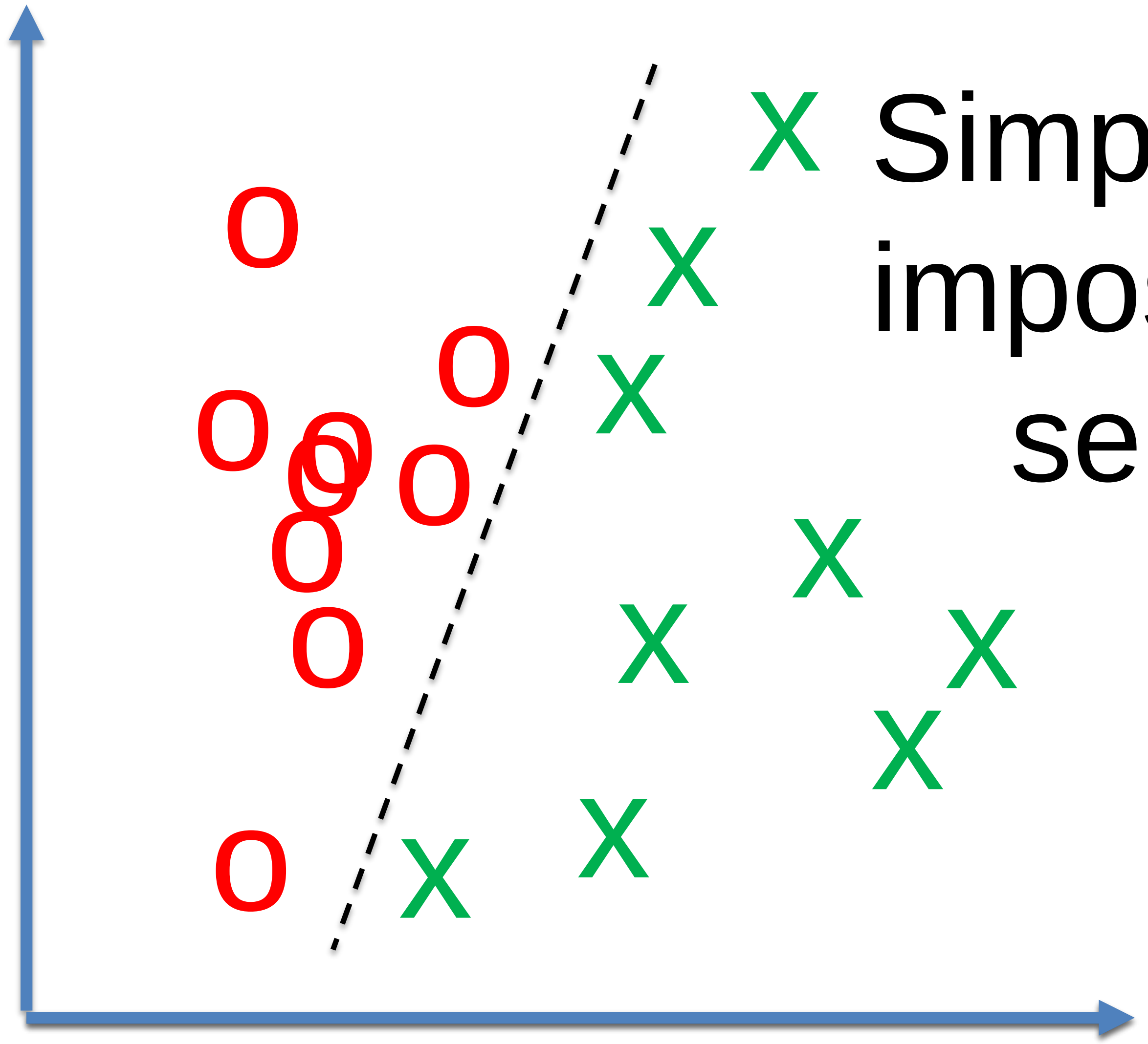
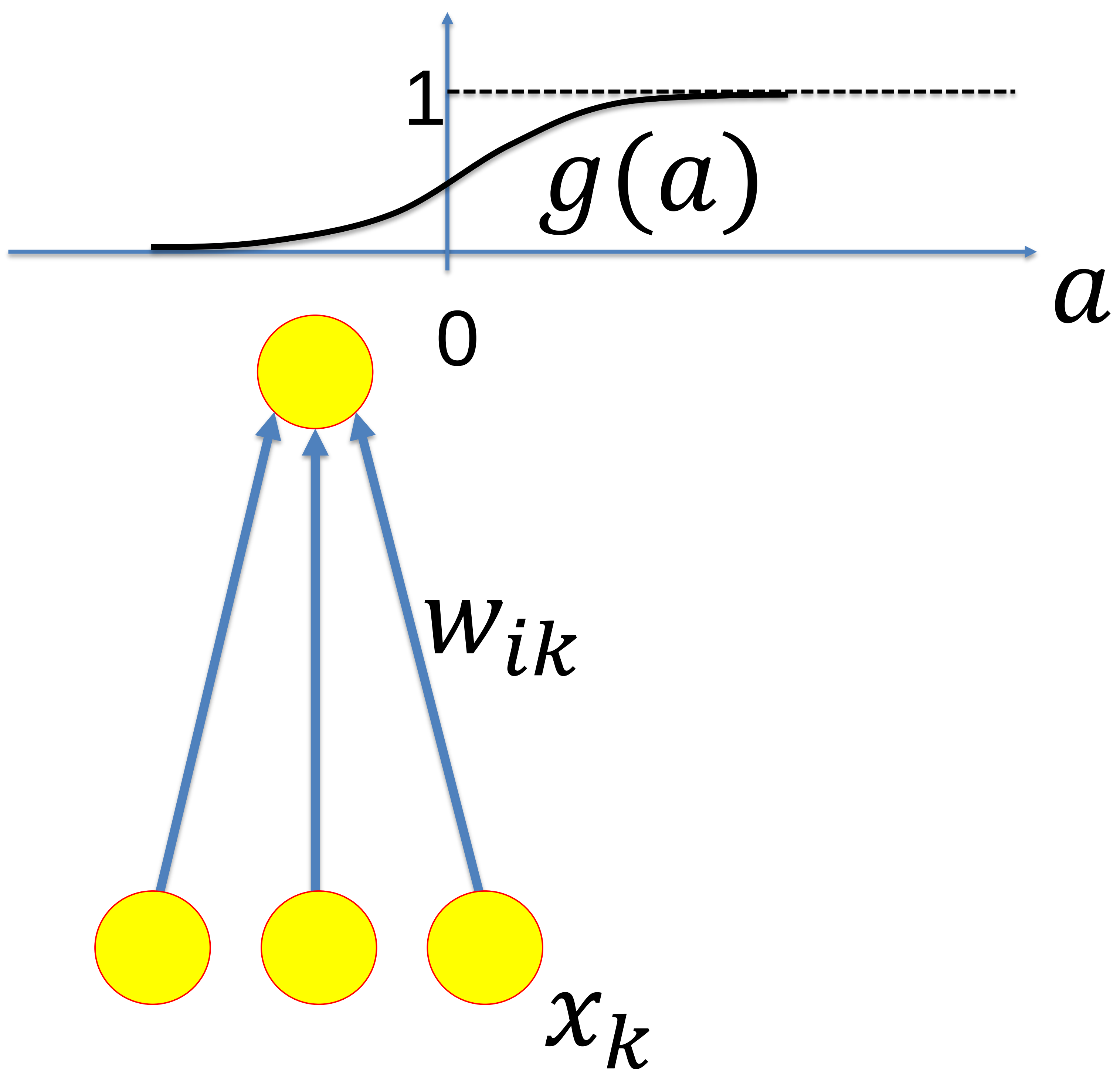


Simple perceptron imposes a linear separation

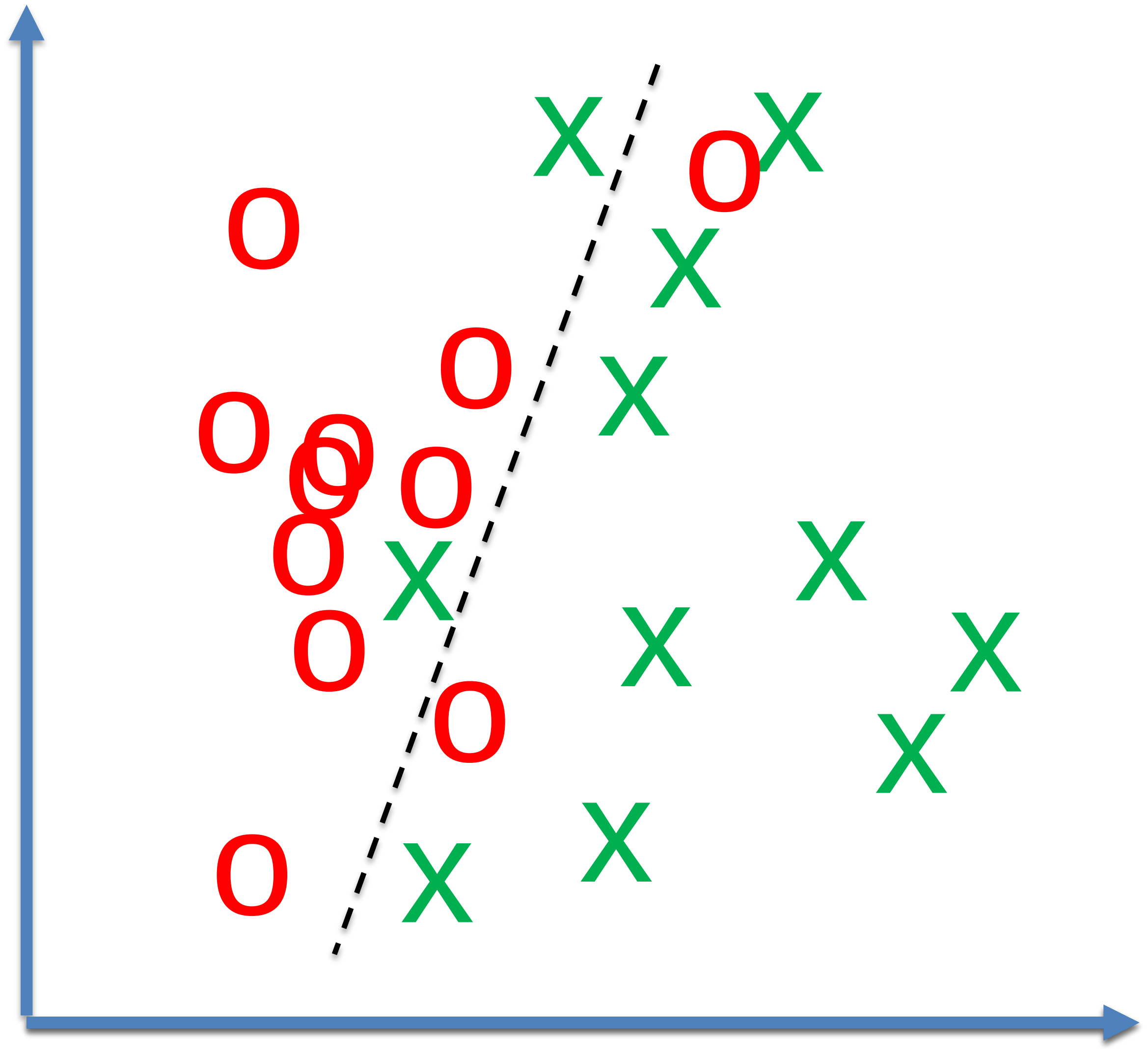
1. Bagging Example: simple perceptron for noisy data

$$\hat{y} \quad = 0.5[1 + tanh(\sum_k w_k \, x_k - \vartheta)]$$



vector $\boldsymbol{x}$

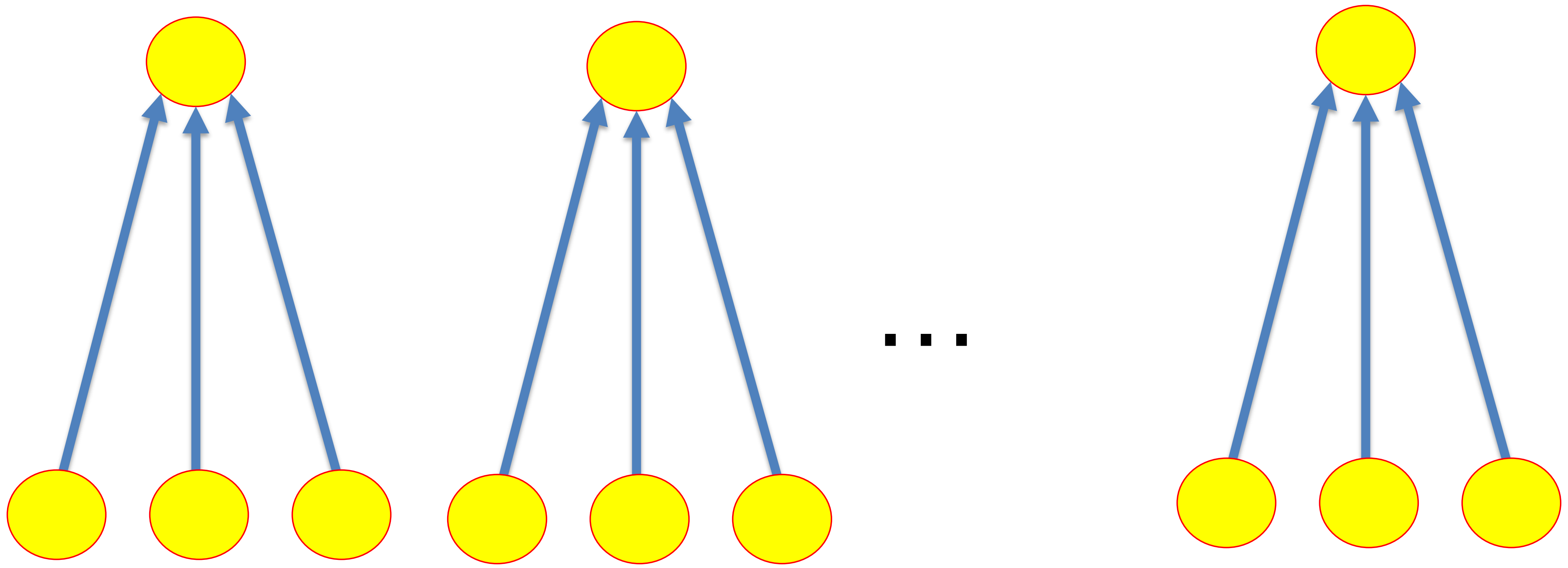
Find best (approximate) linear separation

