Artificial Neural Networks: Lecture 9

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Variants of TD-learning methods and continuous space

Objectives for today:

- TD learning refers to a whole class of algorithms
- **There are many Variations of SARSA**
- **All set up to iteratively solve the Bellman equation**
- **Eligibility traces and n-step Q-learning to extend over time**
- **Continuous space and ANN models**
- **Models of actions and models of value**

Reading for this week:

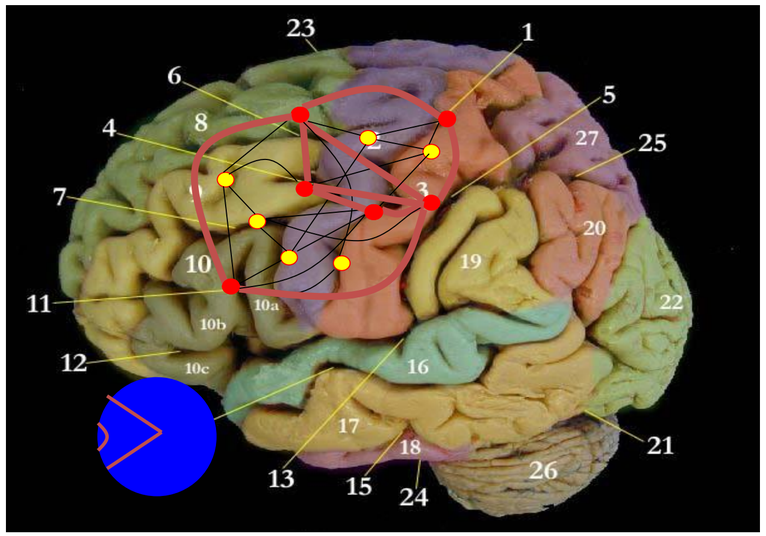
Sutton and Barto, Reinforcement Learning
(MIT Press, 2nd edition 2018, also online)

Chapter: 5.1-5.4 and 6.1-6.3 and 6.5-6.6, and 7.1-7.2 and 9.3

Background reading:

Temporal Difference Learning and TD-Gammon
by Gerald Tesauro (1995) pdf online

1. Review: Artificial Neural Networks for action learning



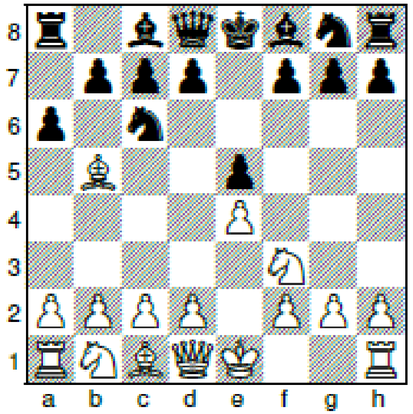
Where is the supervisor?
Where is the labeled data?

Replaced by:
'Value of action'
- 'goodie' for dog
- 'success'
- 'compliment'

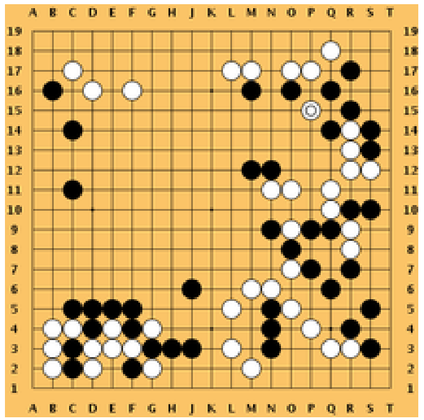
BUT:
Reward is rare:
'sparse feedback' after a long action sequence

1. Review: Deep reinforcement learning

Chess



Go



Artificial neural network (*AlphaZero*) discovers different strategies by playing against itself.

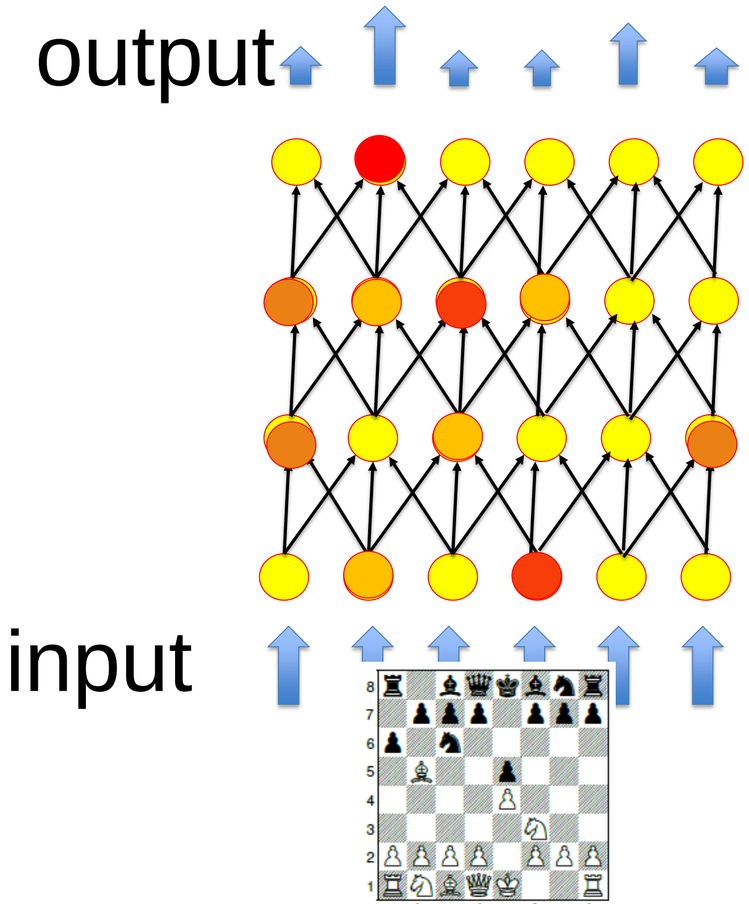
In Go, it beats Lee Sedol

1. Review: Deep reinforcement learning

Network for choosing action

action:

