# Networks out of Control:

## Homework Set 3

## Exercise 1

Suppose that newborn nodes come in groups of $n$ in each period and each adds $m$ undirected links. Suppose that they attach a fraction $f$ of their links uniformly at random to other newborn nodes, and a fraction $1 - f$ to older nodes via preferential attachment. Using a continuous time mean-field approximation as shown in class, find an approximation for the c.d.f. of the degree distribution.

Hint: First let $d_i(t)$ be the degree at time $t$ of a node that was born at time $i$ and use the mean-field approximation to solve for this value.

## Exercise 2

Consider a direct-benefit cascade we saw in class, but on an infinite network – in particular, graphs that have an infinite number of nodes, but each node has finitely many neighbors. We can define the *cascade capacity* $q^\star$ of the network as the largest value of the threshold $q$ for which there exists some finite set of early adopters that can cause a complete cascade. Please explain your answers formally.

1. What is the smallest number of nodes that can trigger a complete cascade on an infinite path?
2. What is the cascade capacity $q^\star$ of an infinite path?
3. Consider an infinite grid (i.e., square lattice) in Euclidean space in which each node is connected to the 8 closest nodes in $\ell_2$ distance. What is the cascade capacity $q^\star$ of this network?
4. For the same infinite grid, does a cascade always spread at the same rate for all $0 \leq q \leq q^\star$?

## Exercise 3

Consider the information cascades model on a network, in which each player only observes the actions of the neighbors who have gone before them. (Please explain your answers formally.)

1. Consider an infinite path (i.e., nodes $\ldots, -2, -1, 0, 1, 2, \ldots$ such that node $i$ is neighbors with $i-1$ and $i+1$), and assume that the nodes go in the following order: 0, $-1$, 1, $-2$, 2, .... Can a cascade form in this network? If so, does a cascade always occur?
2. Now, consider a "thick" infinite path where each node is connected to the 4 closest nodes (i.e., nodes $\ldots, -2, -1, 0, 1, 2, \ldots$ such that node $i$ is neighbors with $i-2$, $i-1$,

$i+1$ and $i+2$), and assume that the nodes go in the following order: 0, $-1$, 1, $-2$, 2, …. Can a cascade form in this network? If so, does a cascade always occur?

3. In the two questions above, is it possible to have infinitely many nodes colored blue **and** infinitely many nodes colored red?