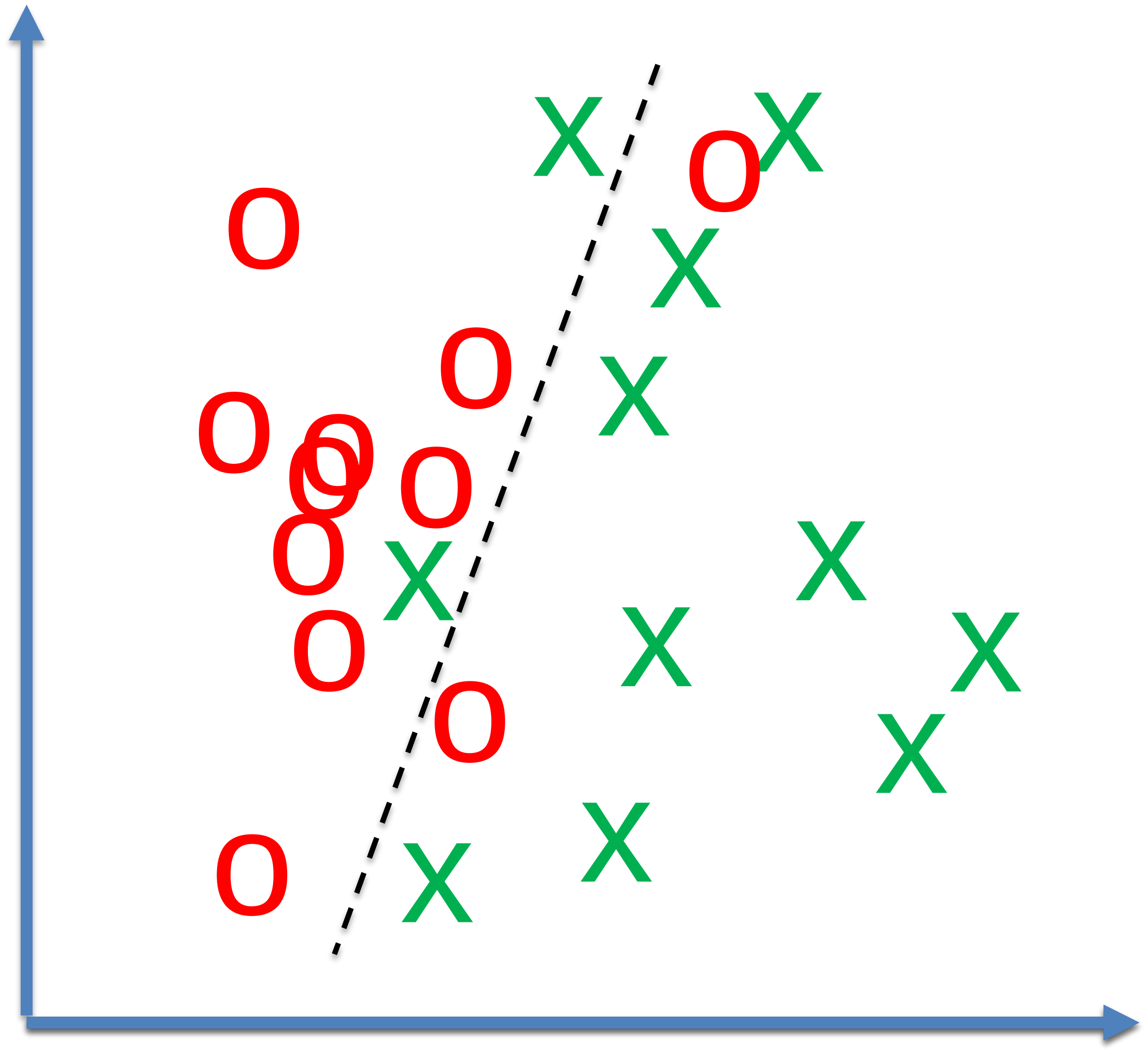


1. Bagging Idea: (i) Repeat variants of your model K times

$$\hat{y} \quad = 0.5[1 + tanh(\sum_k w_k \, x_k - \vartheta)]$$

Find best (approximate) linear separation

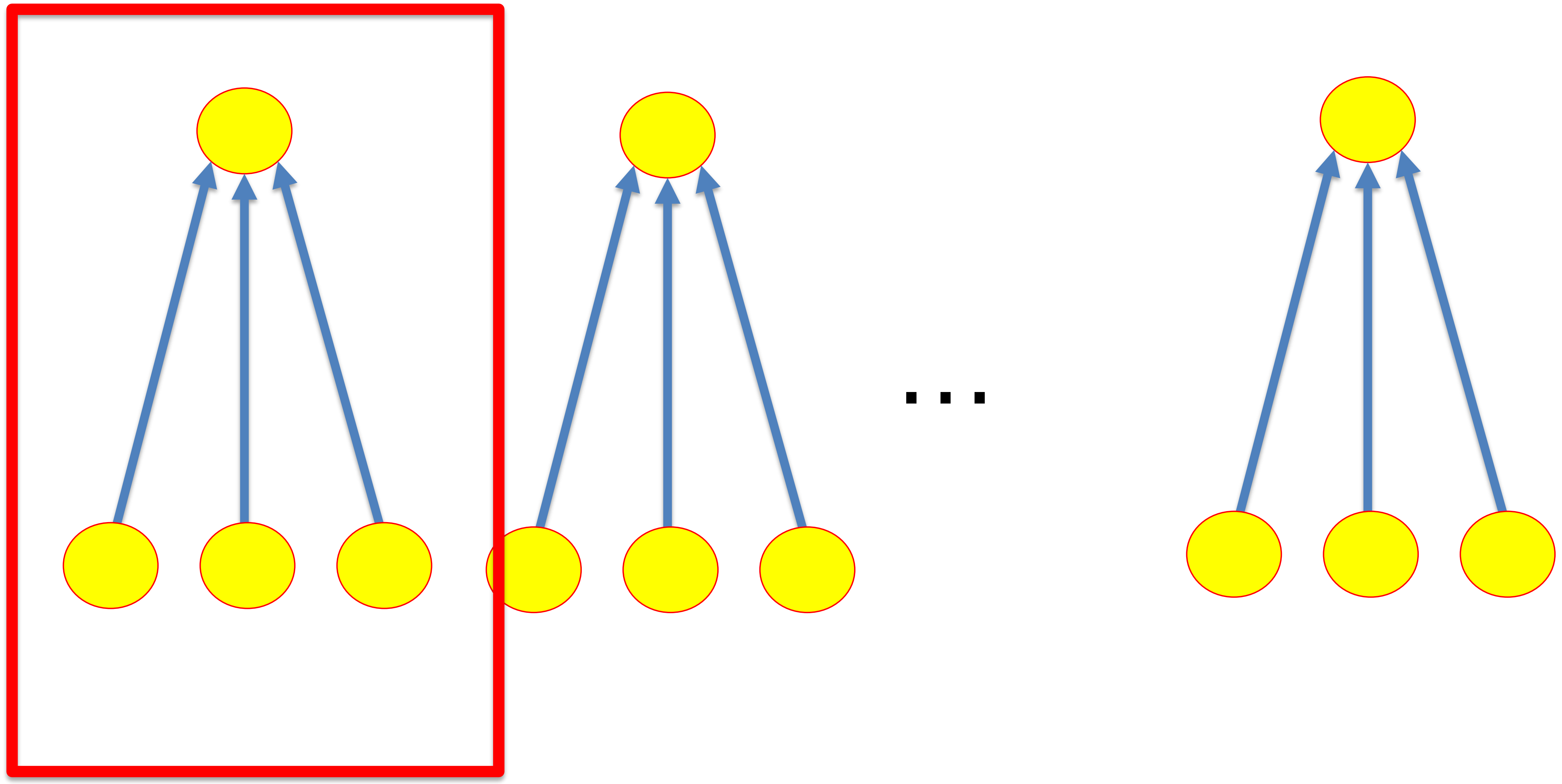


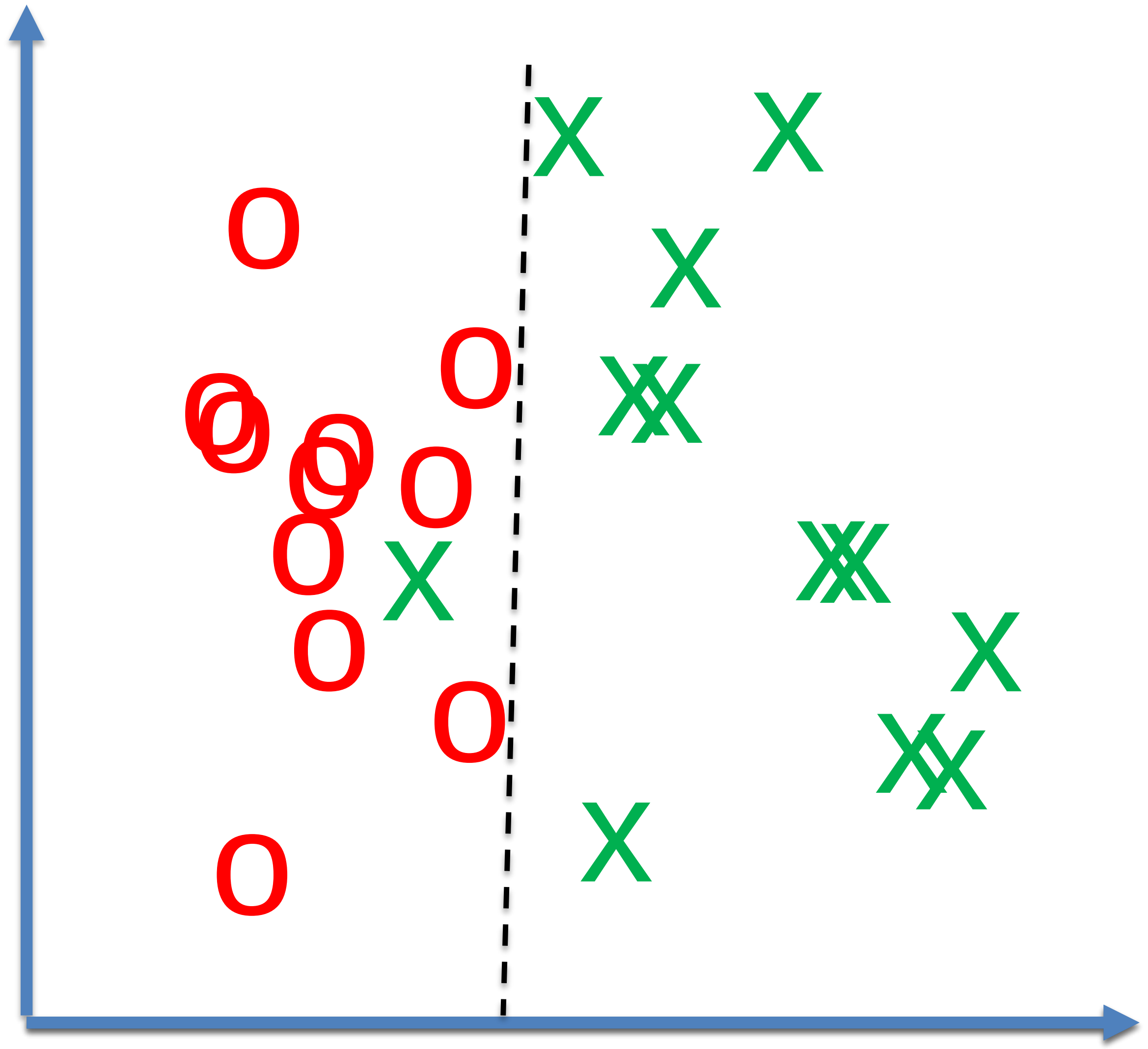


1. Bagging Idea: (ii) Each Variant sees different subsets of data

$$\hat{y}_1 = 0.5[1 + tanh(\sum_k w_k\, x_k - \vartheta)]$$

Find best (approximate) linear separation

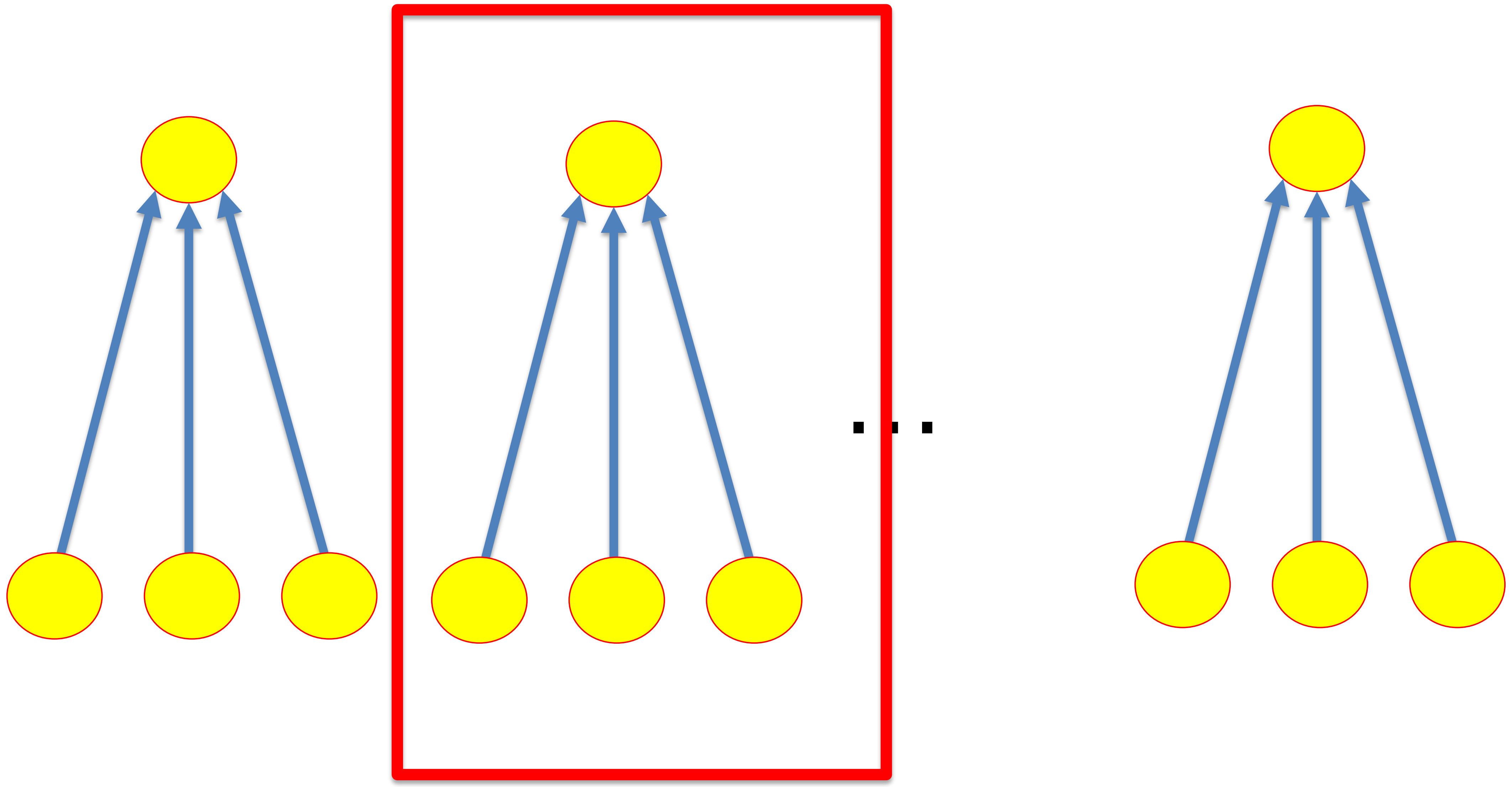


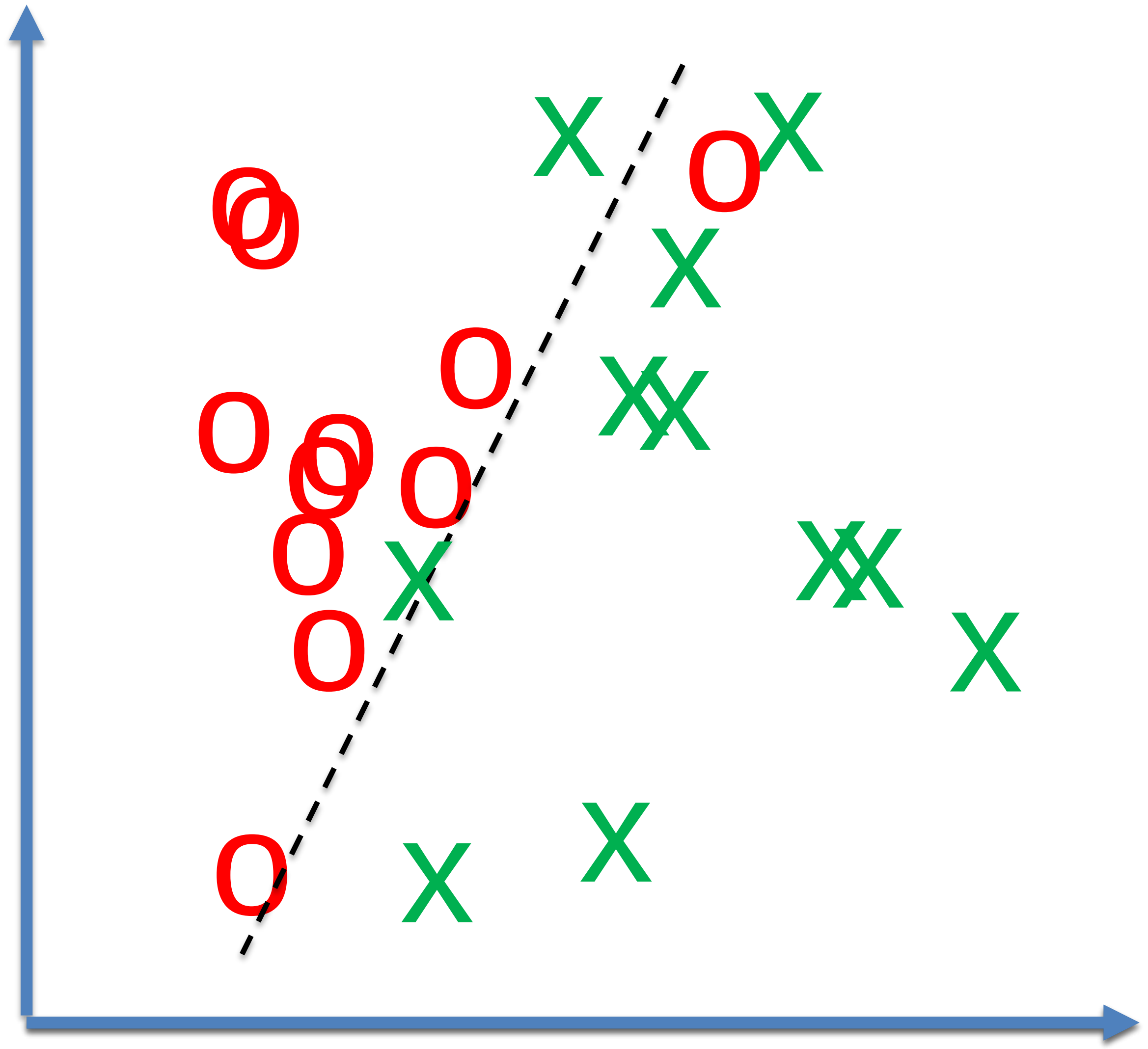


1. Bagging Idea: (ii) Each Variant sees different subsets of data

$$\hat{y}_2 = 0.5[1 + tanh(\sum_k w_k\, x_k - \vartheta)]$$

Find best (approximate) linear separation

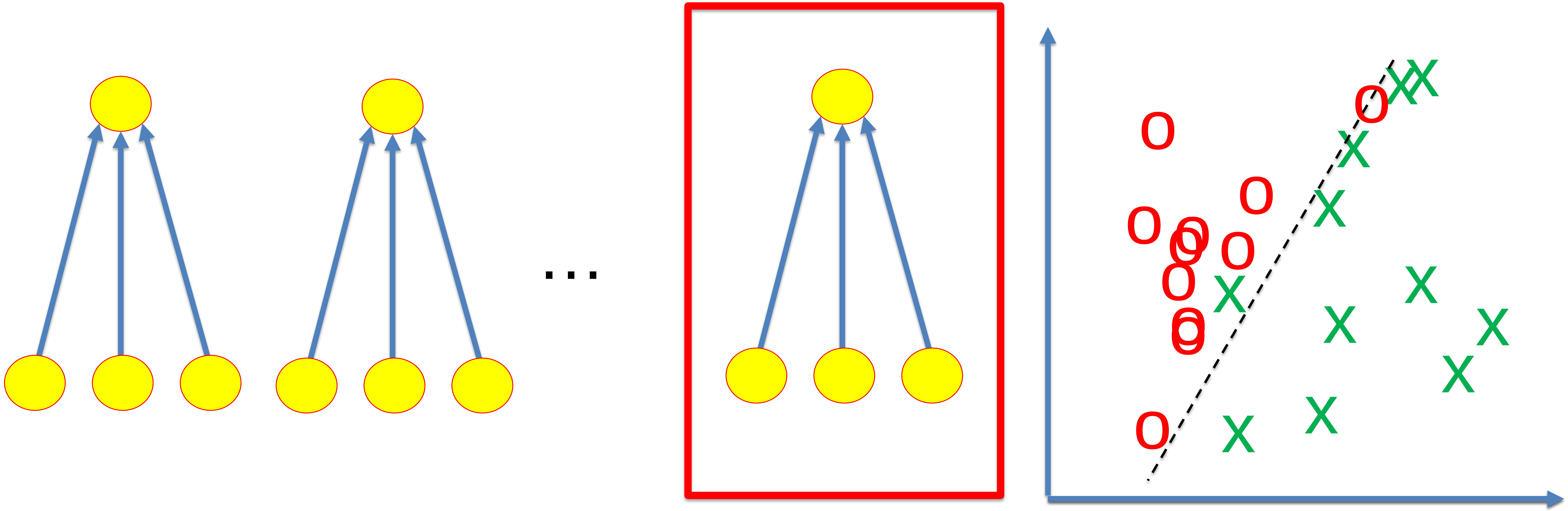




1. Bagging Idea: (ii) Each Variant sees different subsets of data

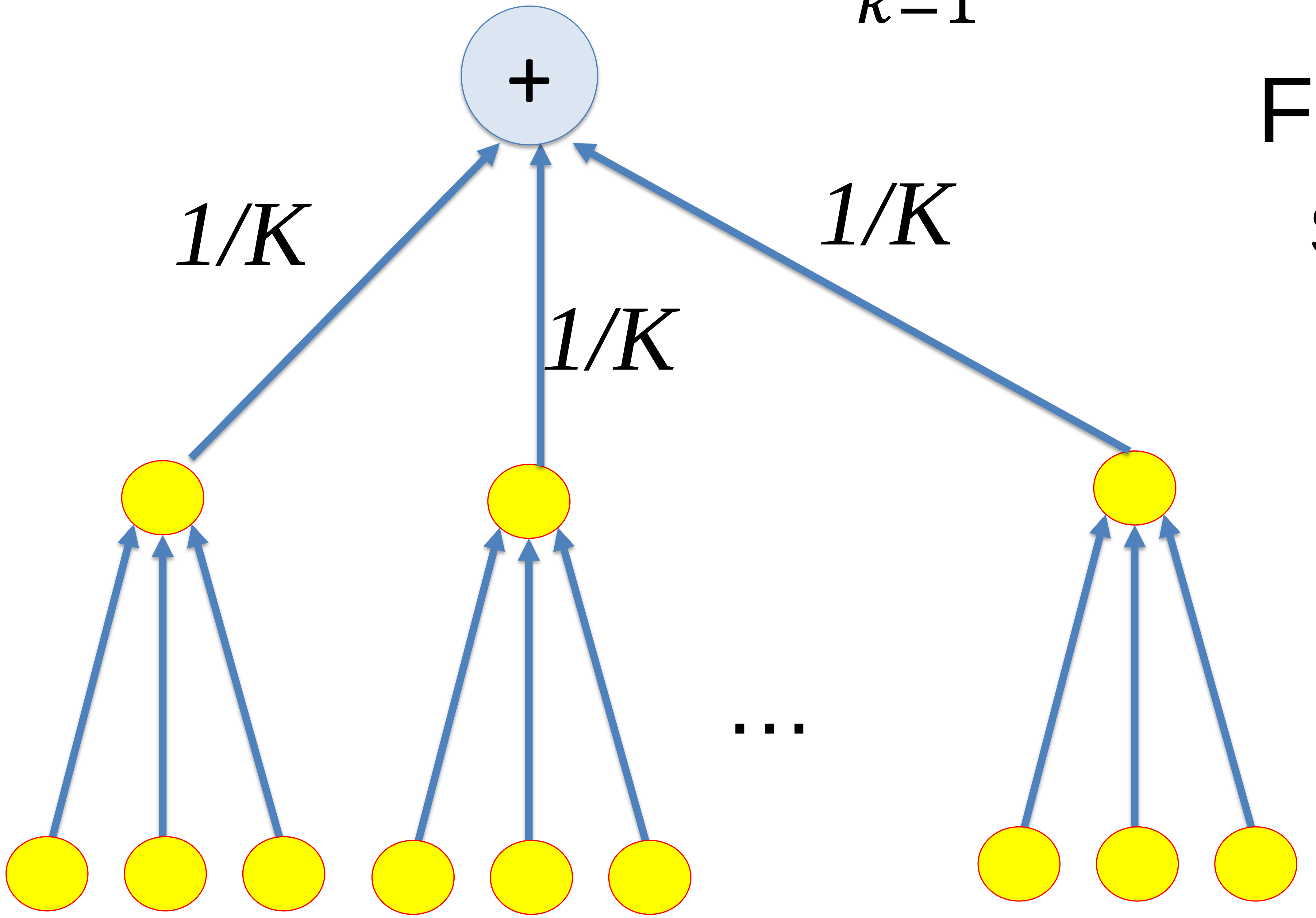
$$\hat{y}_K = 0.5[1 + tanh(\sum_k w_k x_k - \vartheta)]$$

Find best (approximate) linear separation

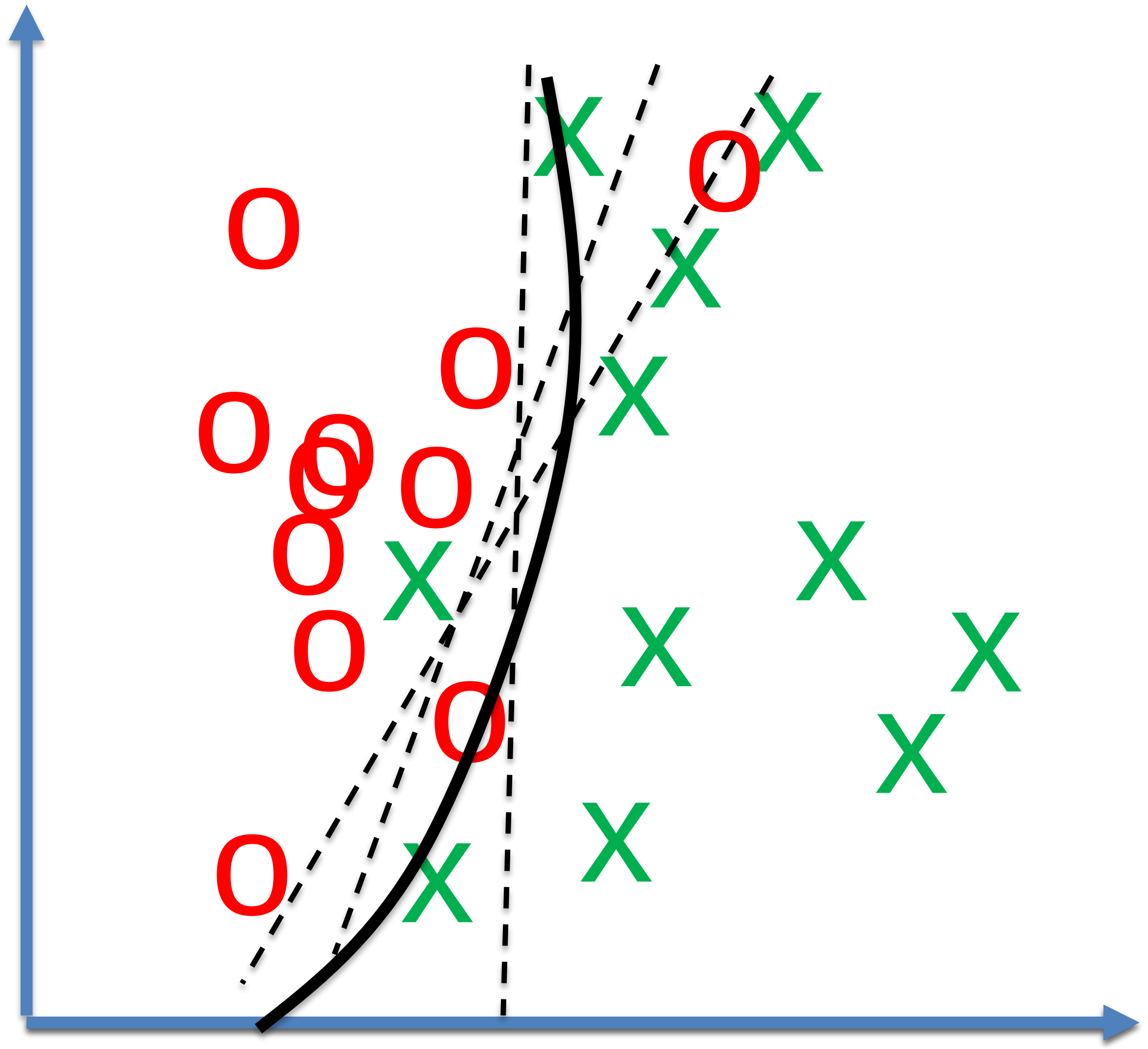


1. Bagging Idea: (iii) Average over all K variants

$$\hat{y}_{bag} = \frac{1}{K}\sum_{k=1}^{K} \widehat{y}_k$$



Find average (nonlinear) separation



1. Bagging : Algorithm

Given: Training data set $\{ \quad (\boldsymbol{x}^{\mu}, t^{\mu}) \quad , \qquad 1 \le \mu \le P1 \quad \}$;

1 Generate K different training sets

for $k=1,\ldots,K$

pick $P1$ times into your data set with replacement

(your can pick the same data point several times)

2 Initialize K different variants of your model

3 Train model k on data set k up to criterion

4 For a future data point (test set)

for $k=1,\ldots,K$

put input $\boldsymbol{x}$ into model k, read out $\hat{y}_k$

5 Report average $\hat{y}_{bag} = \frac{1}{K} \sum_{k=1}^{K} \hat{y}_k$

1. Bagging: Theory

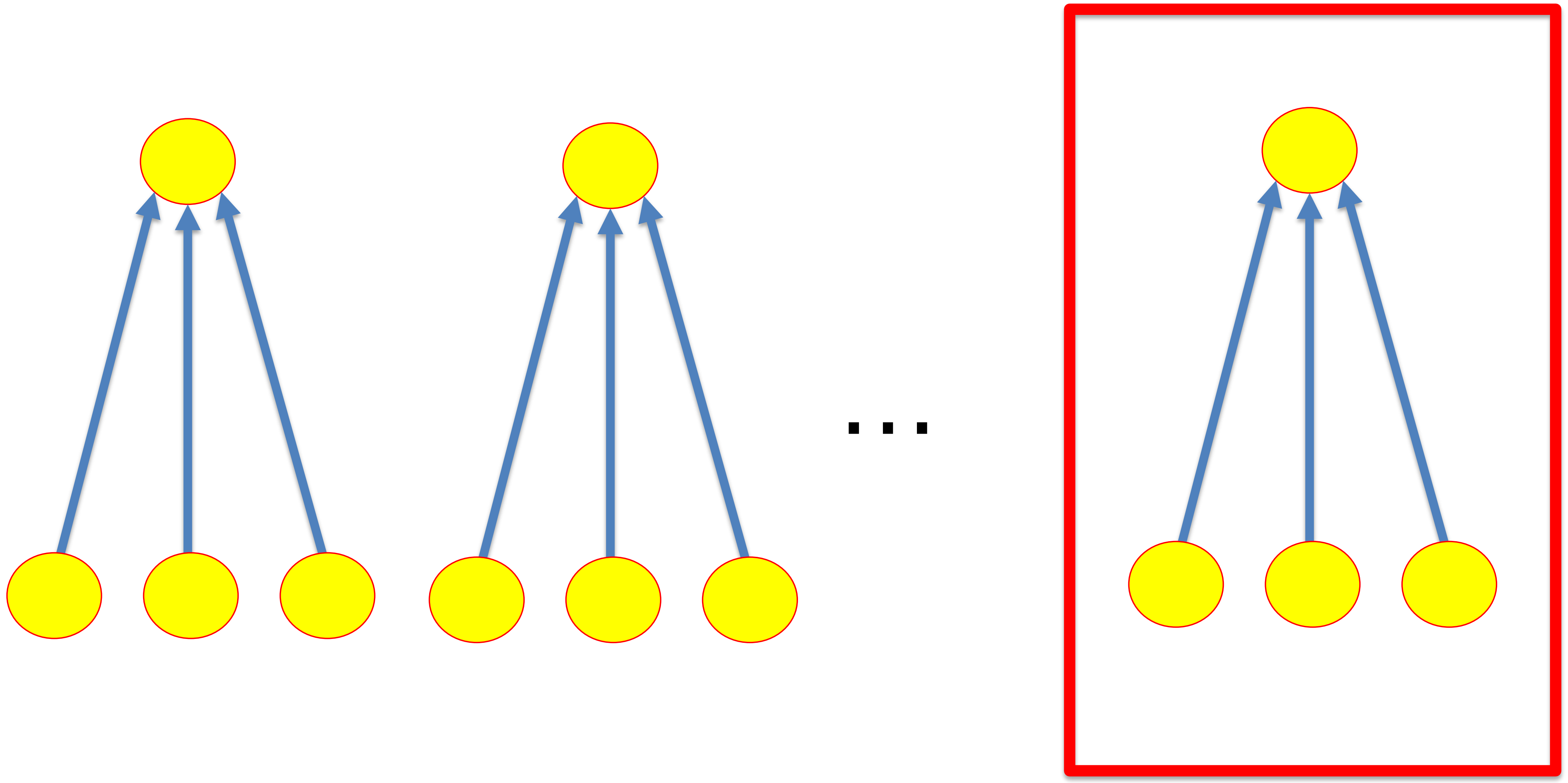
Blackboard: Bagging

Model k

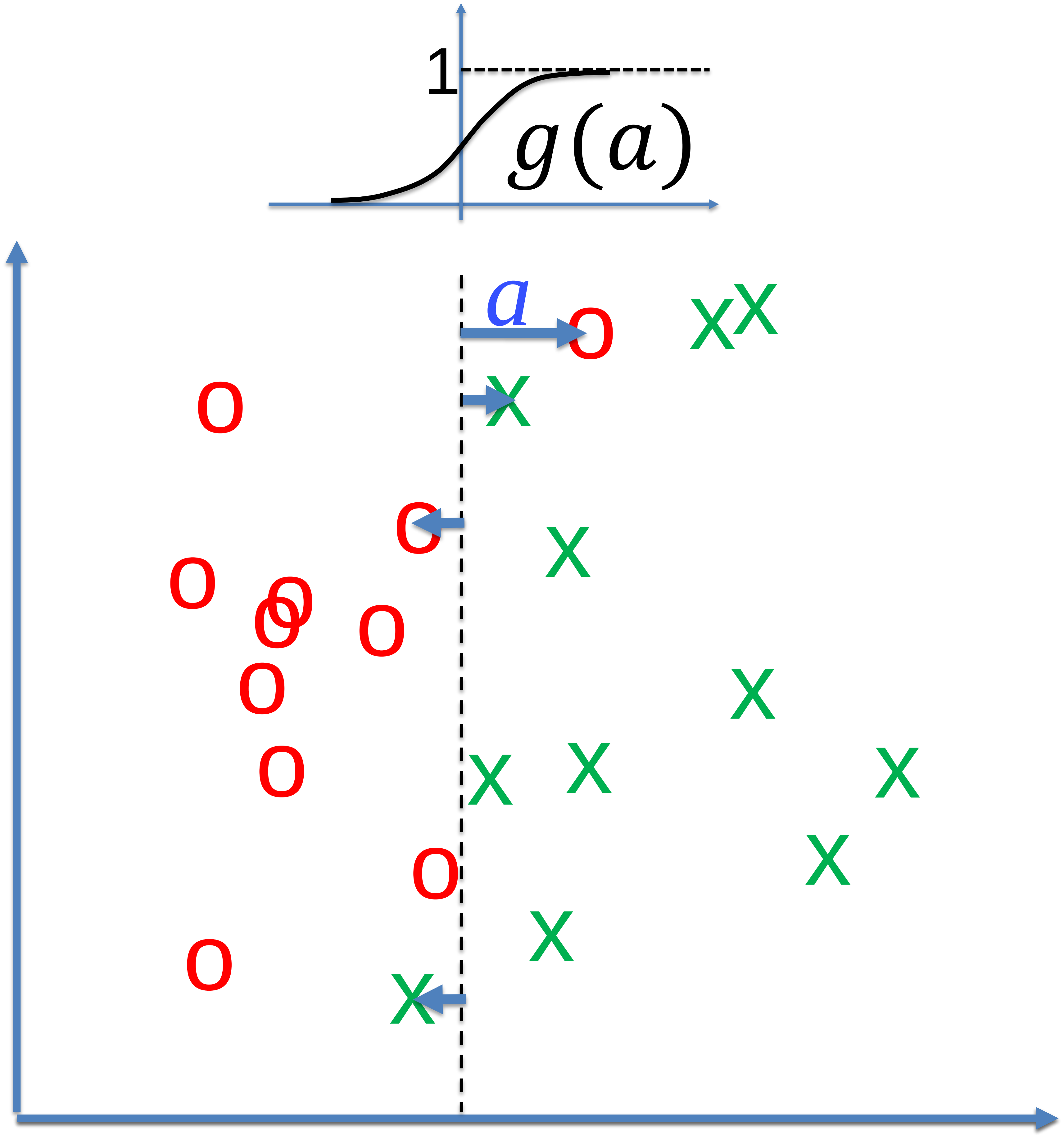
$$\hat{y}_k = 0.5[1 + tanh(\sum_j w_j x_j - \vartheta)]$$

Bagged output

$$\hat{\boldsymbol{y}}_{\mathbf{bag}} = \frac{1}{K} \sum_{k=1}^{K} \hat{y}_k$$



$$\delta_k^\mu = t_k^\mu - \hat{y}_k^\mu = \sigma(a)$$



1. Bagging : Theory

Blackboard: Bagging

Claim: bagged output has smaller quadratic error than a typical individual model

bagged output $\hat{y}_{\mathbf{bag}} = \frac{1}{K}\sum_{k=1}^{K}\hat{y}_k$

1. Bagging : Result

assumption: the average delta-difference, defined as

$$\frac{1}{P}\sum_{\mu=1}^{P}[\delta_k^\mu] = d$$

is the same for all K copies of the model.

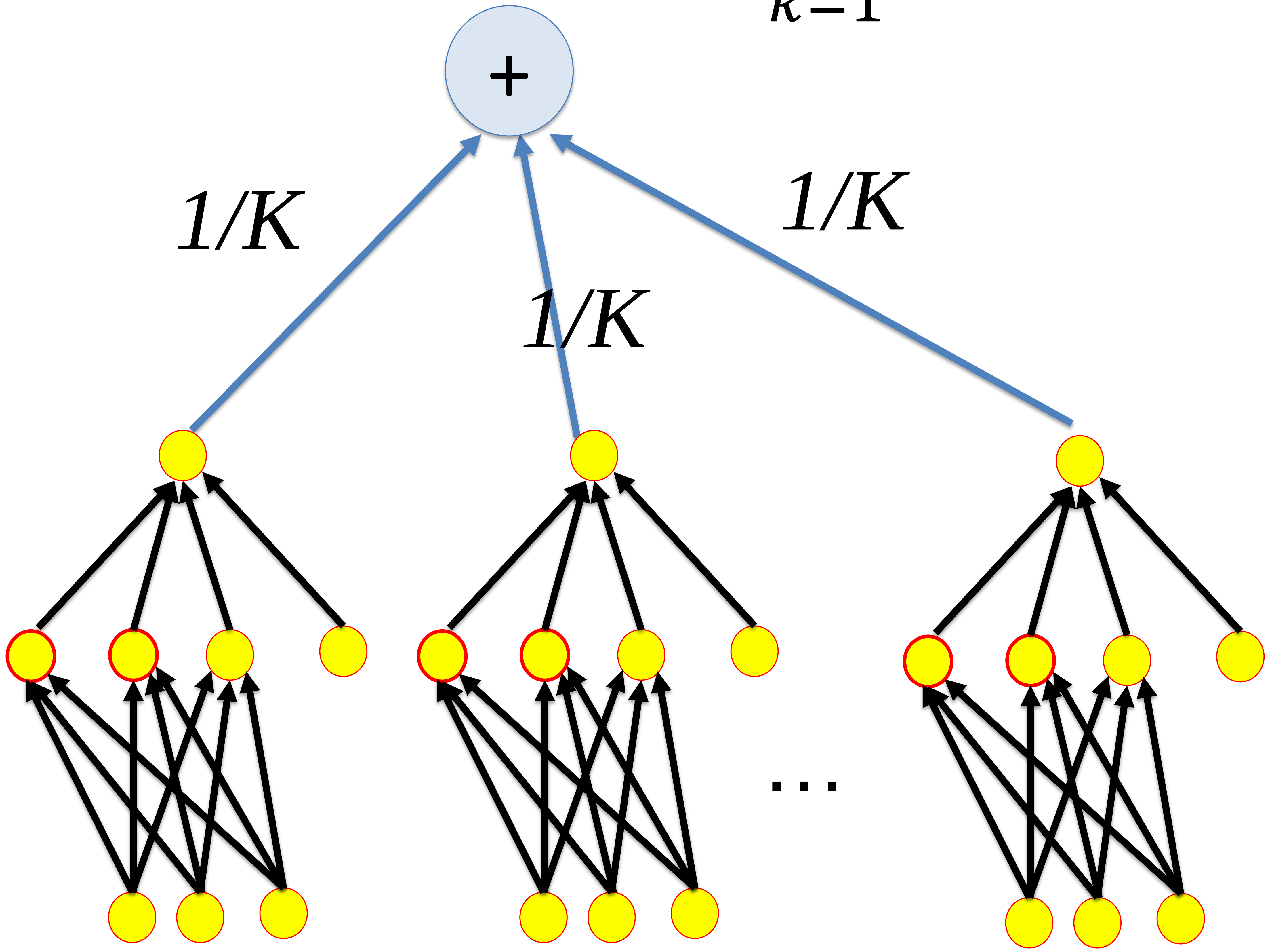
THEN

- bagged output has smaller quadratic error than a typical individual model

- if all K individual models are uncorrelated, the gain in performance scales as 1/K

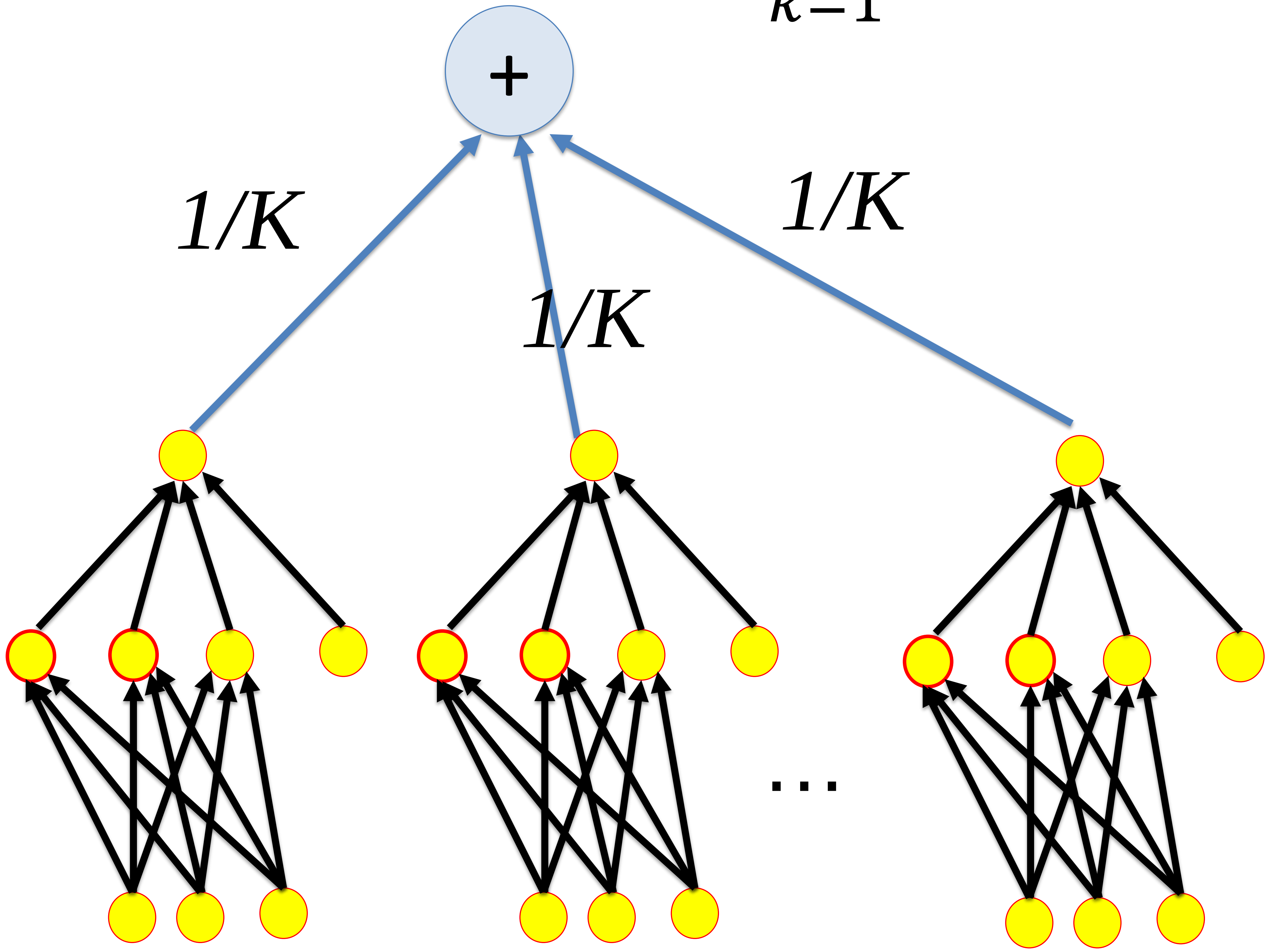
1. Bagging: each of the models can be a deep network

$$\hat{y} = \frac{1}{K}\sum_{k=1}^{K}\widehat{y_k}$$



1. Bagging: each of the models sees a different data set

$$\hat{y} = \frac{1}{K}\sum_{k=1}^{K}\widehat{y}_k$$



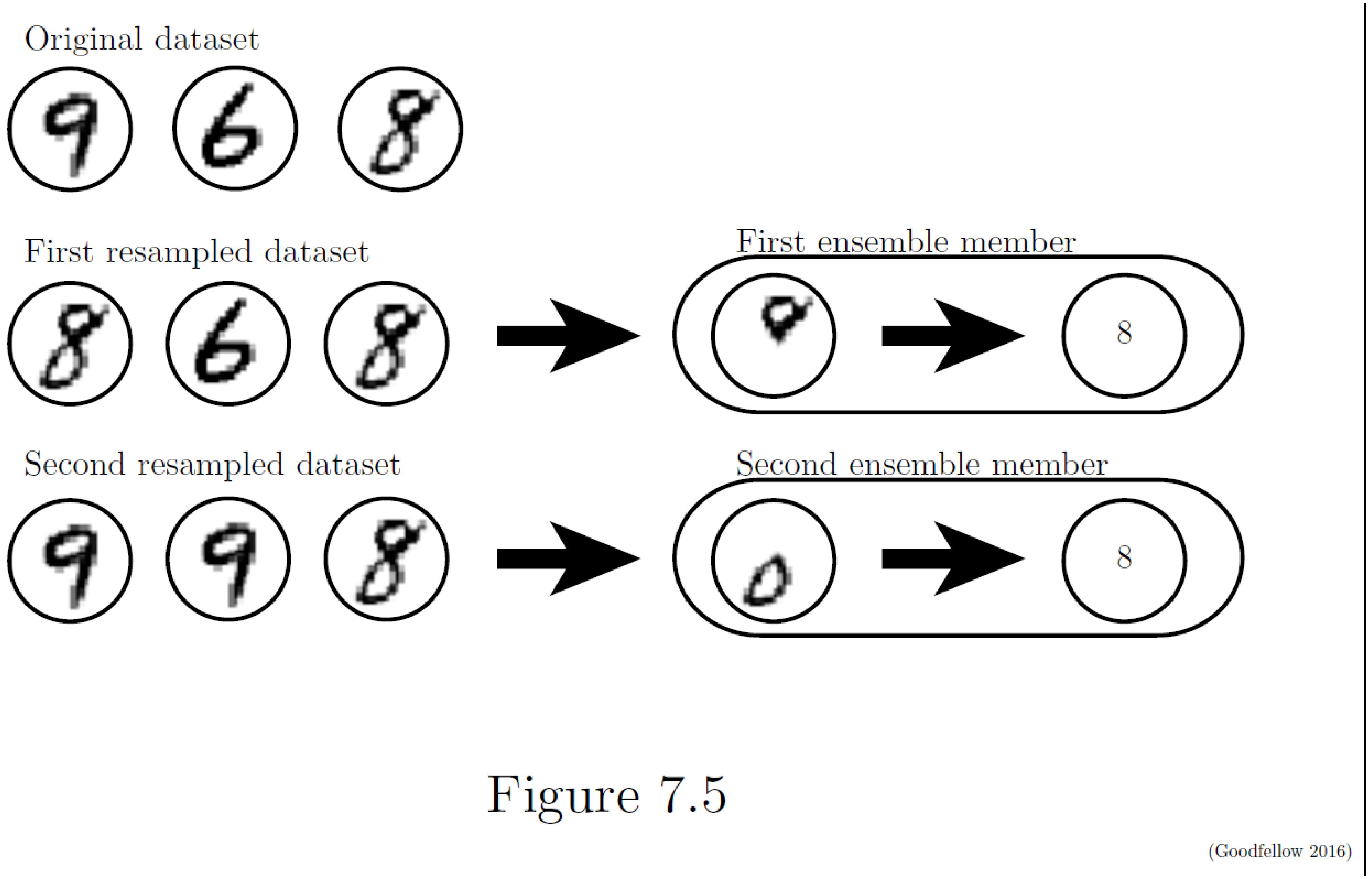


Figure 7.5

Goodfellow et al. 2016

Quiz:

[ ] If you want to win a machine learning competition,
it is better to average the prediction on new data
over ten different models, rather than just using the model
that is best on your validation data.

[ ] If you want to win a machine learning competition,
it is better to hand in 10 contributions (using different author names)
rather than a single contribution

Artificial Neural Networks: Lecture 4

Tricks of the Trade in deep networks

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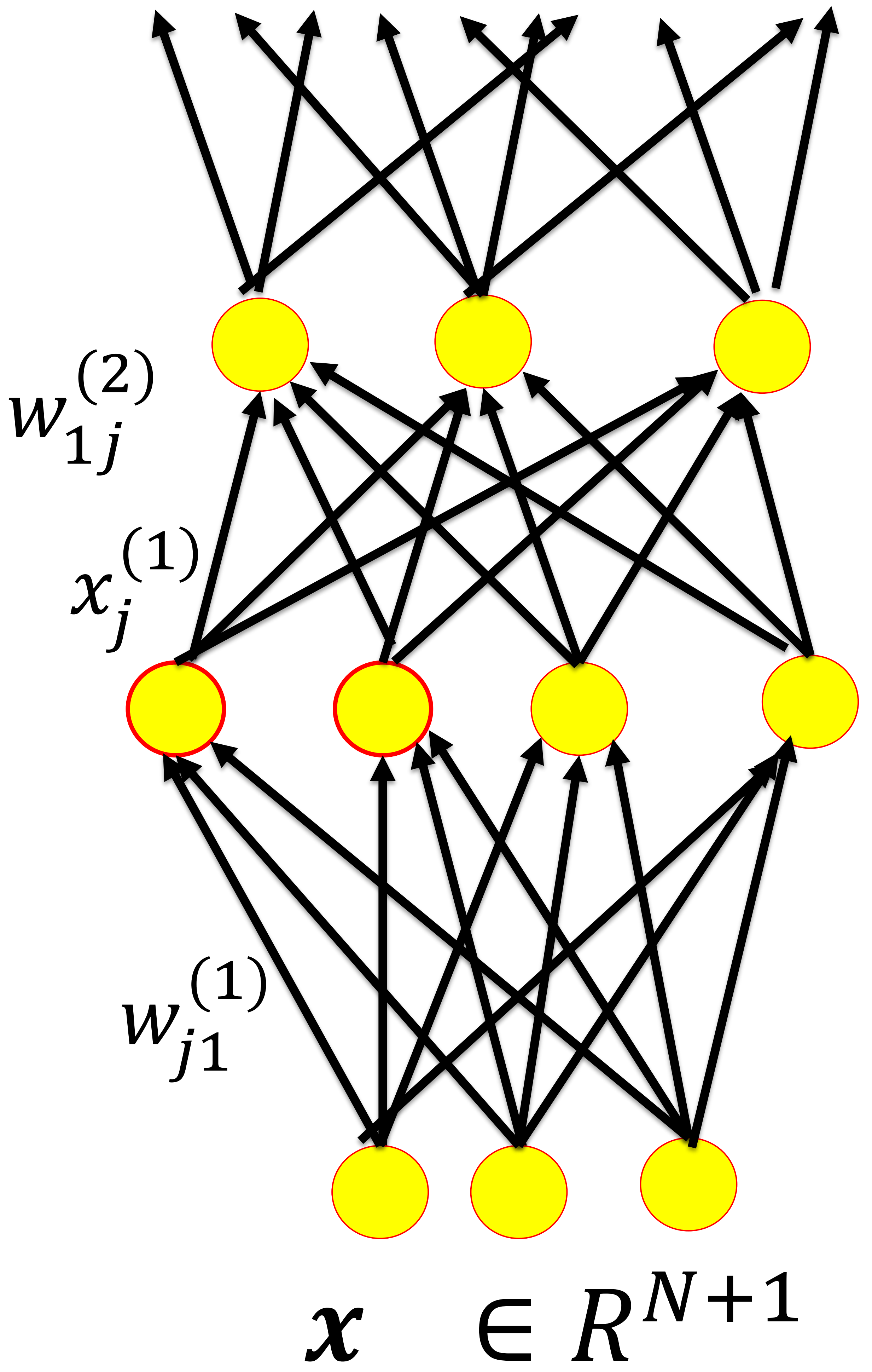
1. Bagging
2. Dropout

2. Dropout: suppress 50 percent of hidden units during training



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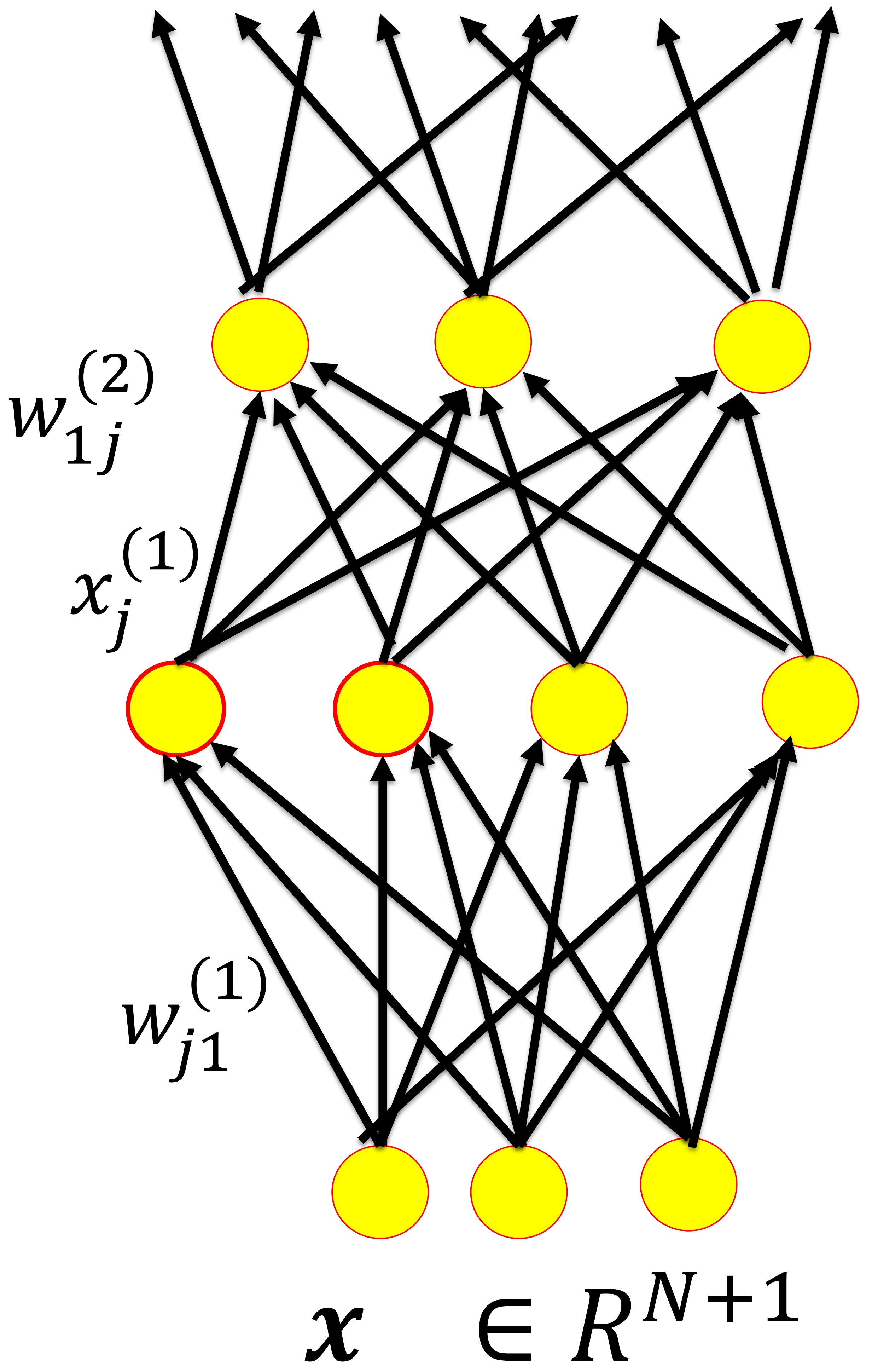

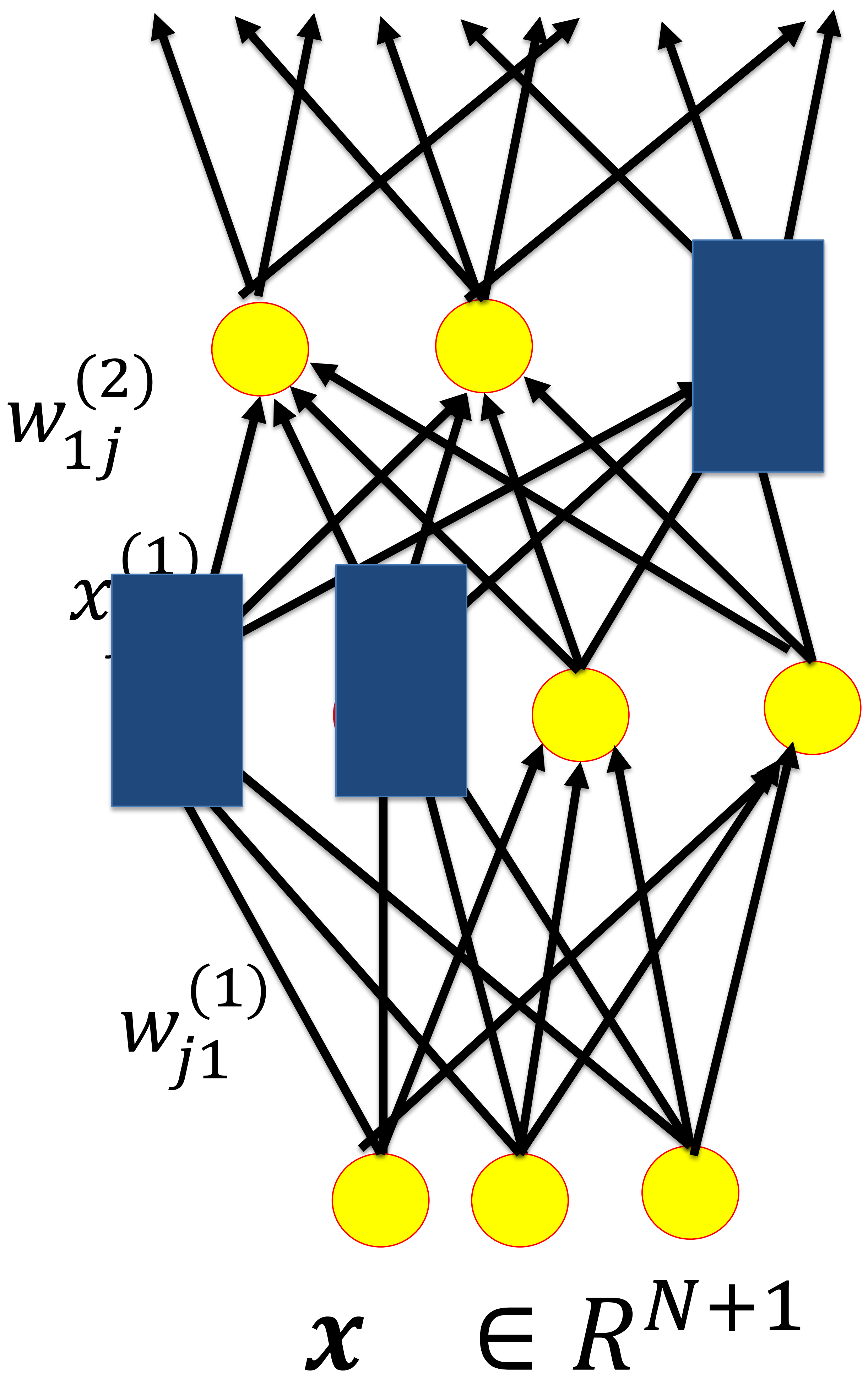

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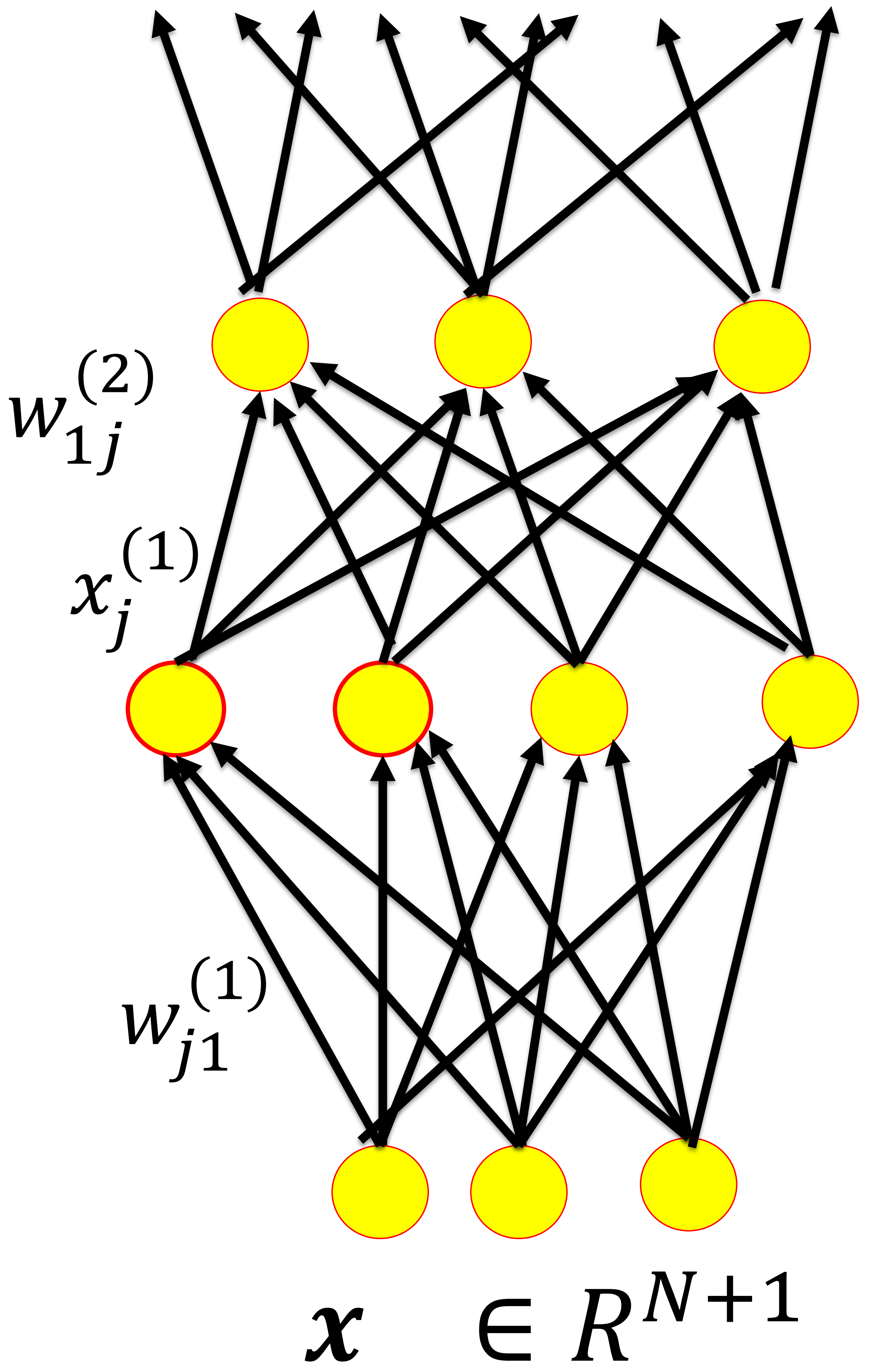

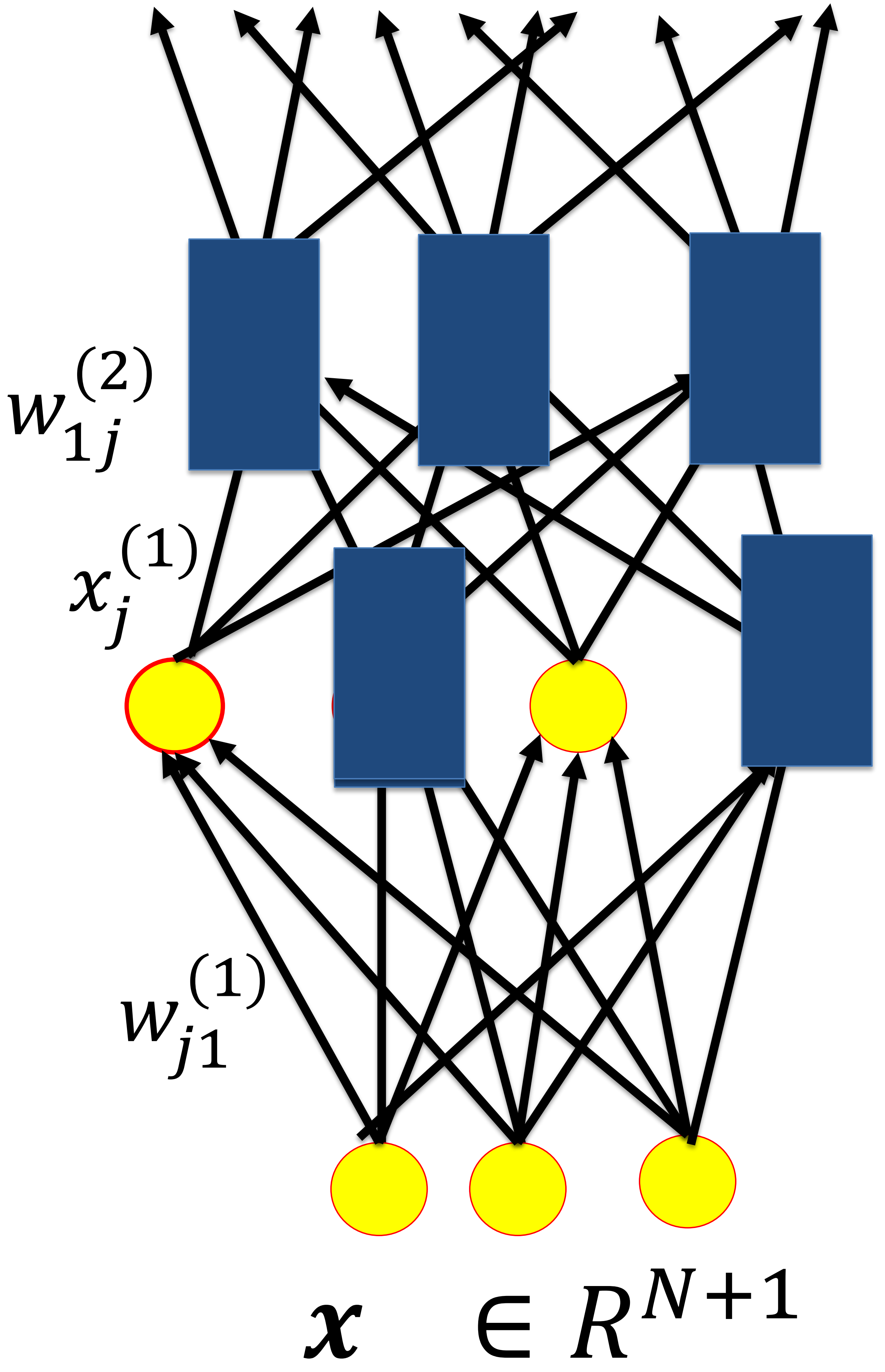

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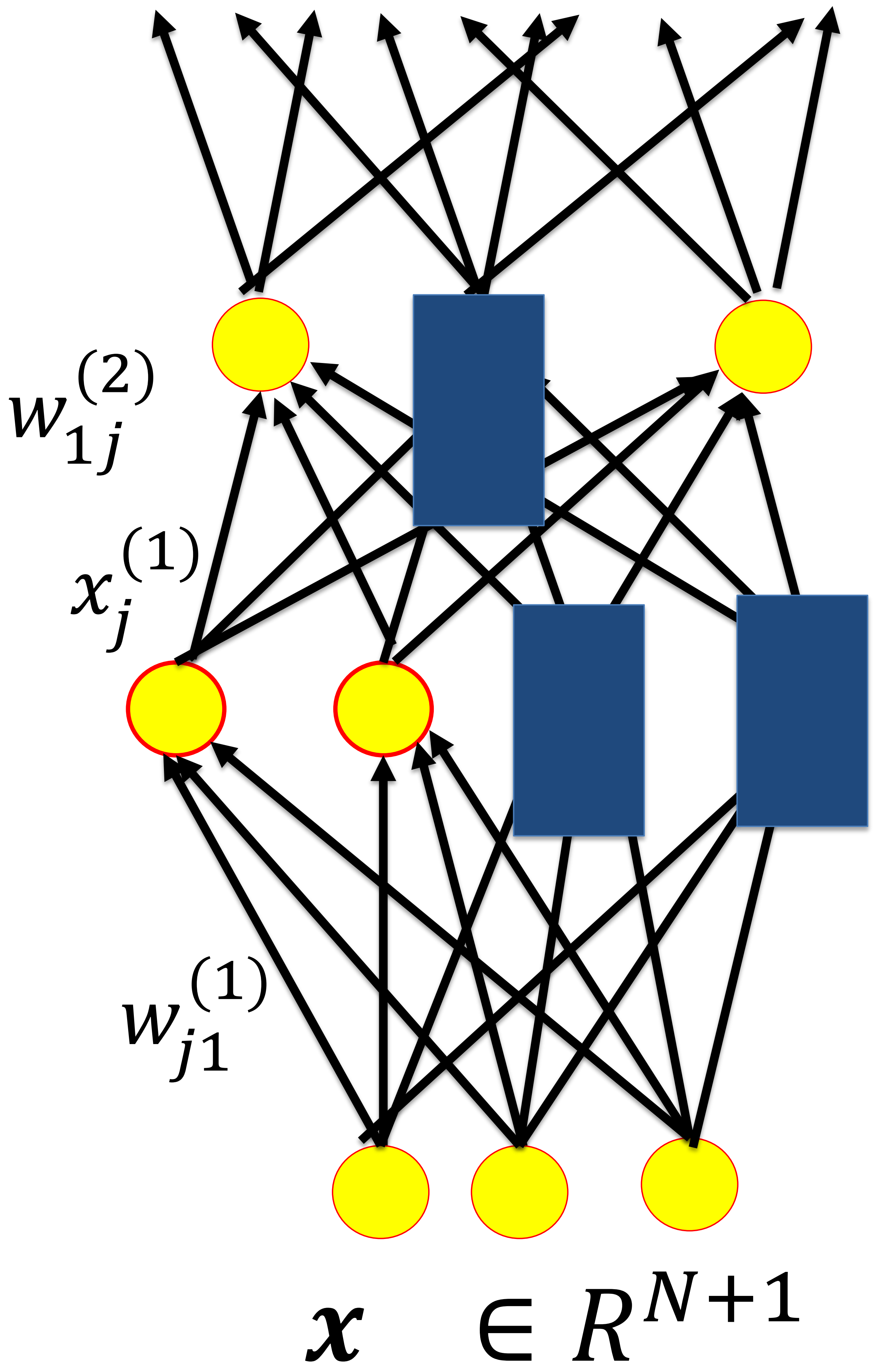
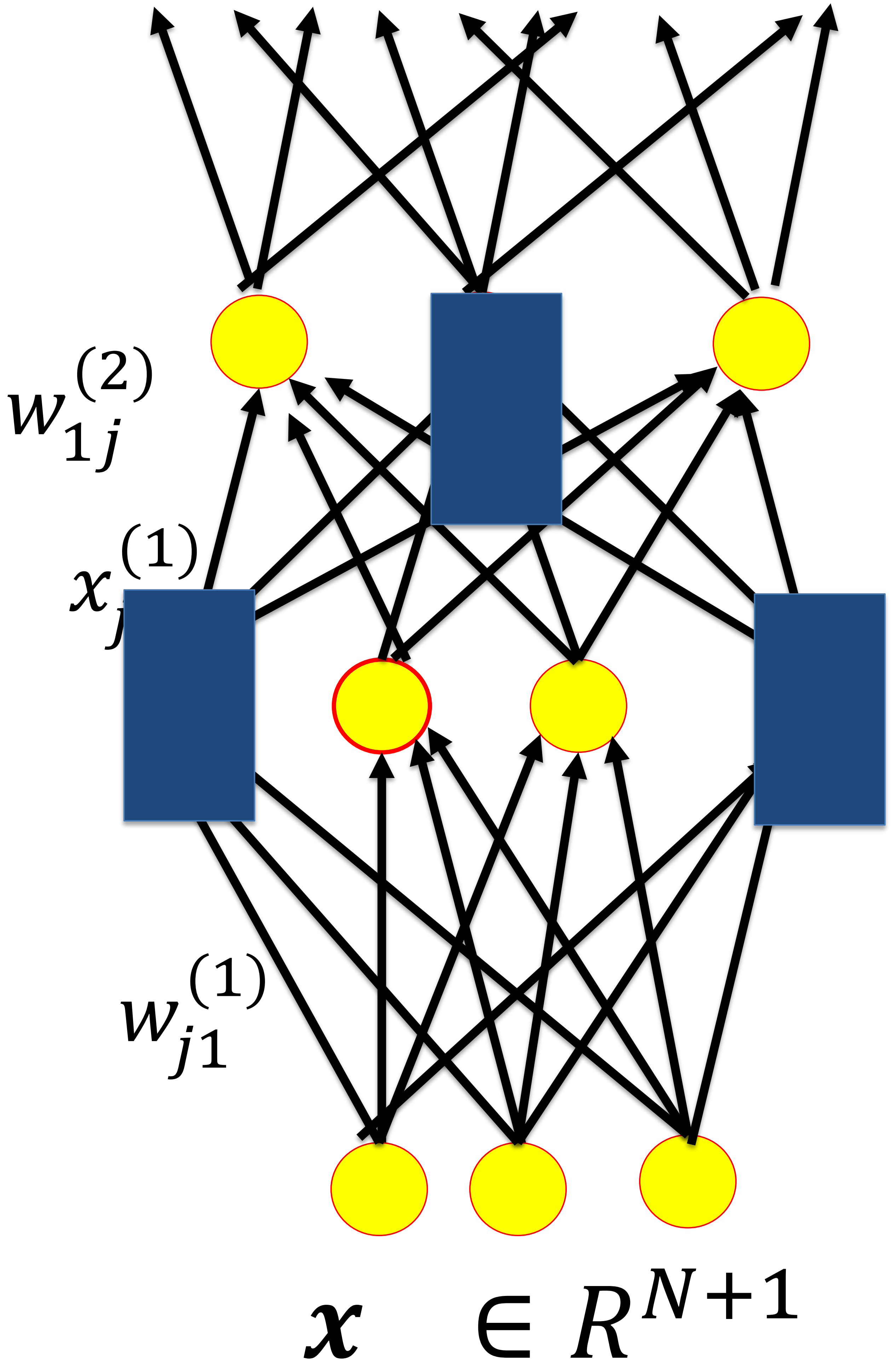
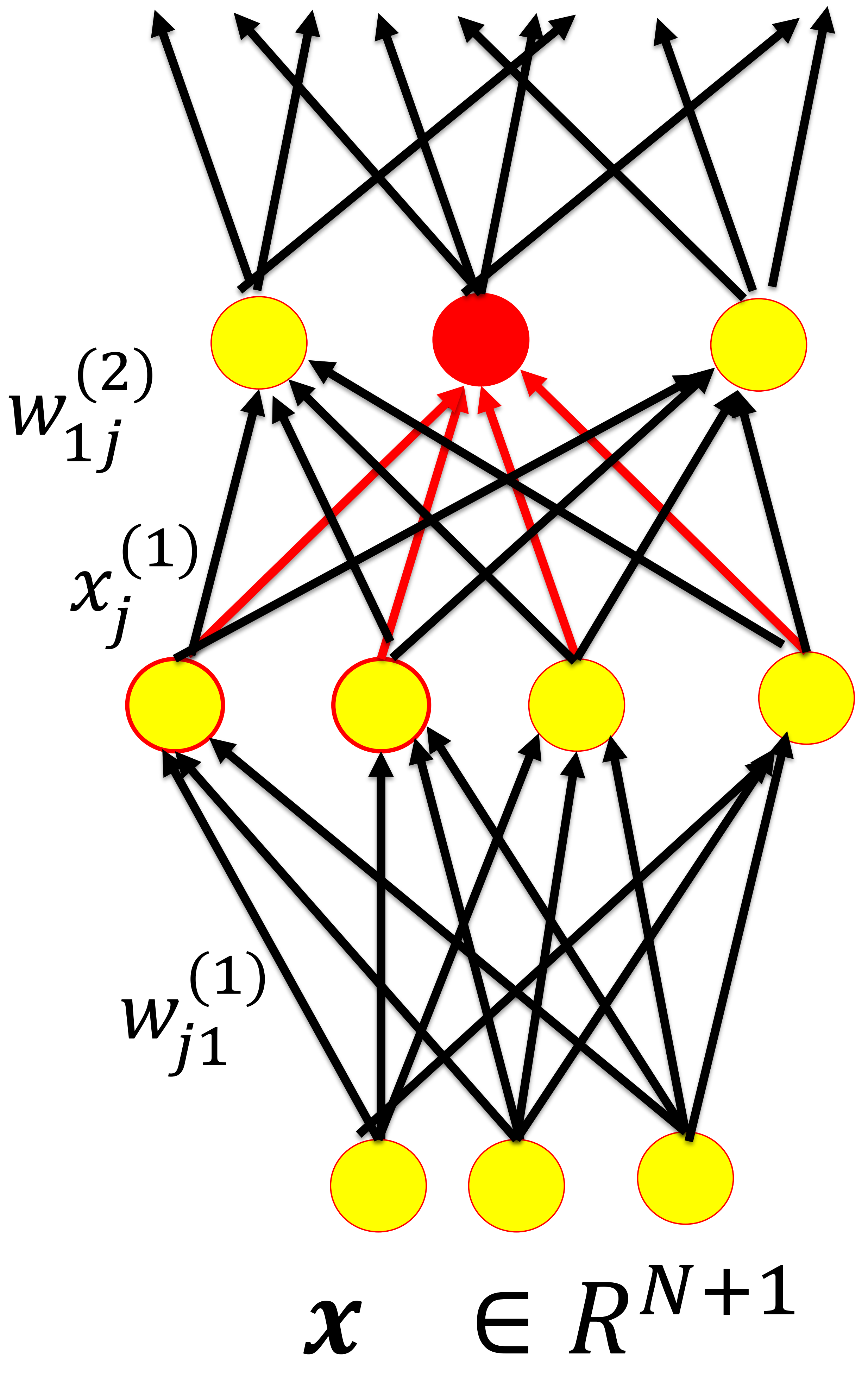




2. Dropout: use full network for validation and test



For test:

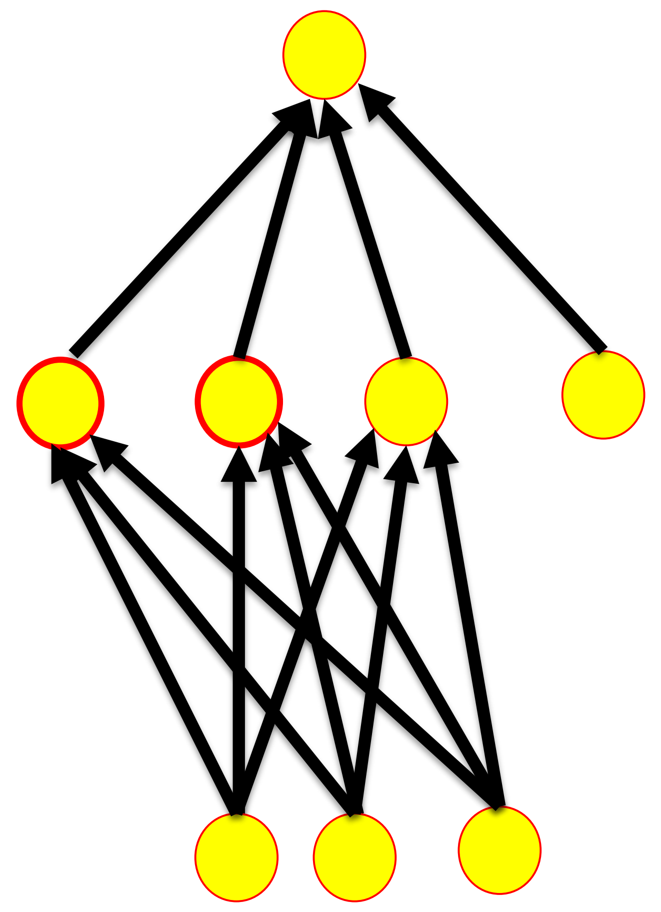
- full network
- but multiply output weights from hidden units by 1/2

→ Total input to each unit is roughly same as during training

2. Dropout: two different interpretations

1. An approximate, but practical implementation of bagging

2. A tool to enforce representation sharing in the hidden neurons



2. Dropout as approximate bagging

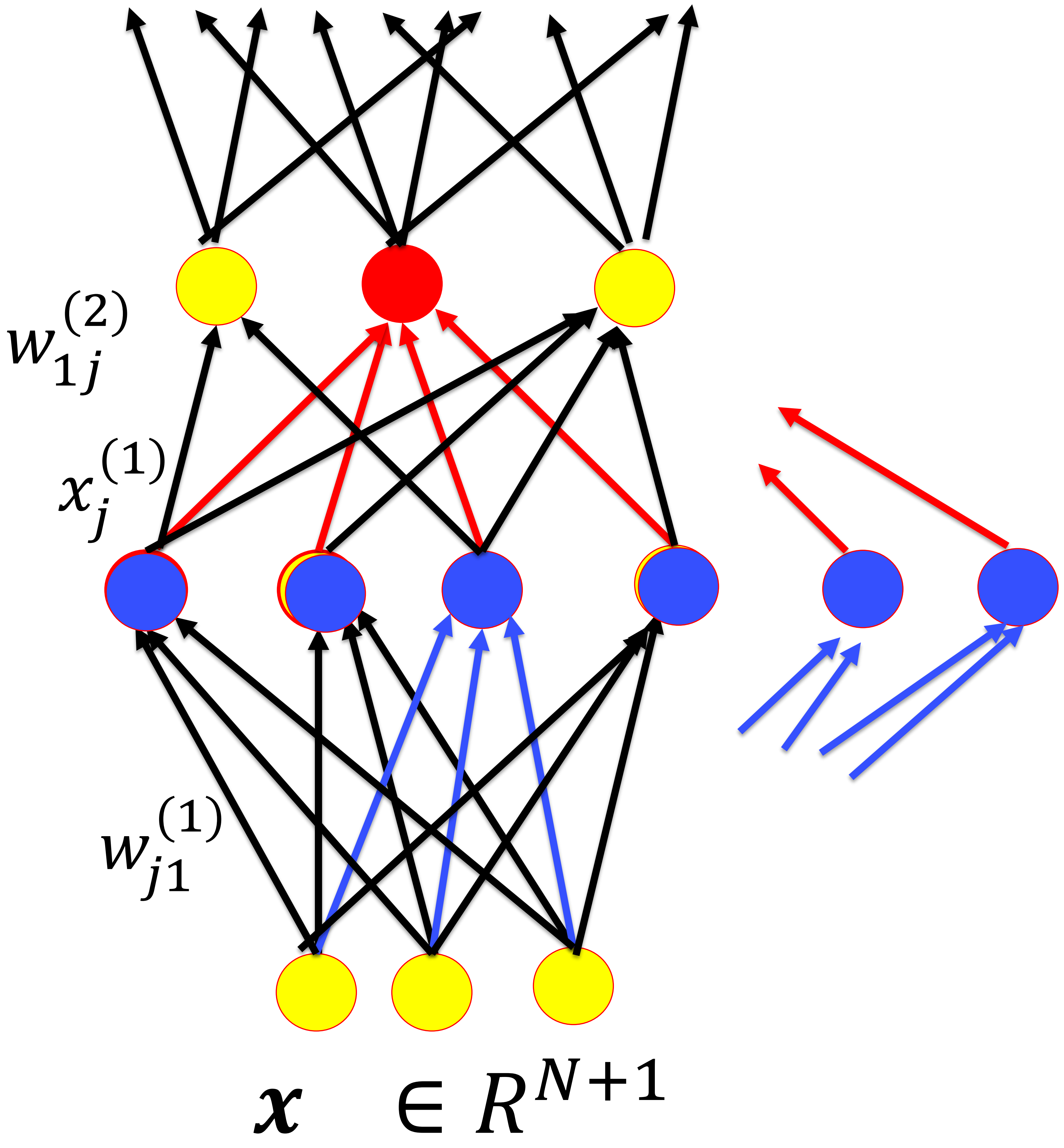
Dropout can be seen as a practical application of the ideas of bagging to deep networks

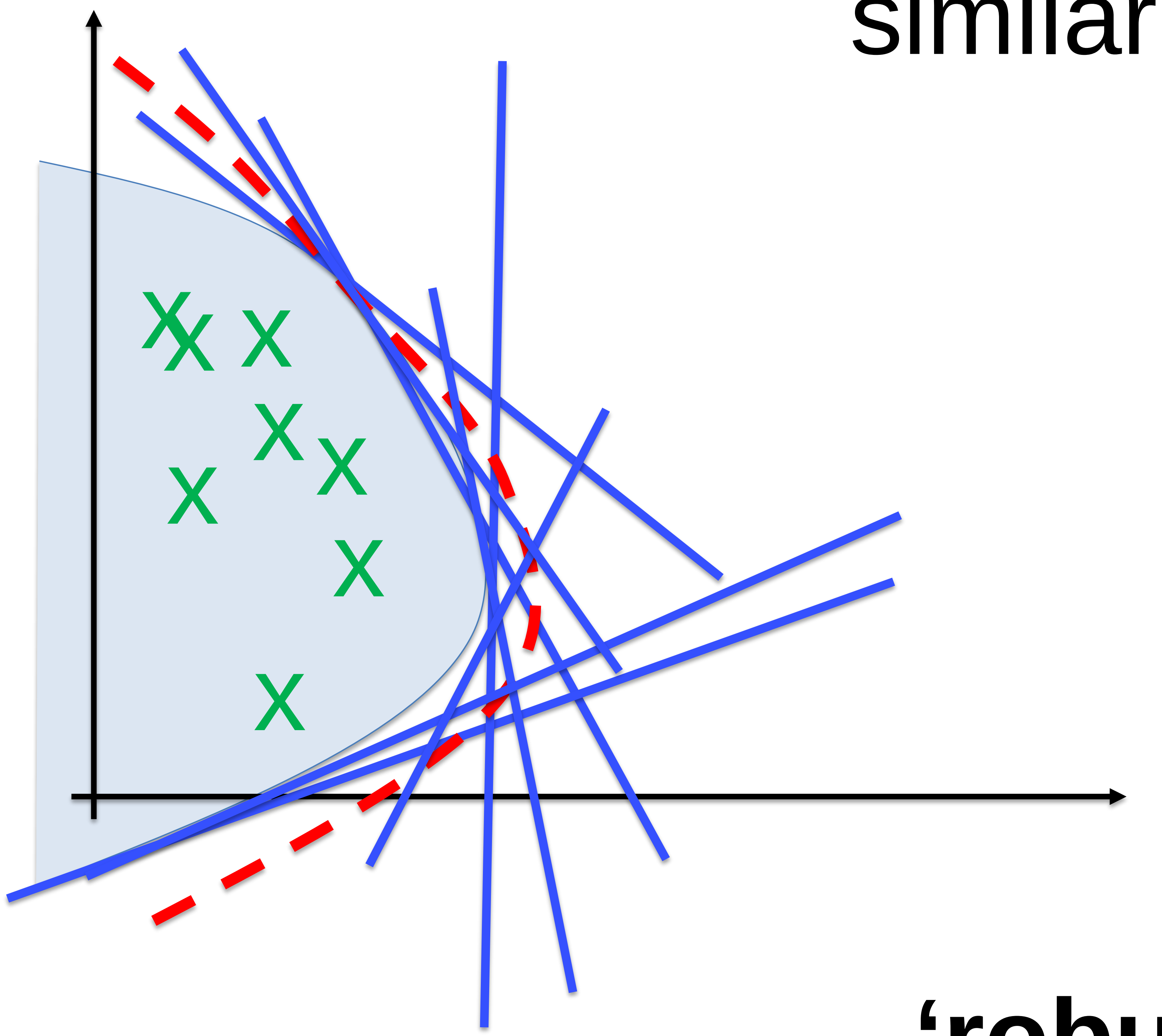
Differences to standard bagging:

- not a fixed data base for each ‘dropout’ configuration
- models not independent: share weights
- output not a ‘sum over model outputs’

2. Dropout as forced feature sharing

Feature sharing:
Take 2 times as many neurons,
But make sure they all solve
similar tasks



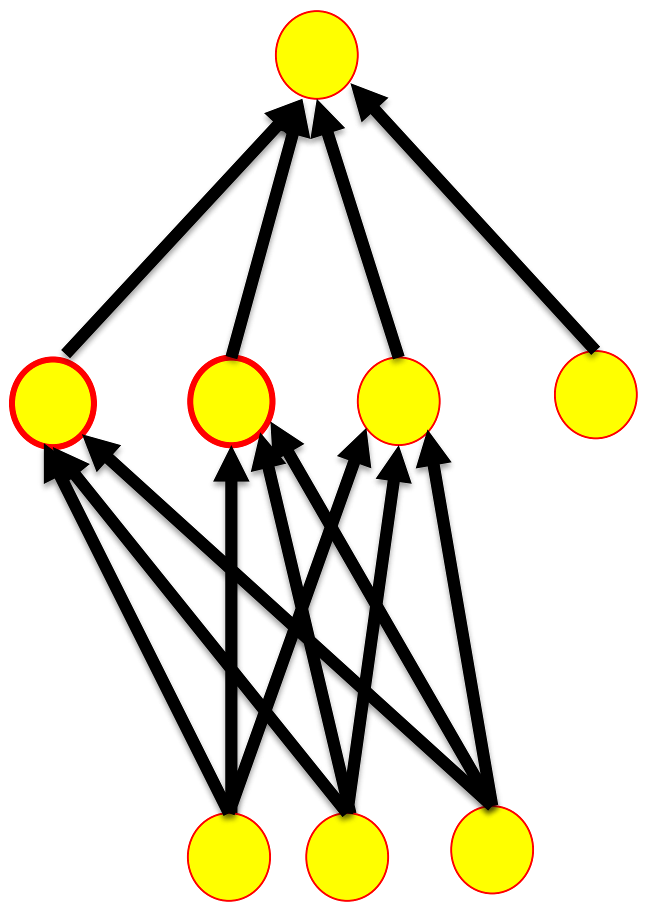


'robust'

2. Dropout: two different interpretations

1. An approximate, but practical
   implementation of bagging

2. A tool to enforce representation sharing
   in the hidden neurons



→ useful regularization method,
→ simple to implement

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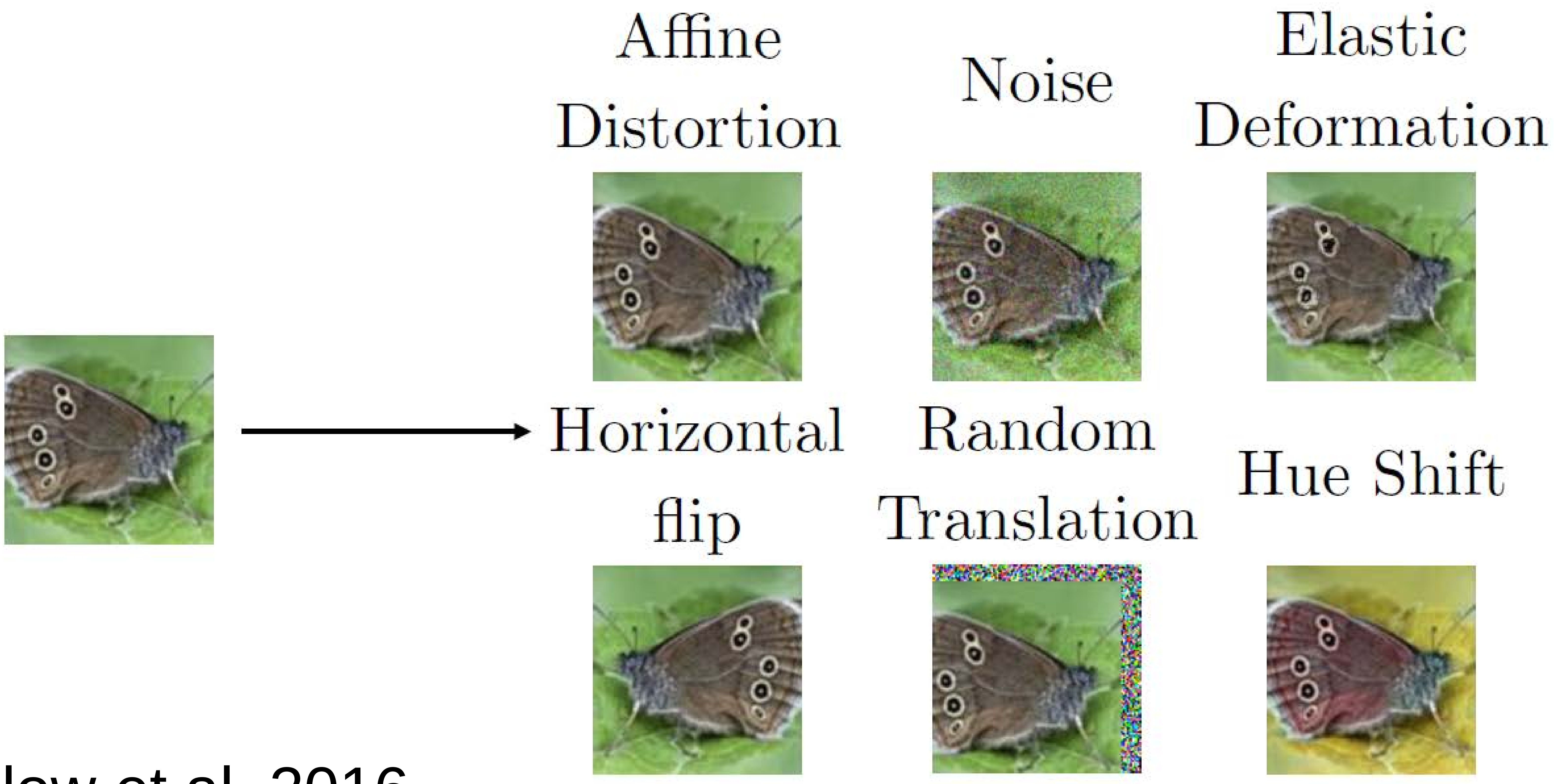
Artificial Neural Networks: Lecture 4

Tricks of the Trade in deep networks

1. Bagging
2. Dropout
3. Other simple regularization methods

3. Other easy regularization methods: dataset augmentation

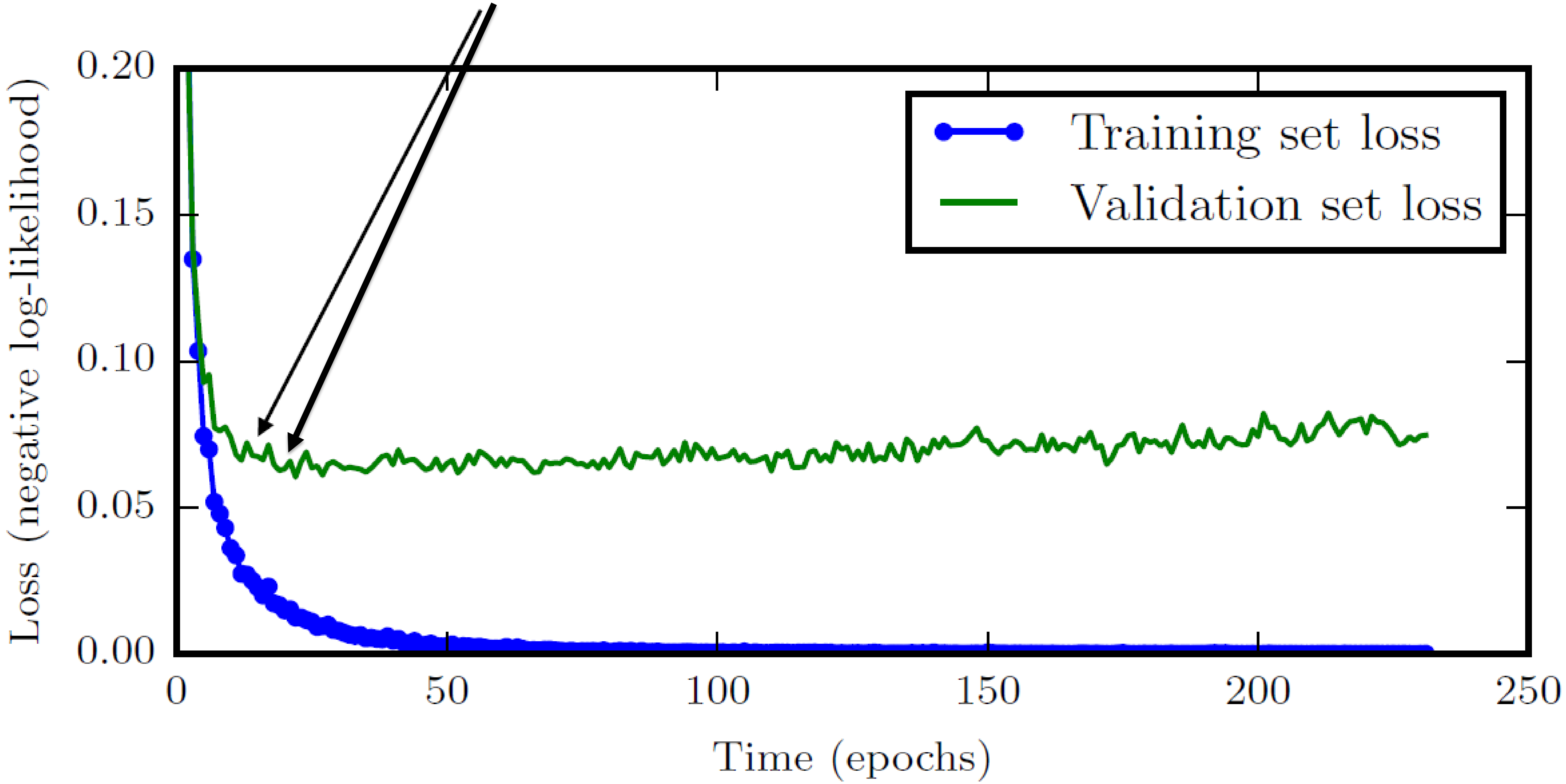
Dataset Augmentation



Goodfellow et al. 2016

3. Other easy regularization methods: early stopping

Go back to **weights** where validation error was minimal



Example: MNIST data base, see Goodfellow et al. 2016

Artificial Neural Networks: Lecture 4

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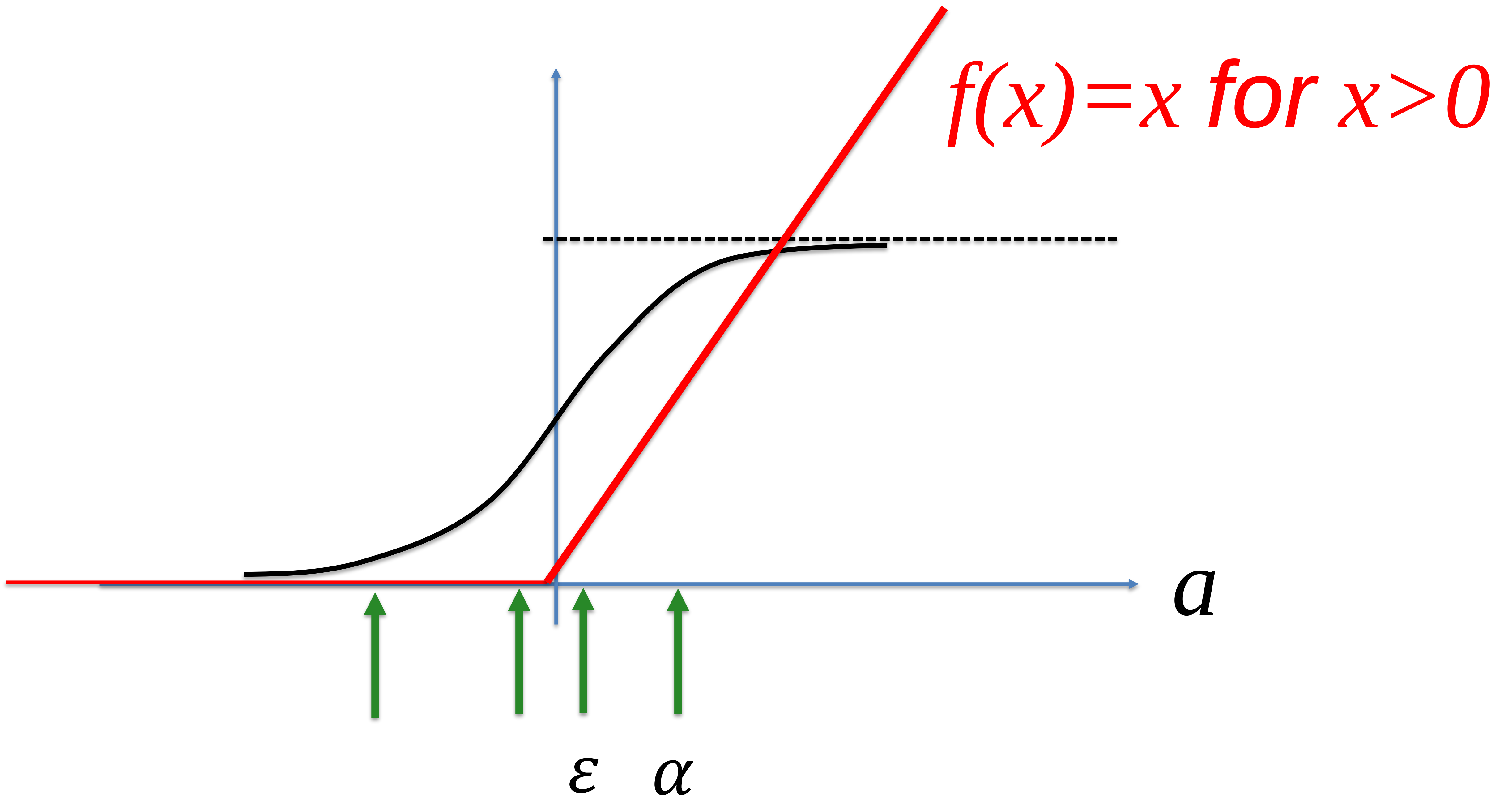
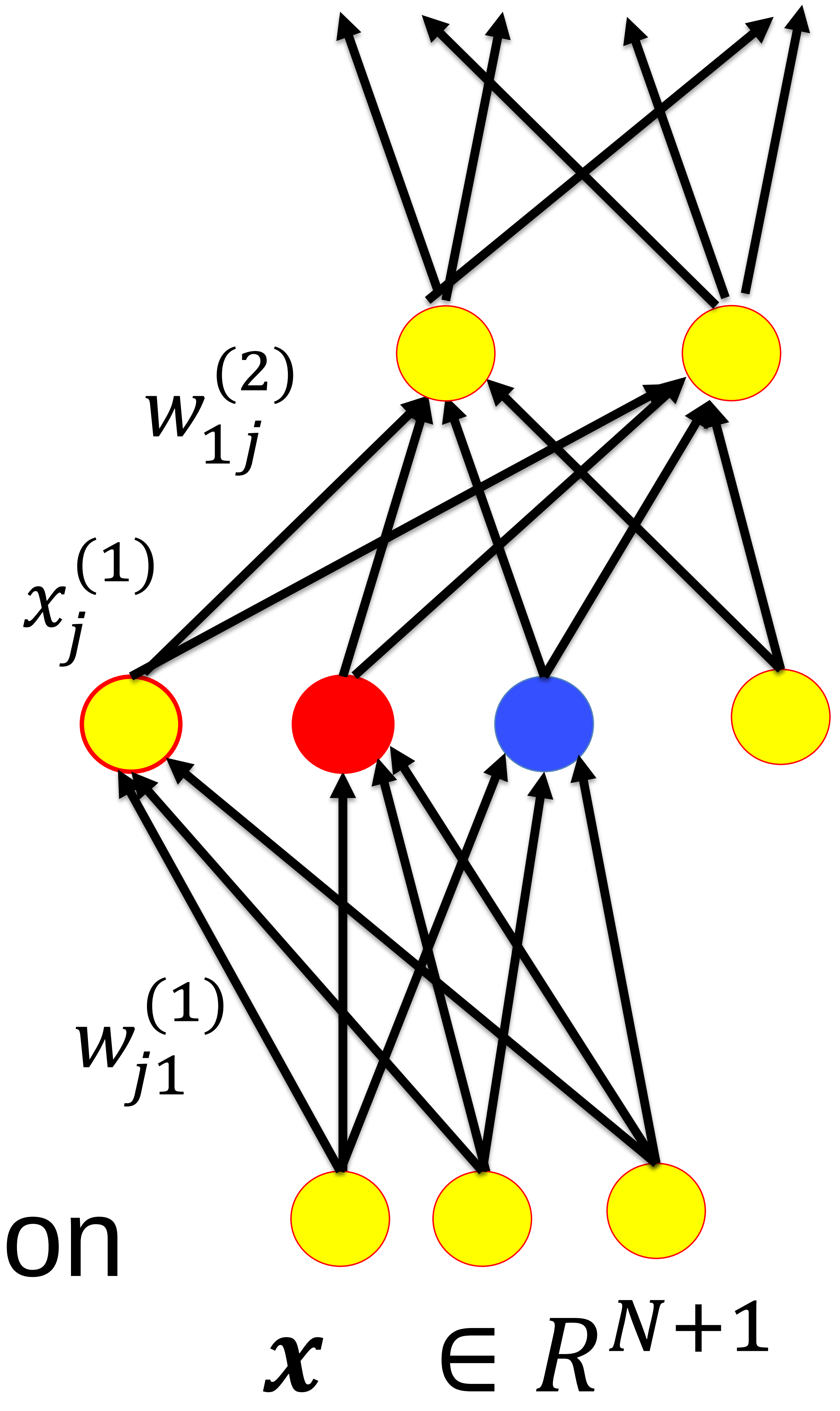
Tricks of the Trade in deep networks

1. Bagging
2. Dropout
3. Other simple regularization methods
4. Weight initialization and choice of hidden units

4. Choice of units





-different patterns give different activation of same neuron (red)

-same input pattern gives different activation of different neurons (red, blue)

4. Initialization (input layer)   Blackboard

Normalization of data base:

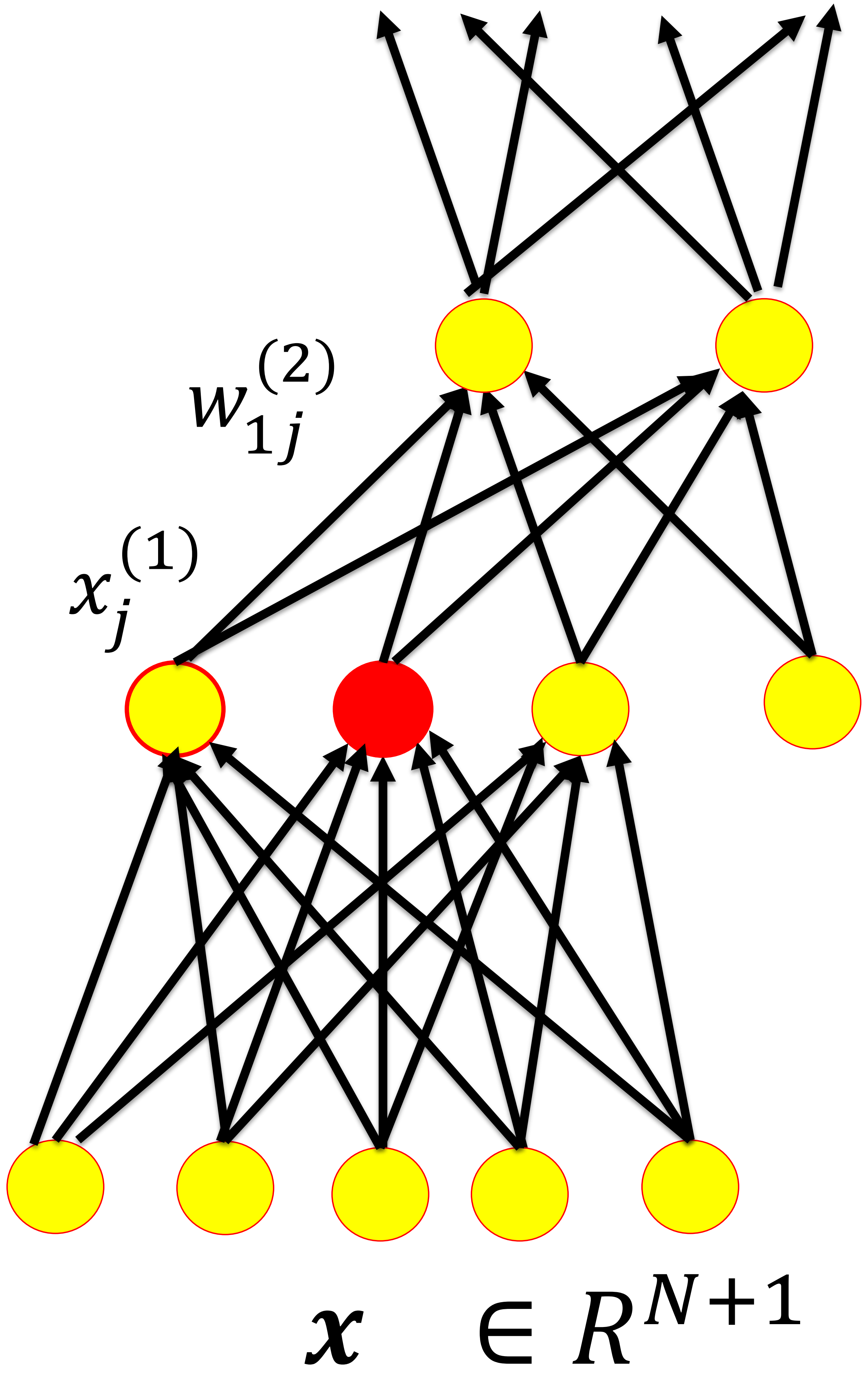
(1) $< x_j > = \frac{1}{P}\sum_{\mu=1}^{P} x_j^{\mu} = 0$

Random initialization of weights:

(2) $< w_{ij}^{(n)} > = 0$

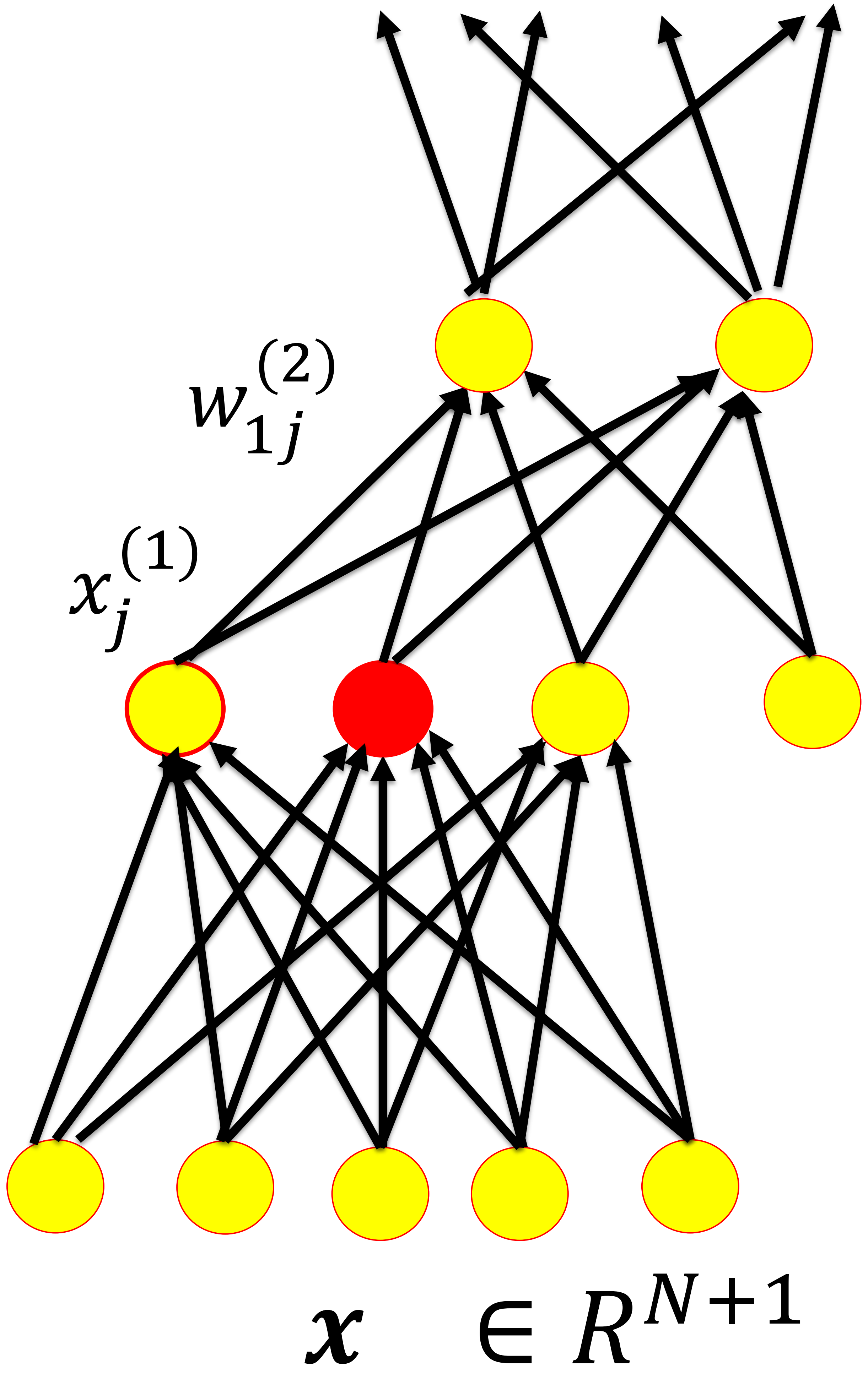
How should you choose the variance?

Claim: square root of *N* is important



Blackboard: Initialization

Claim: square root of *N* is important



4. Initialization (input layer)

Normalization of data base:

(1) $< x_j >= \frac{1}{P} \sum_{\mu=1}^{P} x_j^{\mu}$ =0

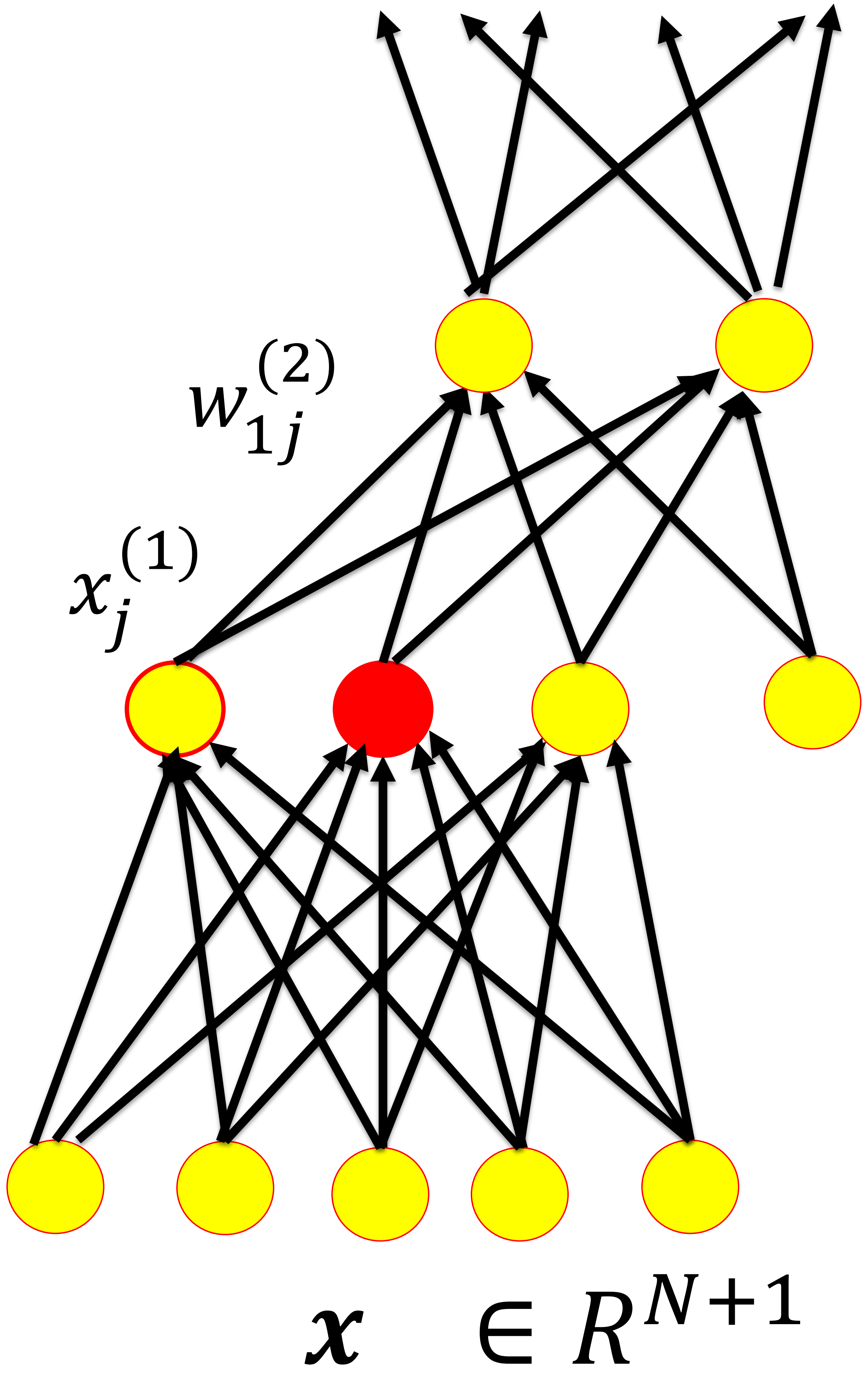
Random initialization of weights:

(2) $< w_{ij}^{(1)} > = 0$

And standard deviation propto $1/\sqrt{N}$

→ **Distribution of $x_j^{(1)}$ in layer 1**

→ **Distribution of $x_j^{(k)}$ in layer k**



BackProp

0. Initialization of weights

1. Choose pattern $\mathbf{x}^\mu$

   input $x_k^{(0)} = x_k^\mu$

2. Forward propagation of signals $x_k^{(n-1)} \longrightarrow x_j^{(n)}$

$$x_j^{(n)} = g^{(n)}(a_j^{(n)}) = g^{(n)}(\sum w_{jk}^{(n)} x_k^{(n-1)}) \tag{1}$$

   output $\hat{y}_i^\mu = x_i^{(n_{\max})}$

3. Computation of errors in output

$$\delta_i^{(n_{\max})} = g'(a_i^{(n_{\max})})\ [t_i^\mu - \hat{y}^\mu] \tag{2}$$

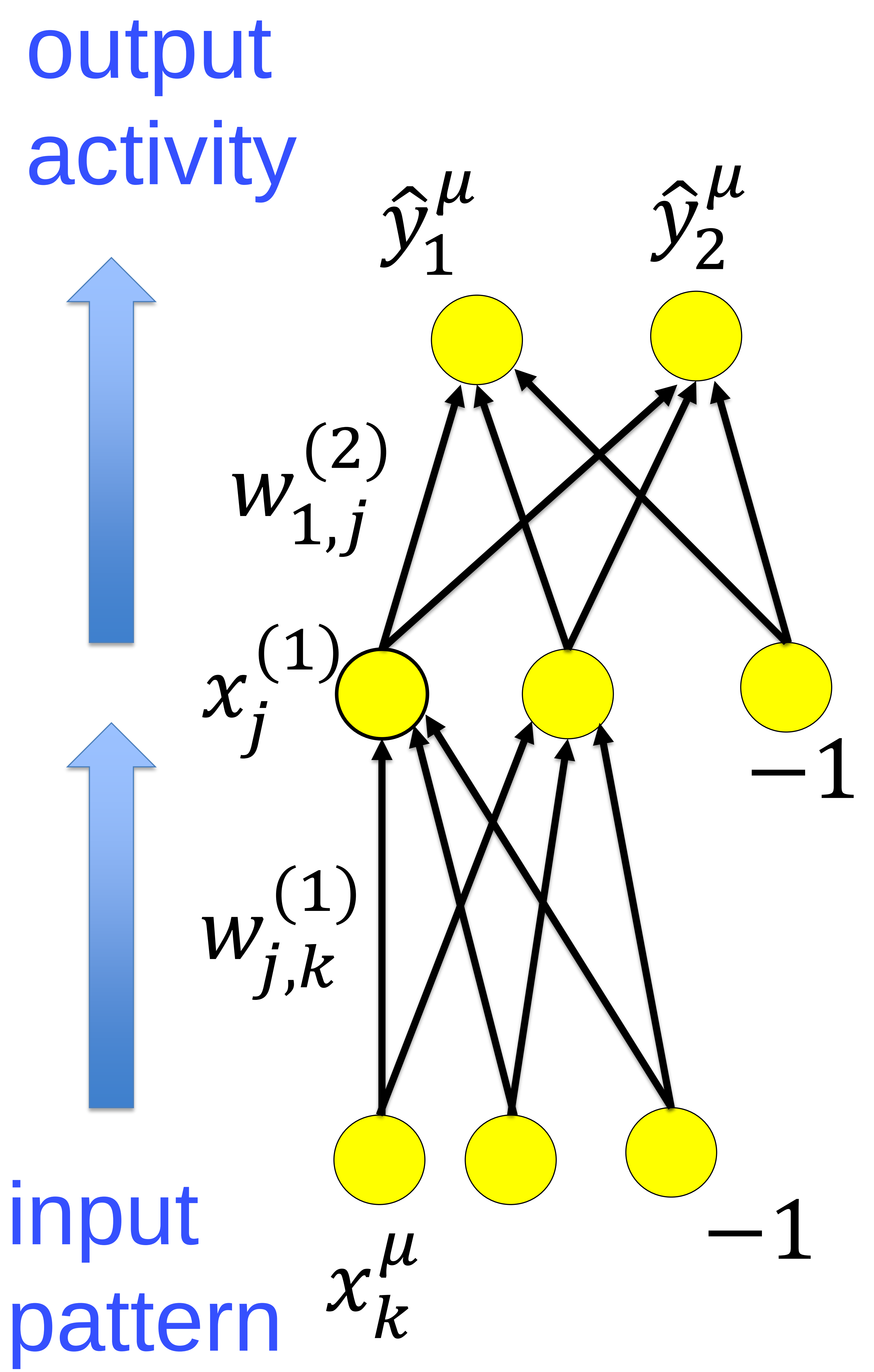
4. Backward propagation of errors $\delta_i^{(n)} \longrightarrow \delta_j^{(n-1)}$

$$\delta_j^{(n-1)} = g'^{(n-1)}(a^{(n-1)}) \sum_i w_{ij}\, \delta_i^{(n)} \tag{3}$$

5. Update weights (for each (i,j) and all layers (n))

$$\Delta w_{ij}^{(n)} = \eta\, \delta_i^{(n)}\, x_j^{(n-1)} \tag{4}$$

6. Return to step 1.



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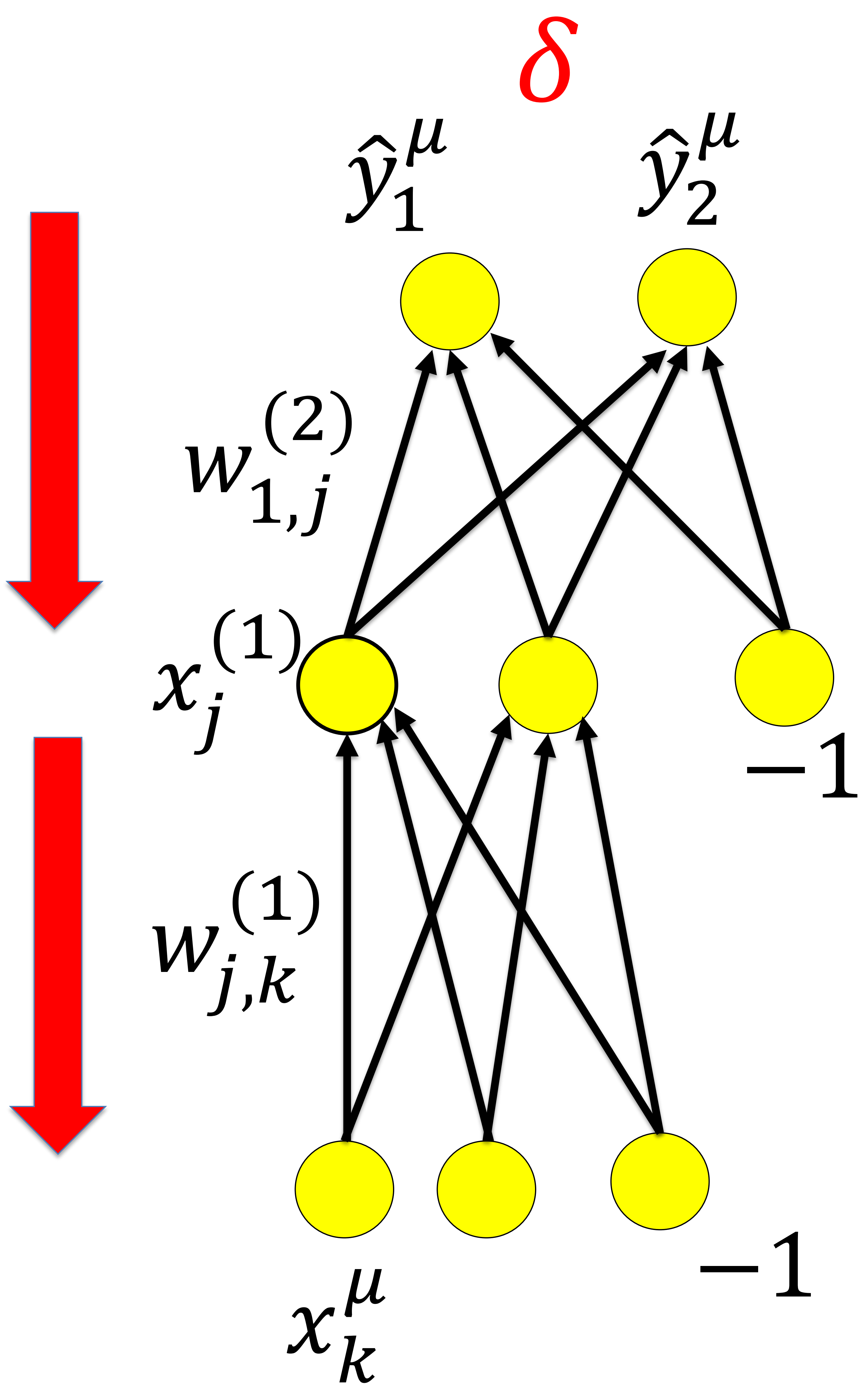
Calculate output error



BackProp

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   input $x_k^{(0)} = x_k^\mu$

2. Forward propagation of signals $x_k^{(n-1)} \longrightarrow x_j^{(n)}$

$$x_j^{(n)} = g^{(n)}(a_j^{(n)}) = g^{(n)}(\sum w_{jk}^{(n)} x_k^{(n-1)}) \tag{1}$$

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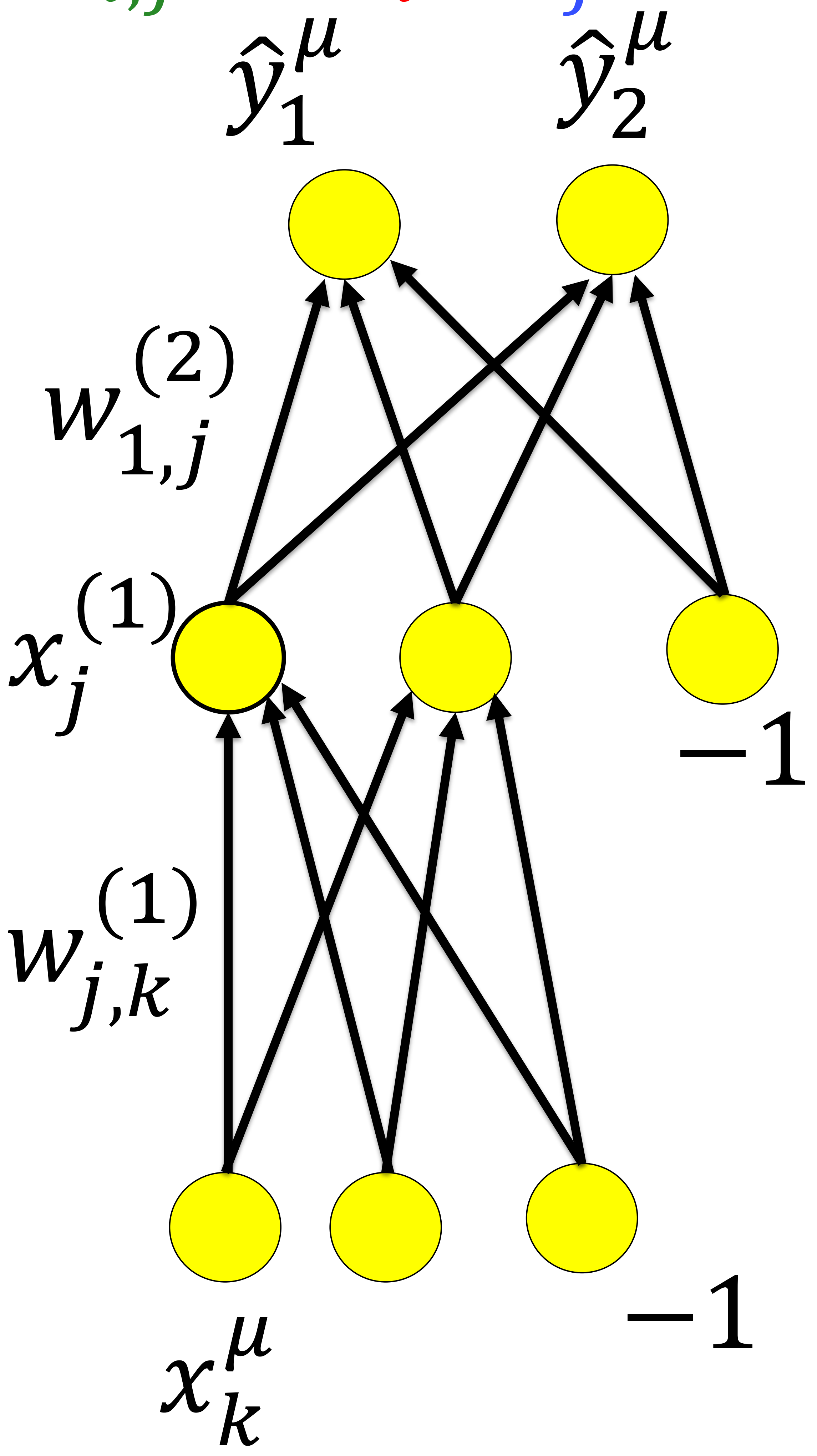
$$\Delta w_{ij}^{(n)} = \eta \, \delta_i^{(n)} \, x_j^{(n-1)} \tag{4}$$

6. Return to step 1.

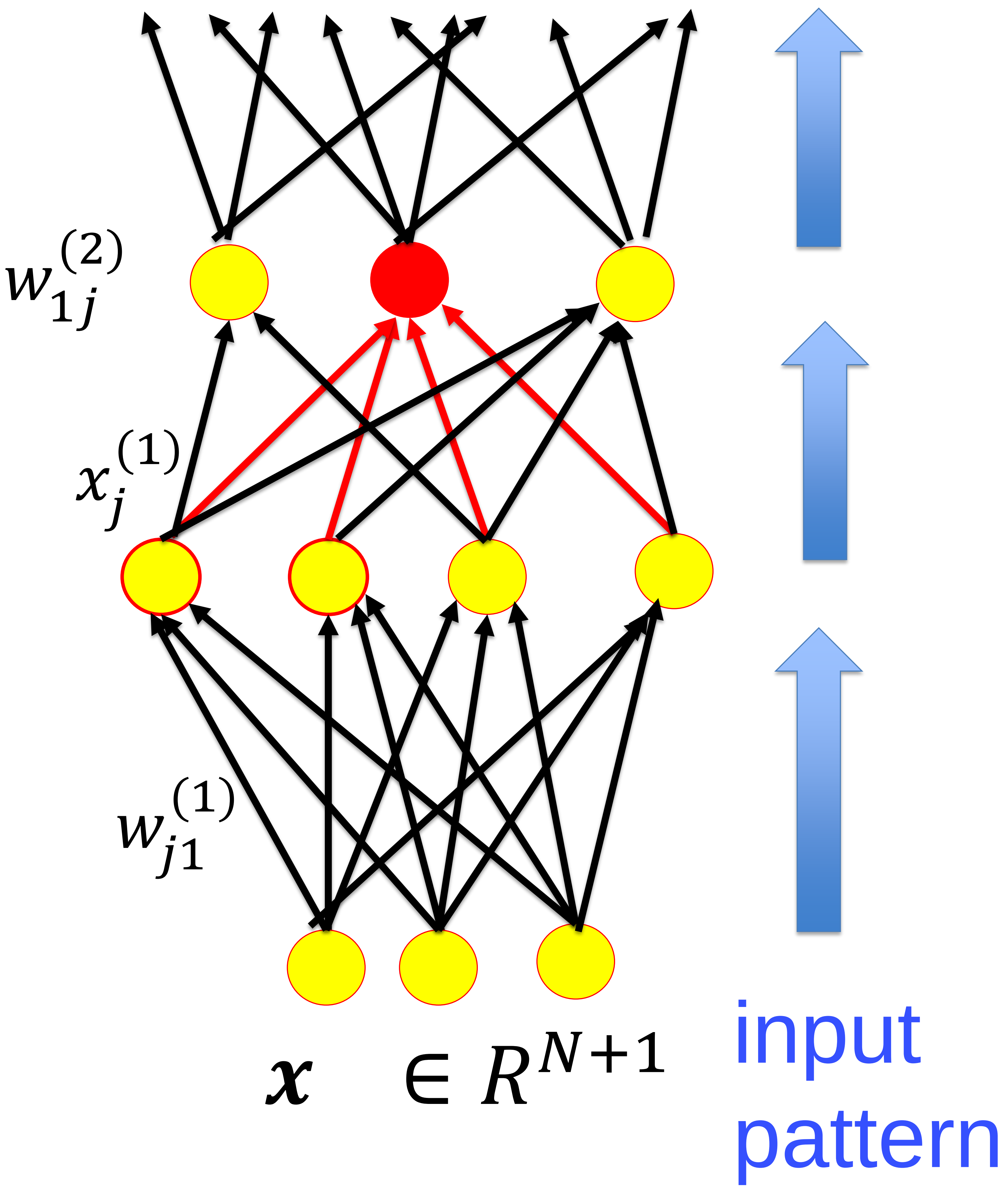
update **all** weights

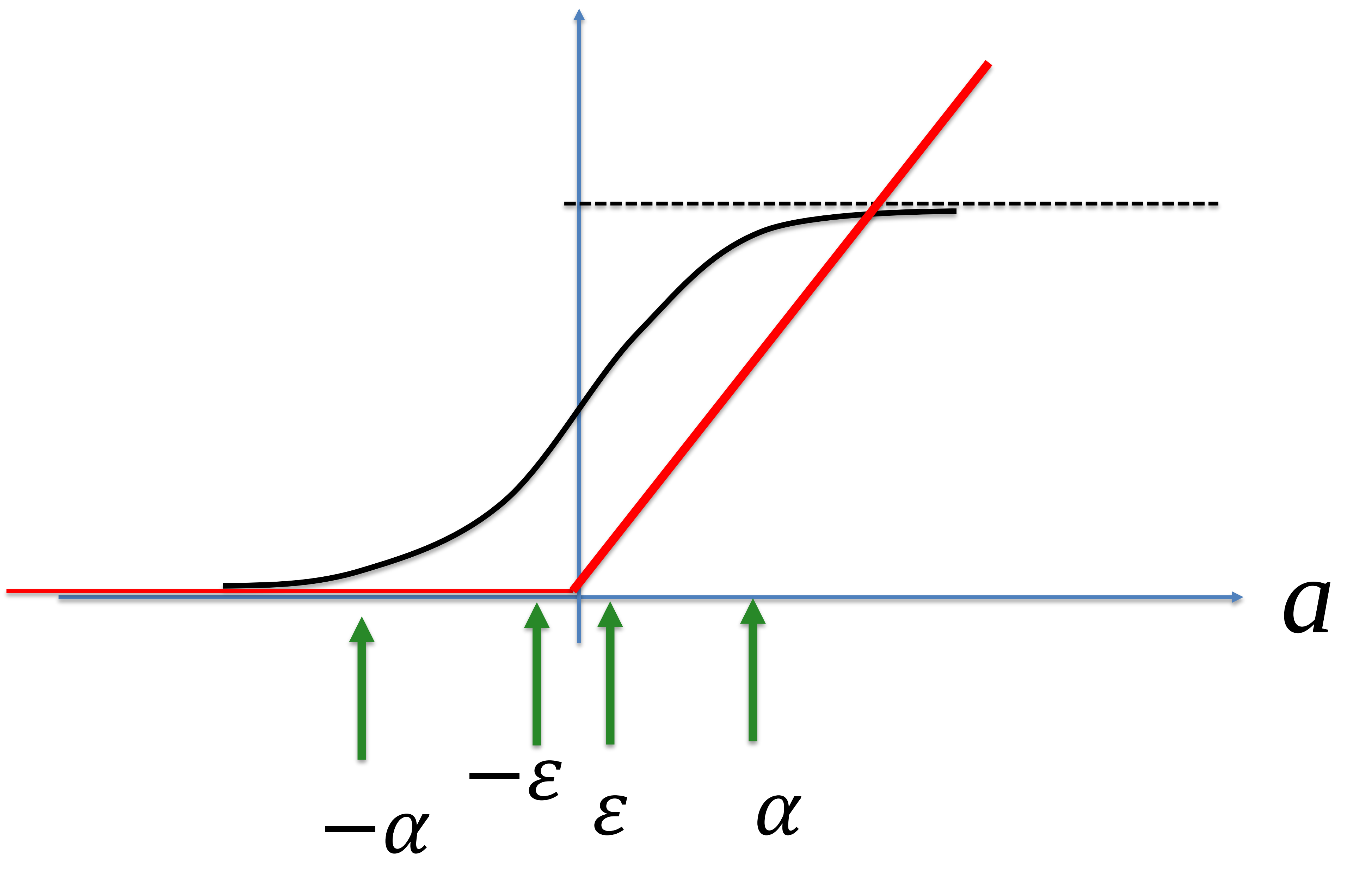
$$\Delta w_{i,j}^{(n)} = \delta_i^{(n)} x_j^{(n-1)}$$



Why does the initatialization or normalization matter in backprop?

4. Forward pass: Linear and nonlinear processing



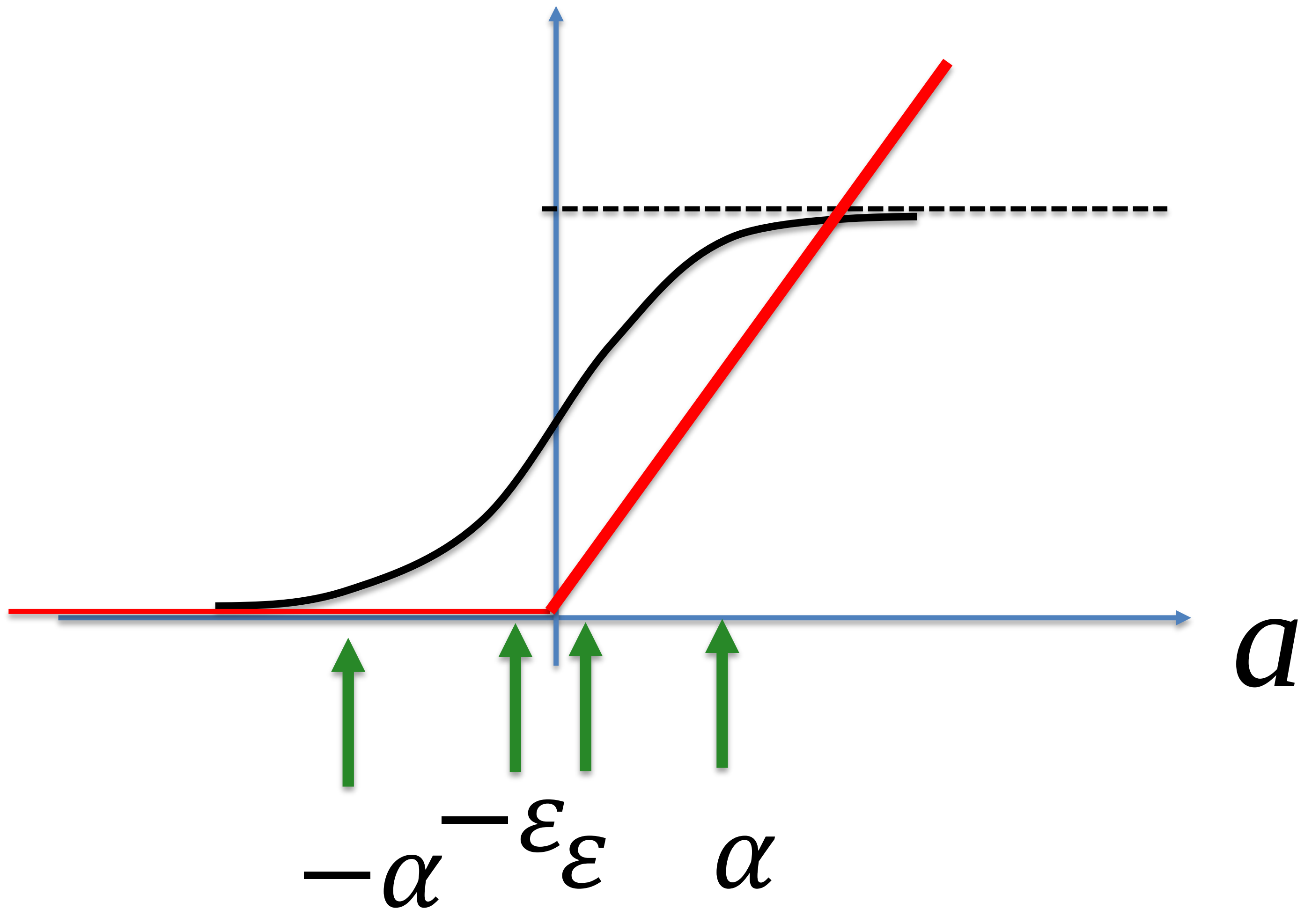


4. Forward pass: Linear and nonlinear processing

Observations:

if all patterns in all layers touch the linear regime of $g(a)$, then the whole network is linear

→ different patterns should touch different regions of $g(a)$.



- this is automatically true for ReLu, if the mean (across patterns) is $a=0$
- this is automatically true for sigmoidals, if the variance (across patterns) is > 2

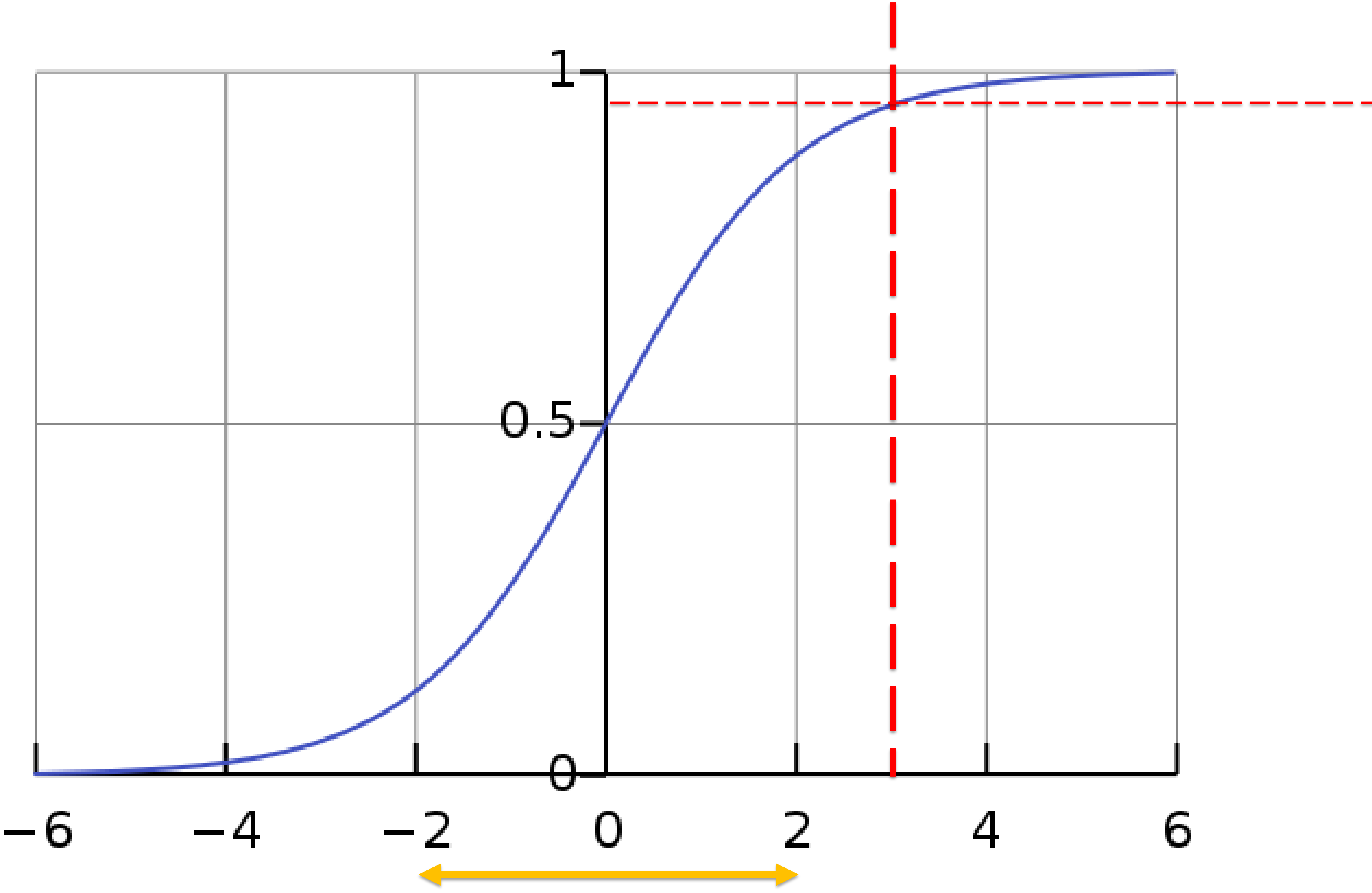
Review. sigmoidal output = **logistic function**

$$g(a) = \frac{1}{1+e^{-a}}$$

Rule of thumb:

for $a = 3$: $g(3) = 0.95$

for $a = -3$: $g(-3) = 0.05$



https://en.wikipedia.org/wiki/Logistic_function

4. Forward pass: exploit nonlinearities ('linearity problem')

To **exploit nonlinearities** of all units in the network, we must

1. Make sure that the **initialization** of weights is well chosen
   → expectation (across patterns) of the activation variable

$$0 = < a_j^{(n)} >; \; a_j^{(n)} = \sum_k w_{j,k}^{(n)} x_k^{(n-1)}$$

   → standard deviation of the activation variable

$a_j^{(n)}$ of order 1.

2. Make sure that **weight updates** do not shift mean (and standard deviation) of distribution too much

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Artificial Neural Networks: Lecture 4

Tricks of the Trade in deep networks

1. Bagging
2. Dropout
3. Other simple regularization methods
4. Choice of hidden units and initialization: ‘linearity problem’
5. Vanishing gradient problem

0. **Initialization of weights**

BackProp

1. **Choose pattern** $\mathbf{x}^{\mu}$

input $x_k^{(0)} = x_k^{\mu}$

2. **Forward propagation of signals** $x_k^{(n-1)} \longrightarrow x_j^{(n)}$

$$x_j^{(n)} = g^{(n)}(a_j^{(n)}) = g^{(n)}(\sum w_{jk}^{(n)} x_k^{(n-1)}) \quad (1)$$

output $\hat{y}_i^{\mu} = x_i^{(n_{\max})}$

3. **Computation of errors in output**

$$\delta_i^{(n_{\max})} = g'(a_i^{(n_{\max})}) \; [t_i^{\mu} - \hat{y}^{\mu}] \quad (2)$$

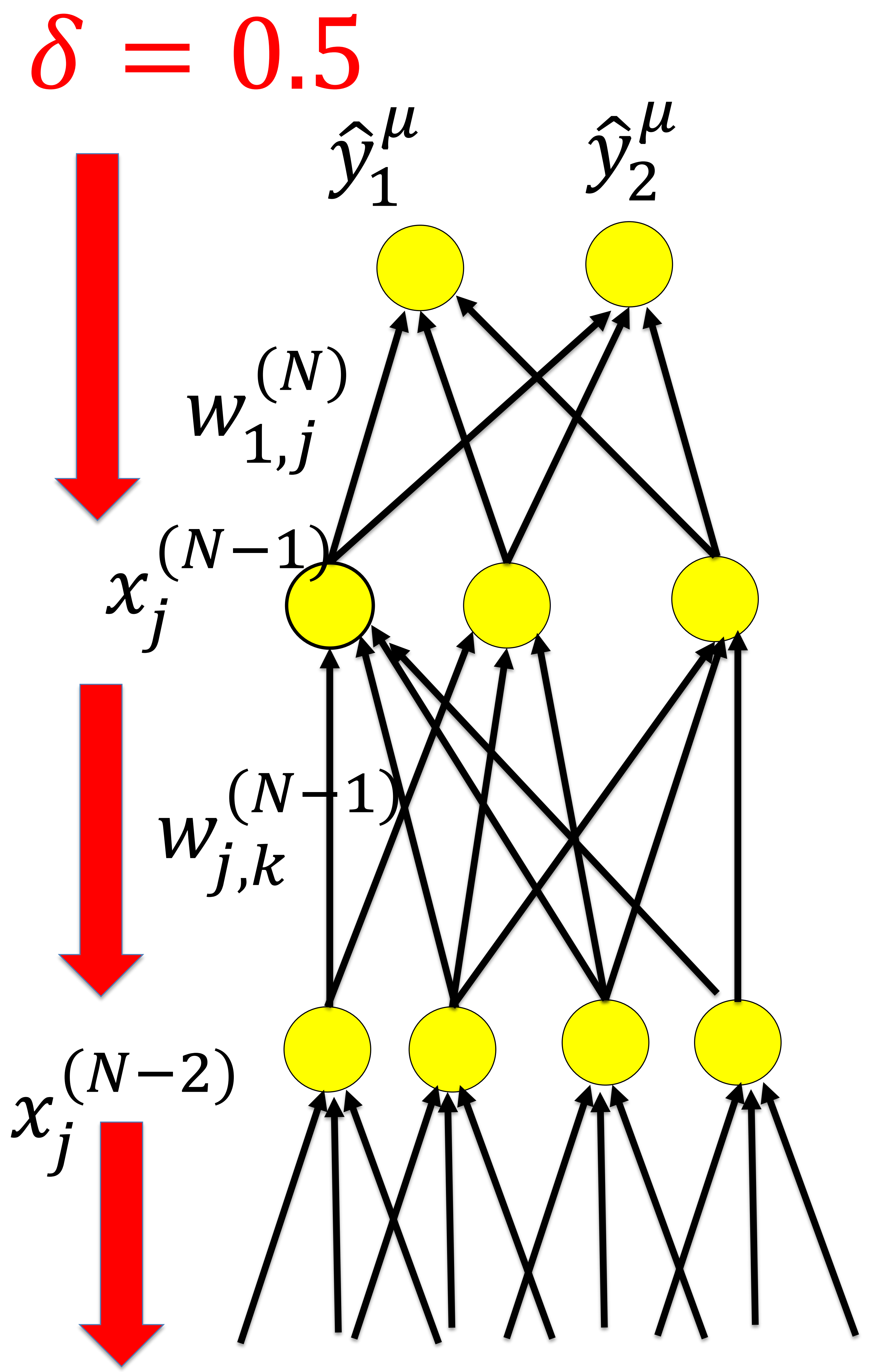
4. **Backward propagation of errors** $\delta_i^{(n)} \longrightarrow \delta_j^{(n-1)}$

$$\delta_j^{(n-1)} = g'^{(n-1)}(a^{(n-1)}) \sum_i w_{ij} \, \delta_i^{(n)} \quad (3)$$

5. **Update weights (for each** (i, j) **and all layers** (n)**)**

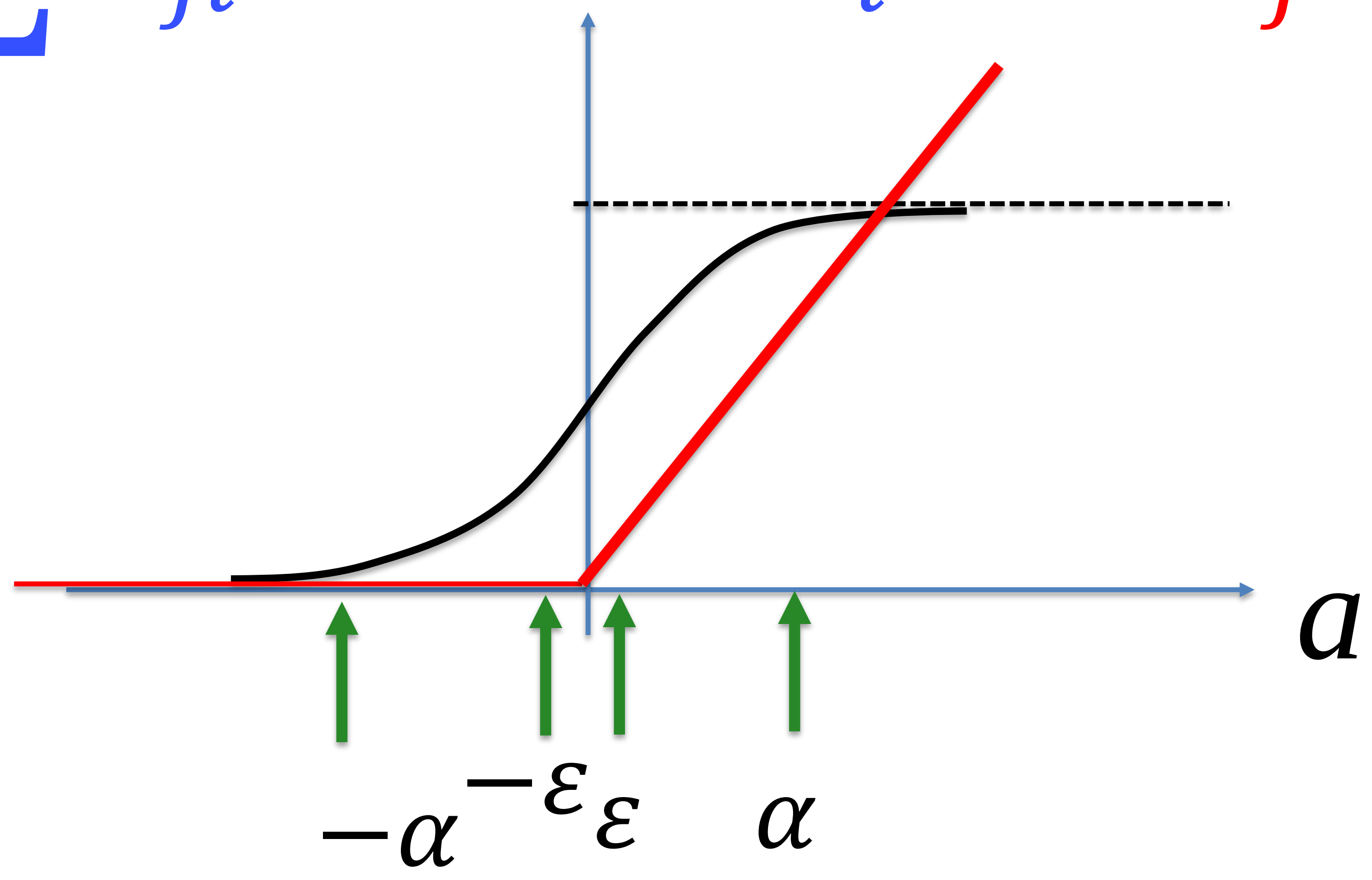
$$\Delta w_{ij}^{(n)} = \eta \, \delta_i^{(n)} \, x_j^{(n-1)} \quad (4)$$

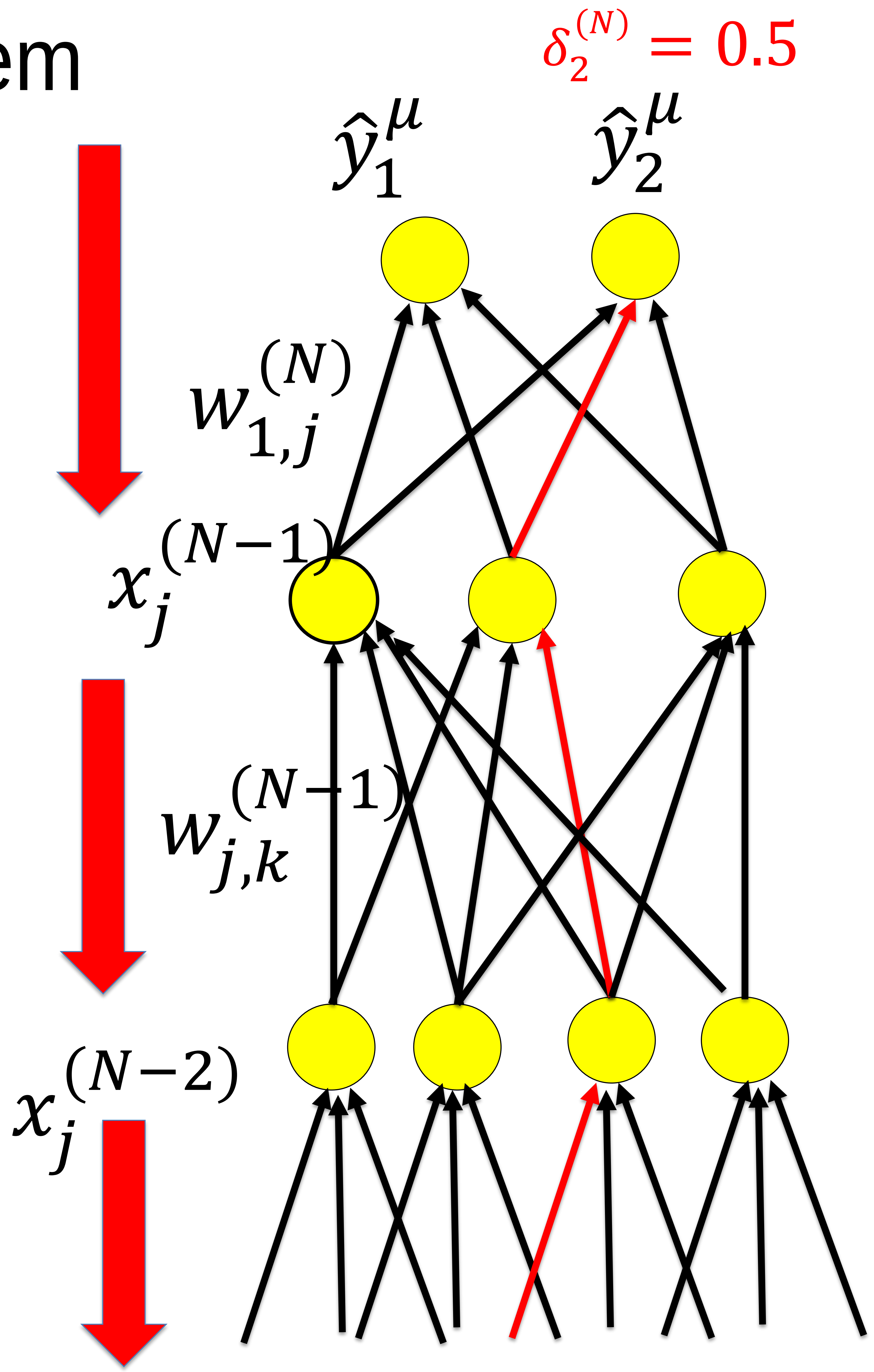
6. **Return to step 1.**



5. Backward pass: Vanishing gradient problem

$$\delta_i^{(n-1)} = \sum_j w_{ji}^{(n)} g'^{(n-1)}(a_i^{(n-1)}) \delta_j^{(n)}$$



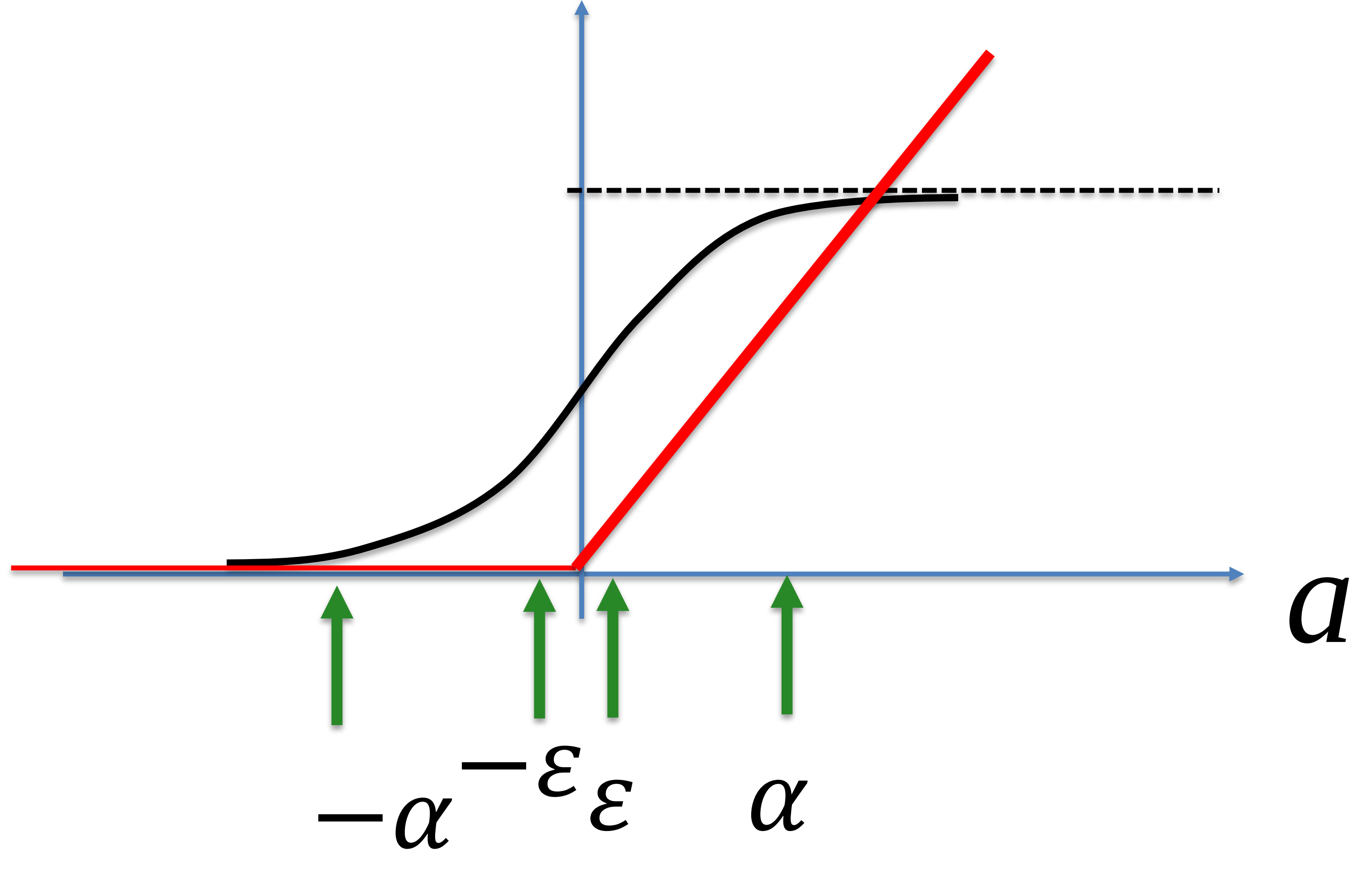


After N layers: each path contributes

$$\delta_i^{(1)} \sim g'^{(1)} g'^{(2)} \ldots g'^{(N-1)} \delta_j^{(N)}$$

Many terms to be summed,
but most terms are tiny if N large

5. Vanishing gradient problem

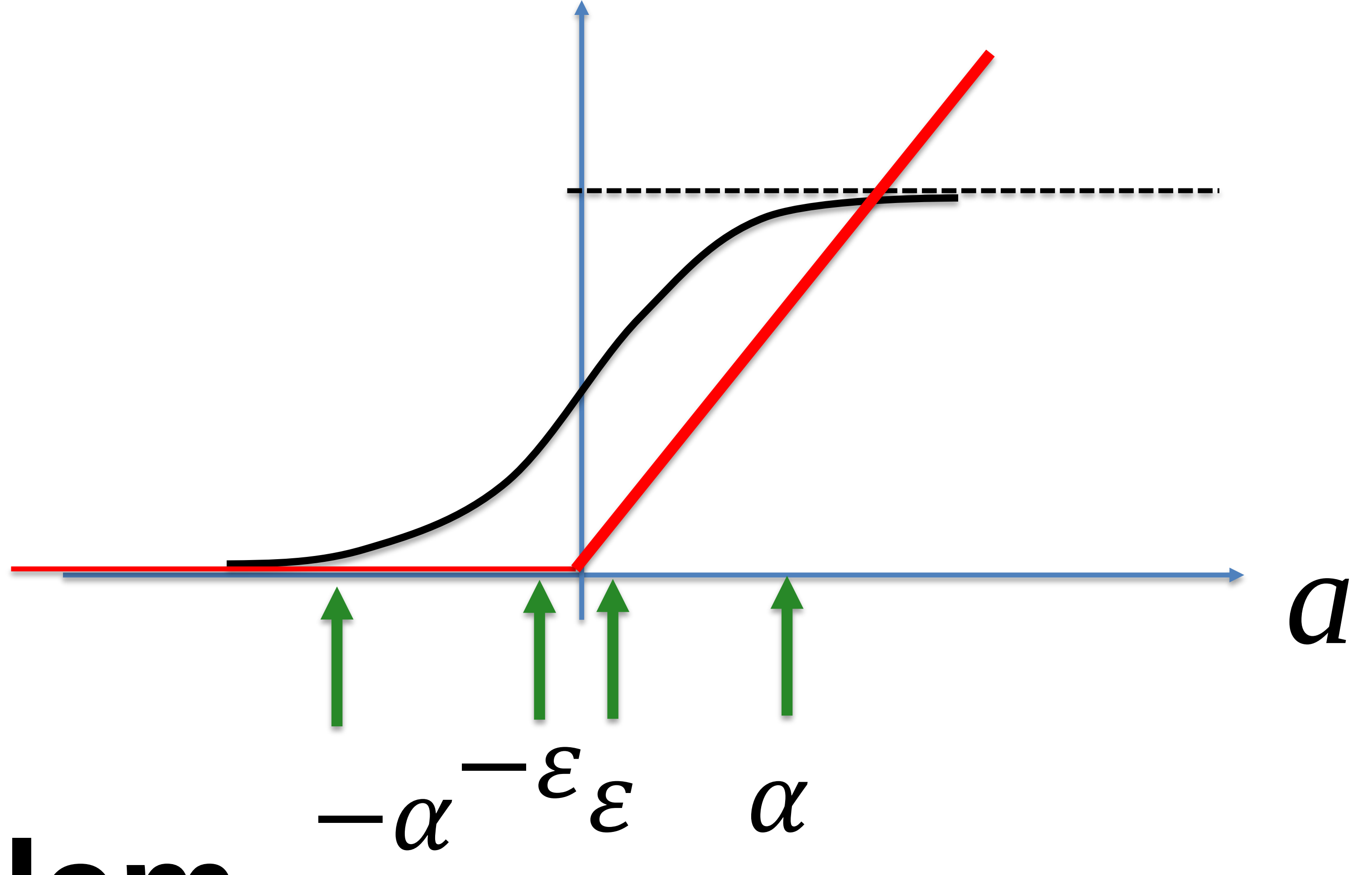


Observations:

- for each single path many terms g'
- g' is small for sigmoidal at $-\alpha$ or $+\alpha$ ($|a|=4$)
- g' vanishes for ReLu if one inactive unit sits in path
- g'=1 for all ReLu on 'active paths'

→ for ReLu highly active forward paths coincide with good gradient transmission on backward path

5. Vanishing gradient problem



Conclusion:

Sucessful forward pass

→ needs to avoid the linearity problem.

('exploit nonlinearities')

Successful backward pass

→ needs to avoid the vanishing gradient problem.

A good hidden units must be good for forward and backward pass!

Artificial Neural Networks: Lecture 4
Tricks of the Trade in deep networks

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EPFL, Lausanne, Switzerland

1. Bagging
2. Dropout
3. Other simple regularization methods
4. Initialization and choice of hidden units are important.
5. Vanishing gradient problem
6. Weight update: mean input and bias problem

6. Weight update step

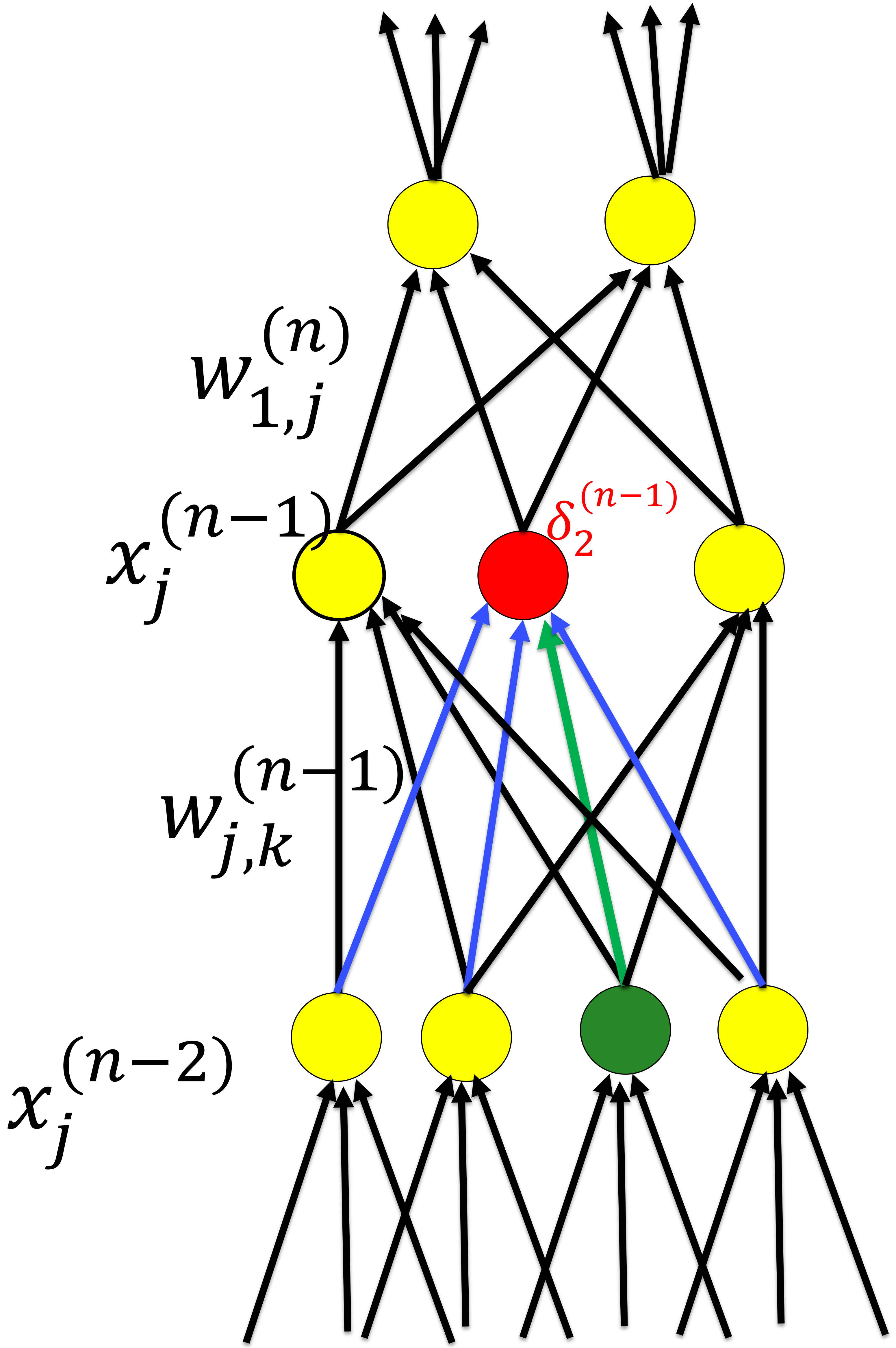
update **all** weights

$$\Delta w_{i,j}^{(n-1)} = \delta_i^{(n-1)} x_j^{(n-2)}$$

Weights onto the same neuron (red) are all updated with same delta

→ if $x_j^{(n-2)}$ are all positive,

all the weights onto red neuron increase or decrease together



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update **all** weights

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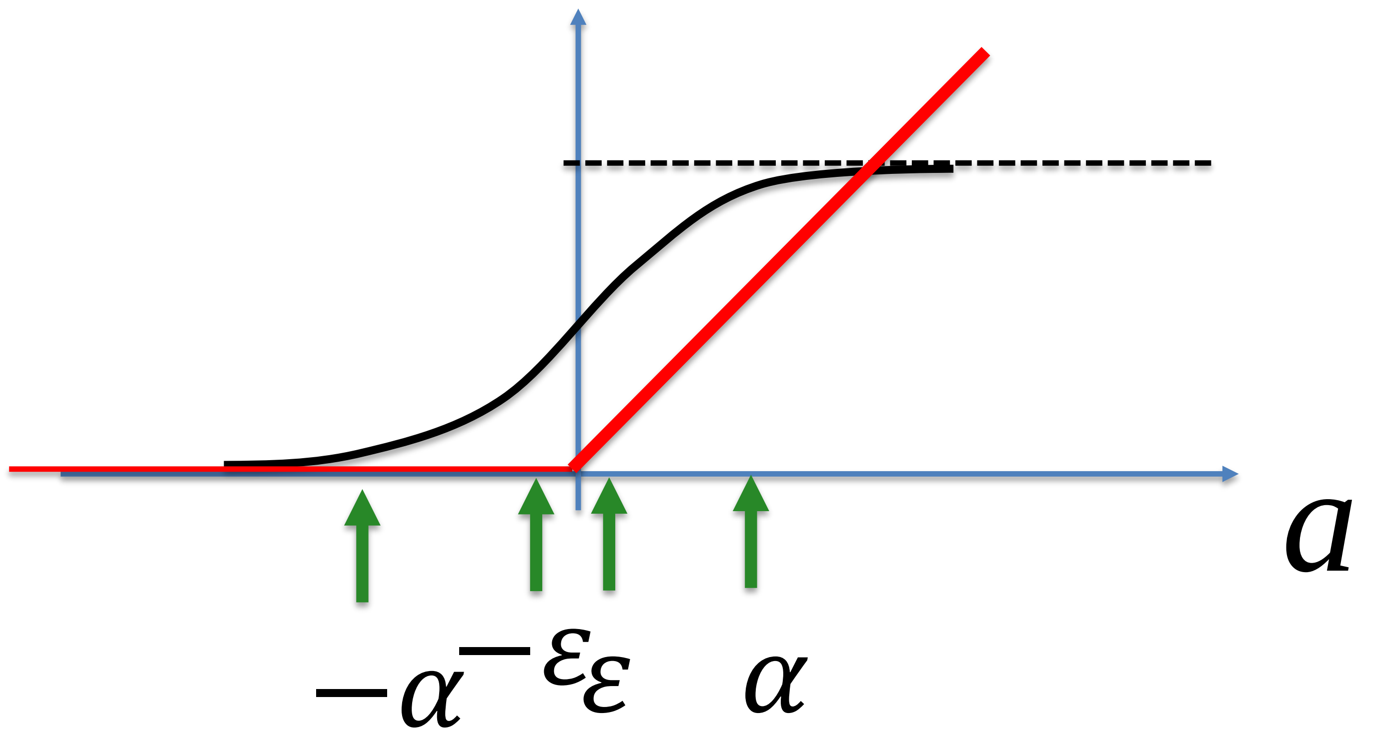
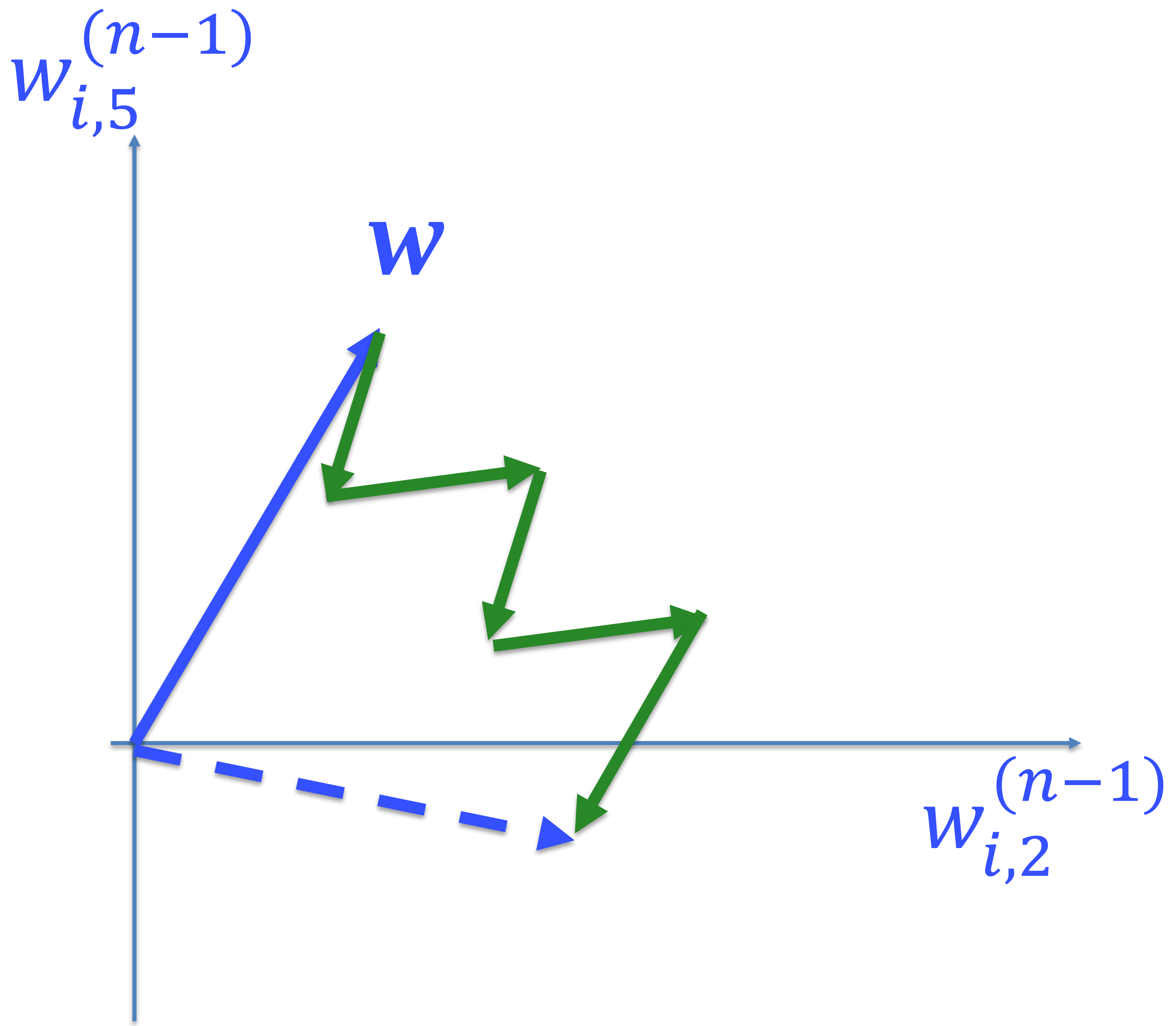




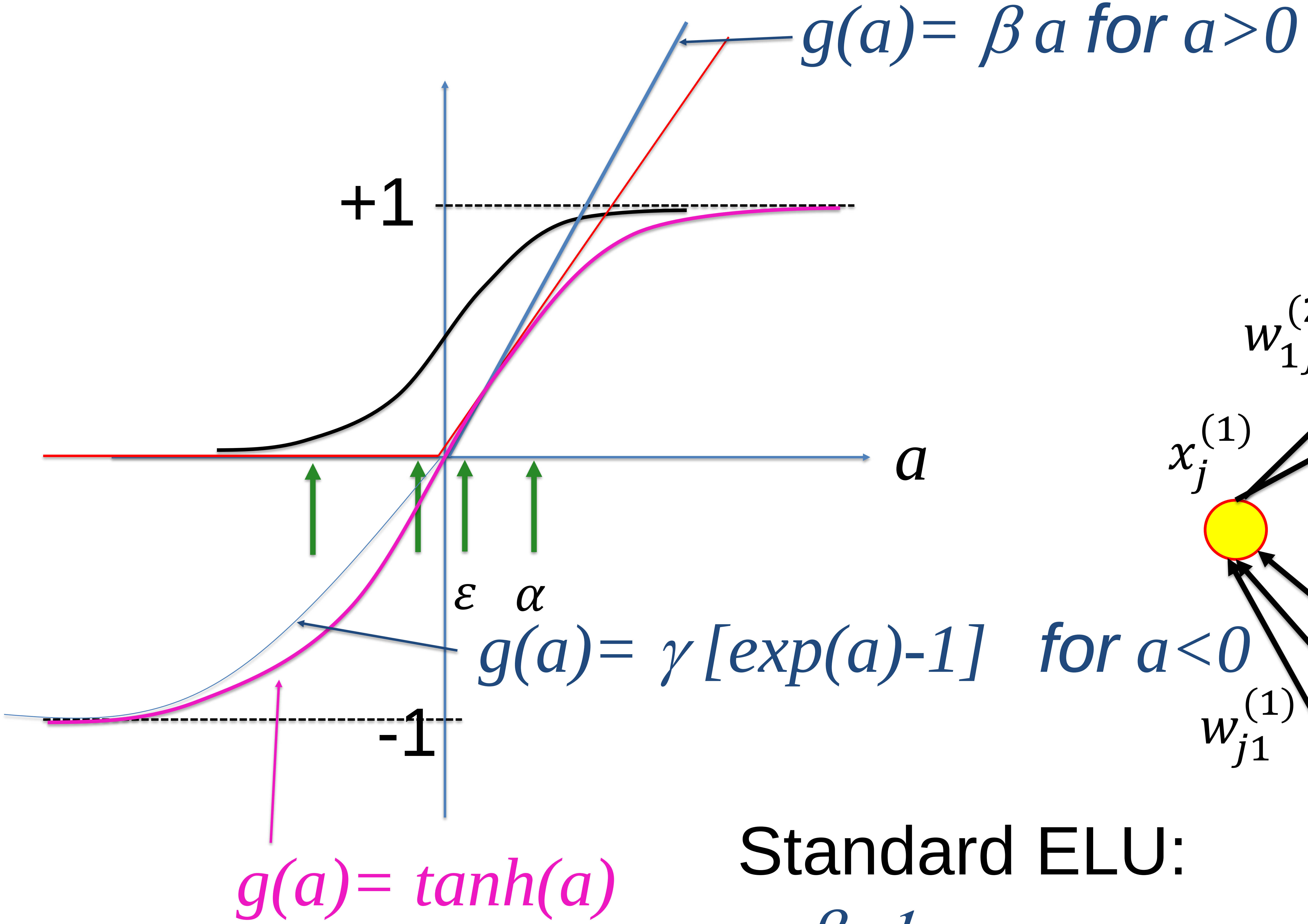
Weights onto the same neuron are all updated with same delta

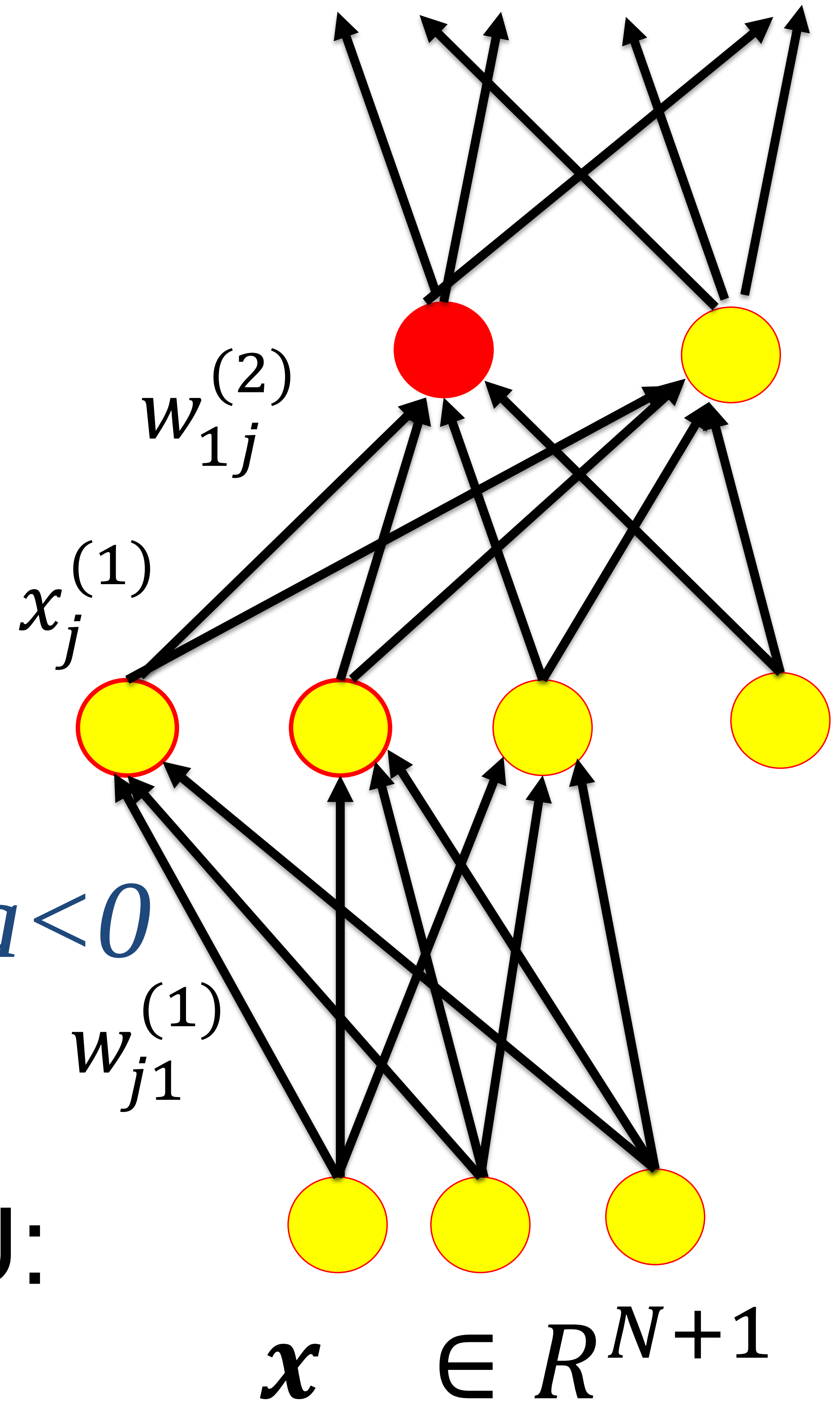
→ Problem for ReLu and other units with non-negative x

→ No problem for tanh

→ No problem for shifted exponential linear Selu

Shifted Exponential Linear (SELU) vs. tanh





Standard ELU:

γ=β=1

6. Bias problem

update **all** weights

$$\Delta w_{i,j}^{(n)} = \delta_i^{(n)} x_j^{(n-1)}$$

Before update

$$a_i^{(n)} = \sum_j w_{ij}^{(n)} x_j^{(n-1)} - \vartheta$$

after update

$$a_i^{(n)} = \sum_j [w_{ij}^{(n)} + \Delta w_{i,j}^{(n)}] x_j^{(n-1)} - \vartheta$$

same sign for all j

non-negative
(for ReLu etc)

Weights onto the same neuron
are all updated with same delta

→ Problem for ReLu and other units with non-negative x

→ The mean changes! ('bias problem')

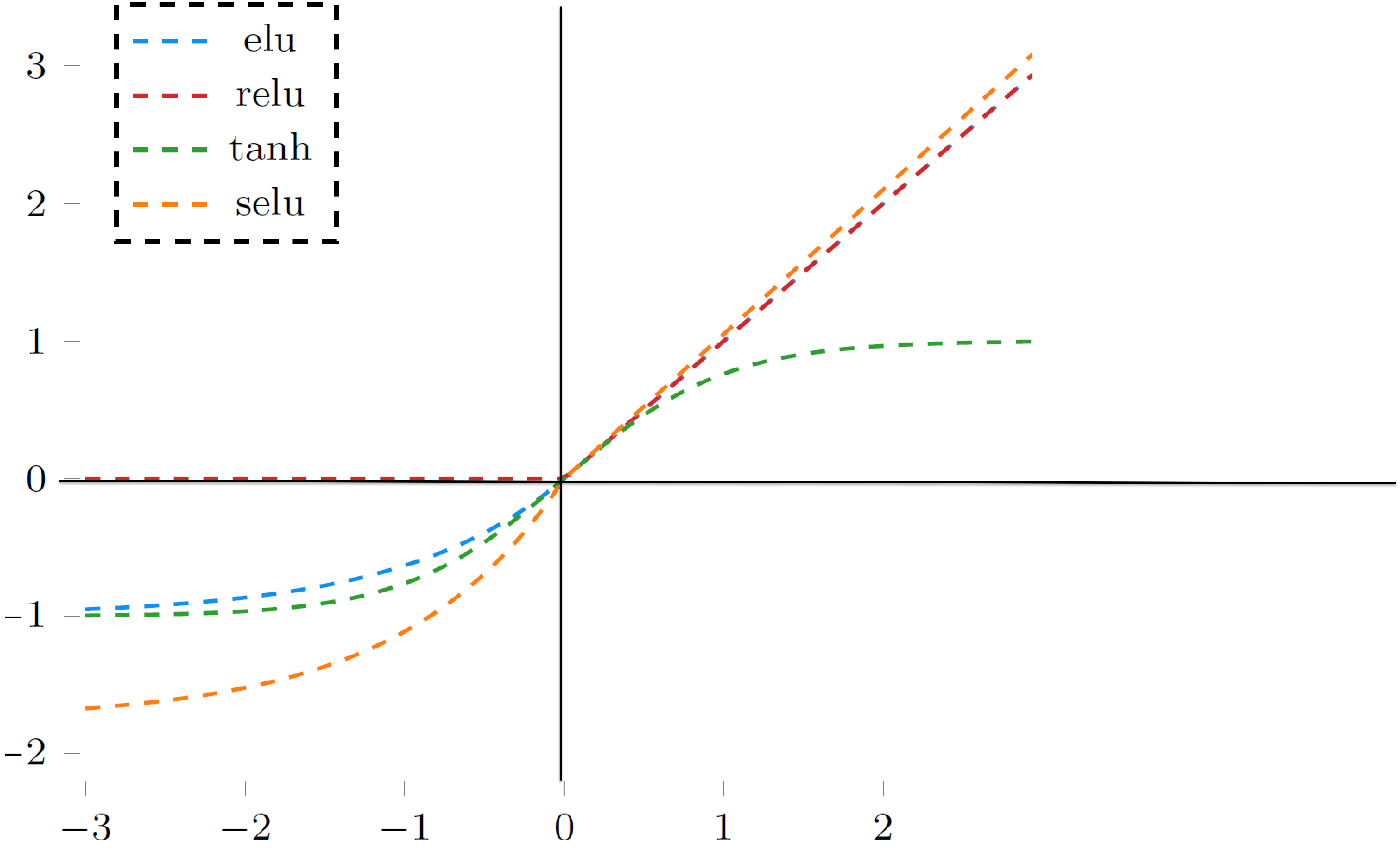
→But controlling the mean was important for correct initialization!

→ Return of vanishing gradient and linearity problem!

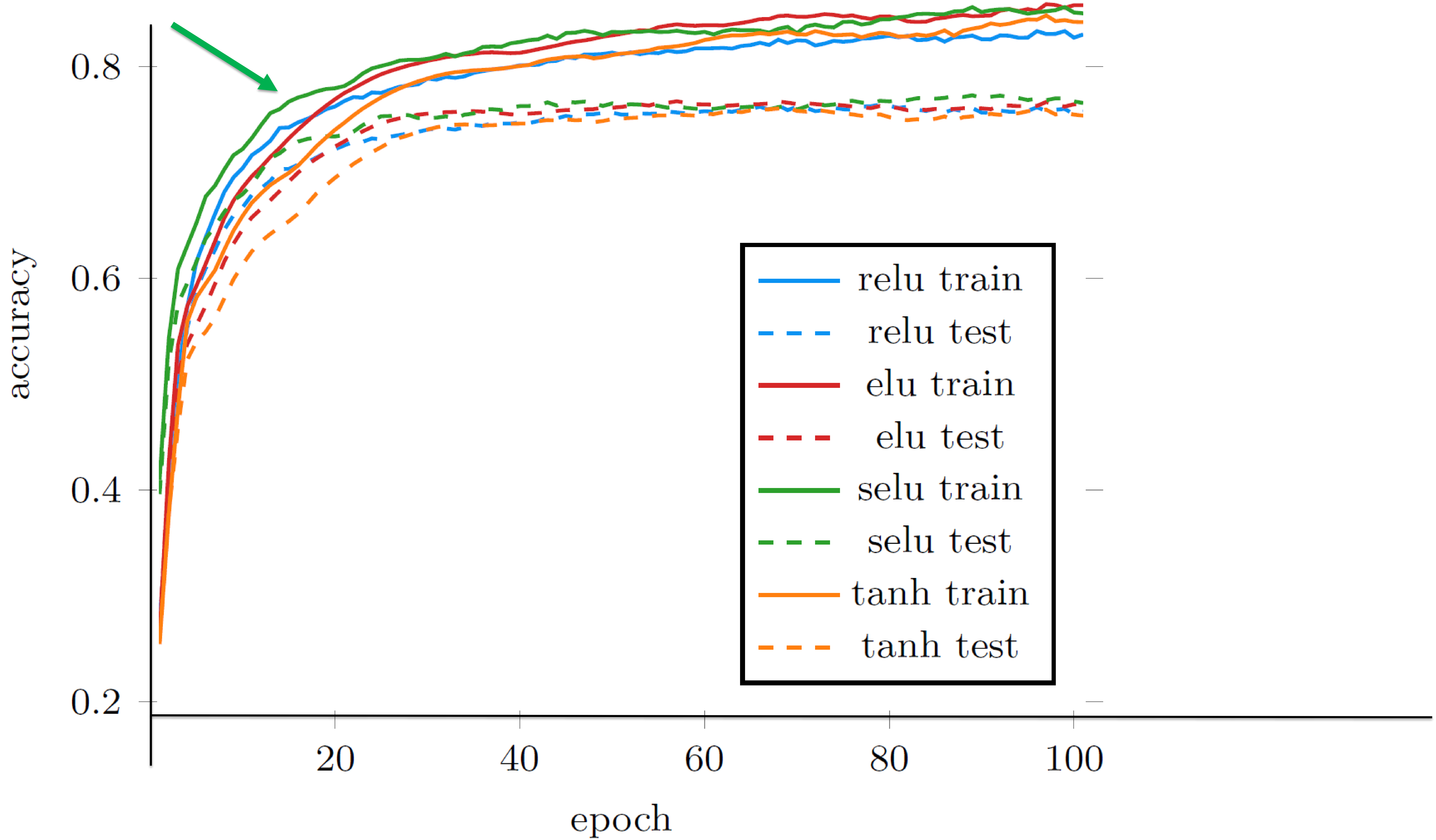
Quiz:

[ ] forward propagation with ReLu leaves only a few active paths
[ ] back propagation with ReLu leaves only a few active paths
[ ] a non-zero weight update step of ReLu shifts most often the mean
[ ] forward propagation with ReLu is always linear on the active paths
[ ] in a ReLu network all patterns are processed with the same linear filter
[ ] in a sigmoidal network with small weights (and normalized inputs) all patterns are processed with the same linear filter
[ ] in a sigmoidal network with big weights, there are active units in the forward pass that contribute a vanishing gradient in the backward path
[ ] in a network with SELU, there are active units in the forward path which contribute a vanishing gradient in the backward path
[ ] a non-zero the weight update step of SELU shifts the mean

Shifted Exponential Linear vs. tanh



Shifted Exponential Linear (SELU)



6. Conclusion

- initialization is important so as to exploit nonlinearities
- choice of hidden unit is important in **initial phase** of training
- ReLu has disadvantages in keeping the mean
  → batch normalization
- Tanh has problems with vanishing gradient
- Sigmoidal has problems with vanishing gradient **and** mean
- SELU solves all problems and is currently best choice

Paper: Klaumbauer, …, Hochreiter (2017)
Self-normalizaing neural networks
https://arxiv.org/pdf/1706.02515.pdf

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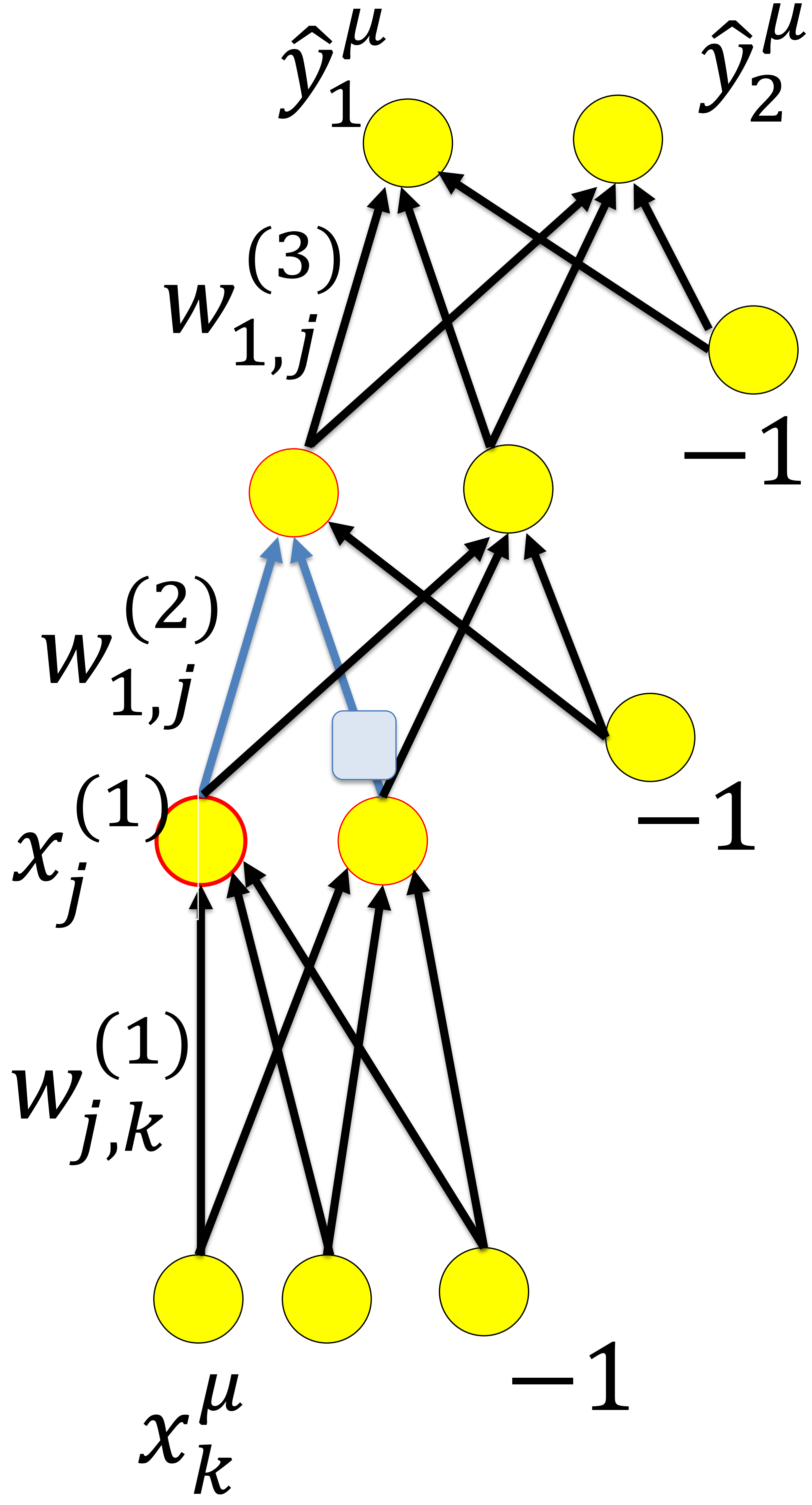
Artificial Neural Networks: Lecture 4
Tricks of the Trade in deep networks

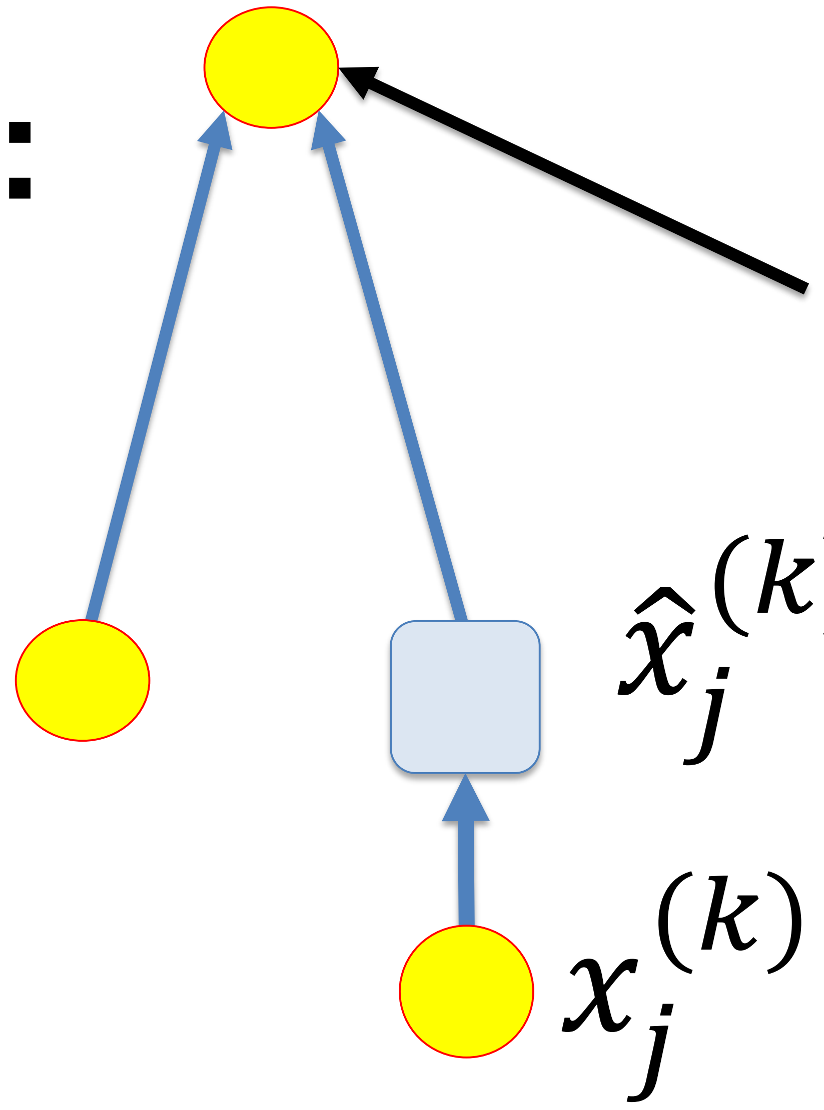
1. Bagging
2. Dropout
3. Other simple regularization methods
4. Hidden units: linearity problem (exploit nonlinearities)
5. Hidden units: Vanishing gradient problem
6. Weight update: bias problem
7. Batch normalization

7. Batch normalization: Idea



Normalize input on each input line

Zoom:



$$\hat{x}_j^{(k)} = \frac{x_j^{(k)} - E[x_j^{(k)}]}{\sqrt{Var[x_j^{(k)}]}}$$

7. Batch normalization

Ioffe&Szegedi, 2015

Work with minibatch:
Normalize per minibatch

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1...m}\}$;
Parameters to be learned: γ, β

Output: $\{y_i = \mathrm{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

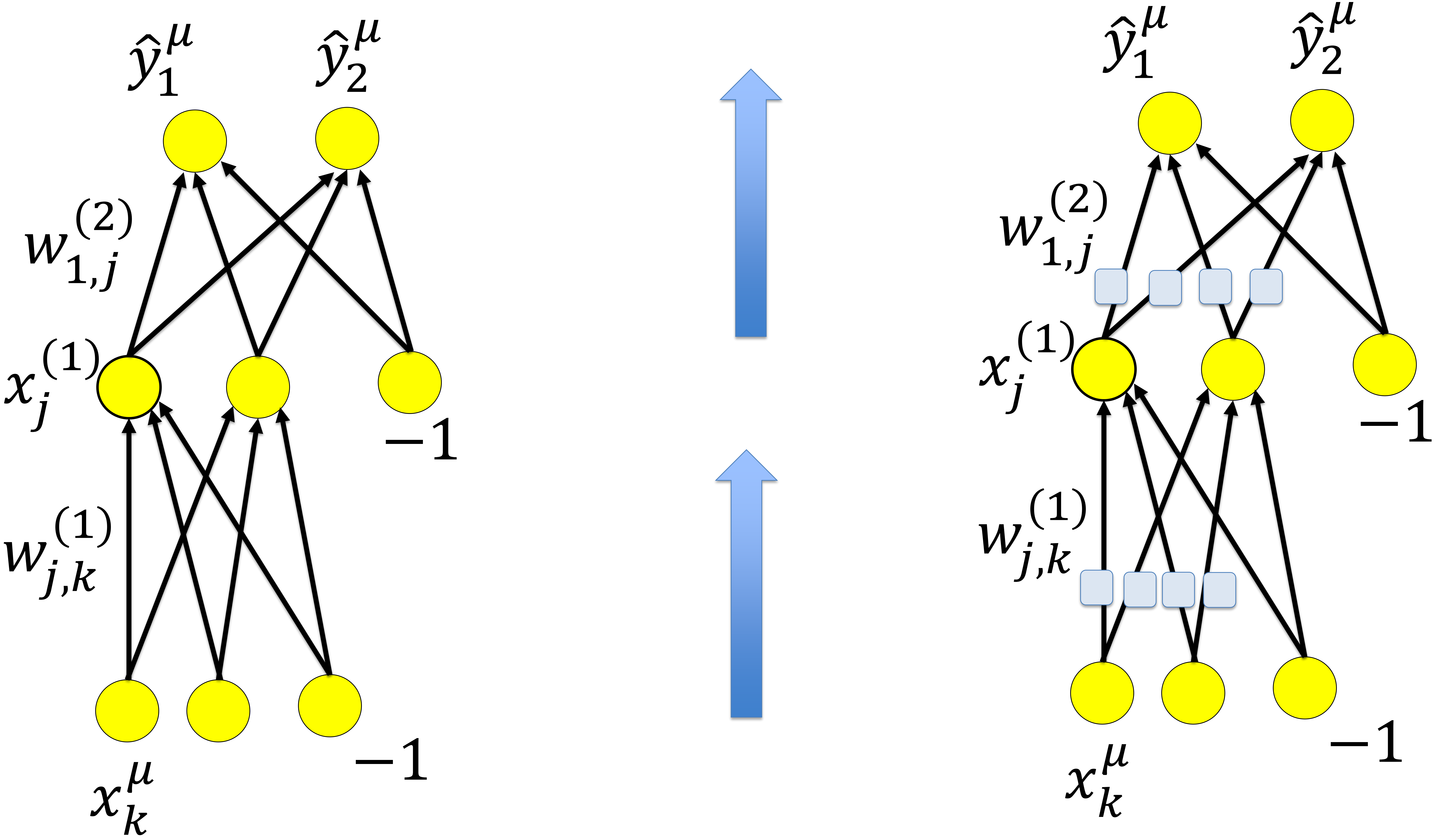
$$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_{\mathcal{B}})^2 \qquad \text{// mini-batch variance}$$

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \qquad \text{// normalize}$$

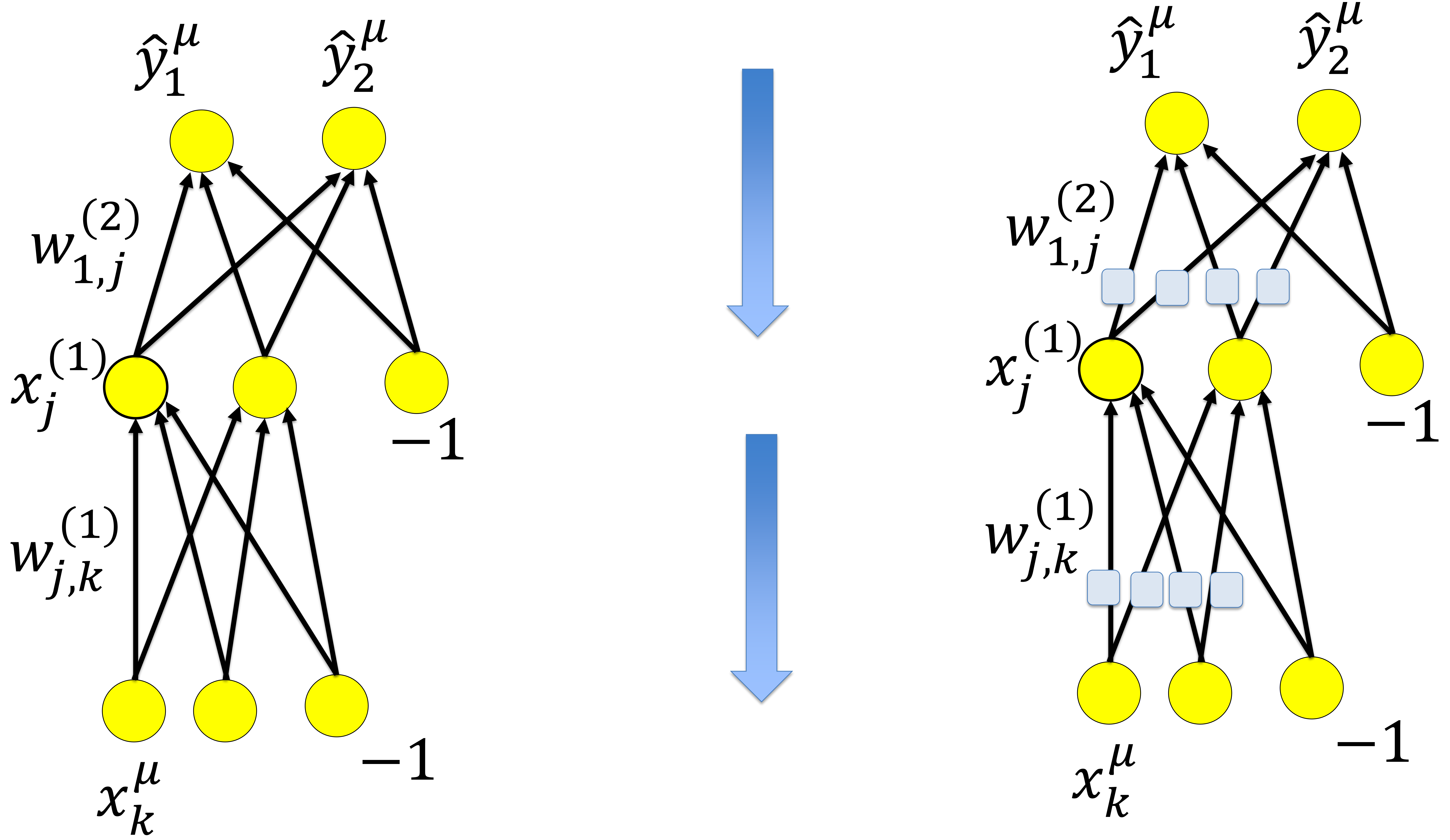
$$y_i \leftarrow \gamma \widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

Algorithm 1: Batch Normalizing Transform, applied to activation x over a mini-batch.

7. Batch normalization Ioffe&Szegedi, 2015



7. Batch normalization Ioffe&Szegedi, 2015



7. Batch normalization

Ioffe&Szegedi, 2015

Input: Network N with trainable parameters Θ; subset of activations $\{x^{(k)}\}_{k=1}^{K}$

Output: Batch-normalized network for inference, $N_{\mathrm{BN}}^{\mathrm{inf}}$

$N_{\mathrm{BN}}^{\mathrm{tr}} \leftarrow N$ // Training BN network

for $k = 1 \ldots K$ **do**

  Add transformation $y^{(k)} = \mathrm{BN}_{\gamma^{(k)},\beta^{(k)}}(x^{(k)})$ to $N_{\mathrm{BN}}^{\mathrm{tr}}$ (Alg. 1)

  Modify each layer in $N_{\mathrm{BN}}^{\mathrm{tr}}$ with input $x^{(k)}$ to take $y^{(k)}$ instead

end for

end for

Train $N_{\mathrm{BN}}^{\mathrm{tr}}$ to optimize the parameters Θ $\{\gamma^{(k)}, \beta^{(k)}\}_{k=1}^{K}$

$N_{\mathrm{BN}}^{\mathrm{inf}} \leftarrow N_{\mathrm{BN}}^{\mathrm{tr}}$ // Inference BN network with froze // parameters

for $k = 1 \ldots K$ **do**

  // For clarity, $x \equiv x^{(k)}, \gamma \equiv \gamma^{(k)}, \mu_{\mathcal{B}} \equiv \mu_{\mathcal{B}}^{(k)}$, etc

  Process multiple training mini-batches $\mathcal{B}$, each size m, and average over them:

$$\mathrm{E}[x] \leftarrow \mathrm{E}_{\mathcal{B}}[\mu_{\mathcal{B}}]$$

$$\mathrm{Var}[x] \leftarrow \frac{m}{m-1}\mathrm{E}_{\mathcal{B}}[\sigma_{\mathcal{B}}^2]$$

  In $N_{\mathrm{BN}}^{\mathrm{inf}}$, replace the transform $y = \mathrm{BN}_{\gamma,\beta}(x)$ wi

$$y = \frac{\gamma}{\sqrt{\mathrm{Var}[x]+\epsilon}} \cdot x + \left(\beta - \frac{\gamma\,\mathrm{E}[x]}{\sqrt{\mathrm{Var}[x]+\epsilon}}\right)$$

end for

Algorithm 2: Training a Batch-Normalized Network

7. Batch normalization

Ioffe&Szegedi, 2015

Necessary for ReLu and other unbalanced hidden units

Normalization step in forward pass is also taken care of during backward pass

Objectives for today:

- Bagging: multiple models help always to improve results!
- Dropout: two interpretations
  - (i) a practical implementation of bagging
  - (ii) forced feature sharing
- BackProp: Initialization, nonlinearity, and symmetry
- What are good units for hidden layers?
  - problems of vanishing gradient and shift of mean
  - → solved by Shifted exponential linear (SELU)
- Batch normalization → necessary for ReLu

The end