Advance king



Today:

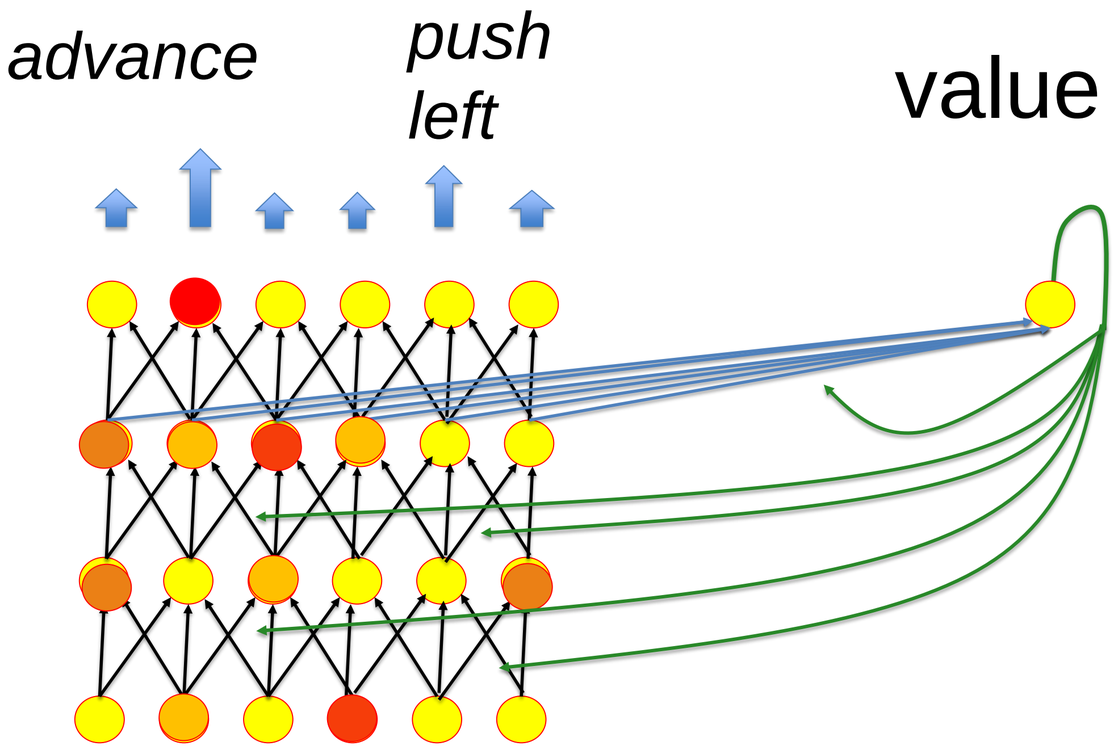
- How can we set-up such a network?
- What is the error function?
- How can we optimize weights?

(previous slide)
The basic idea of Reinforcement Learning (RL) was introduced in a previous lecture. Today we make a first step to link RL to artificial neural networks.

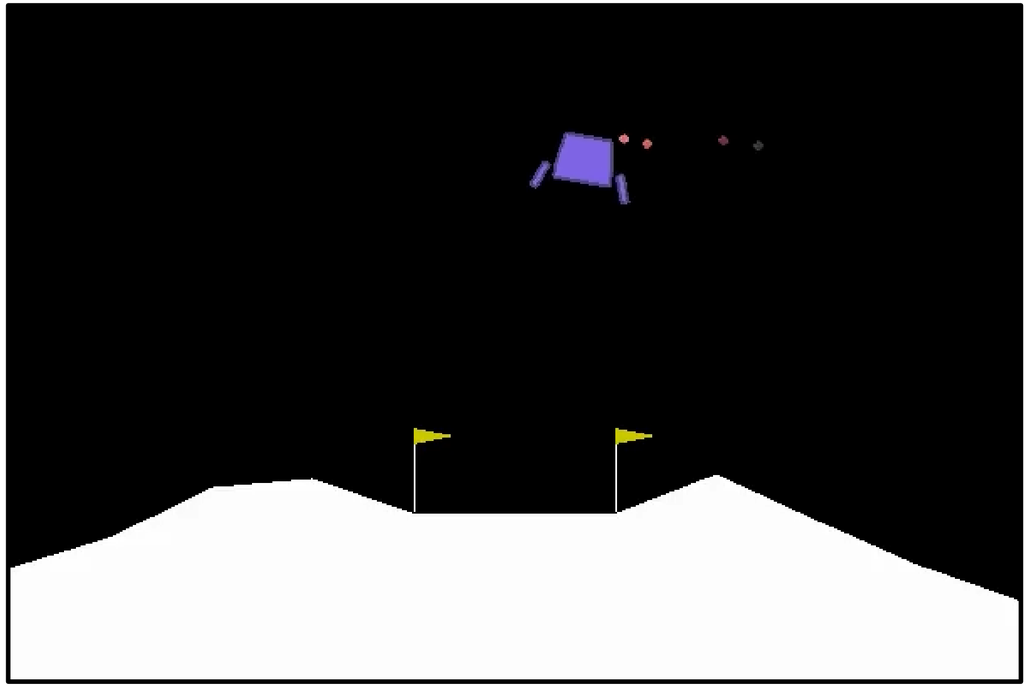
Training in networks is via an error-function – so what is the error function for RL?
And how can we optimize the weights?

1. Deep Reinforcement Learning: Lunar Lander (miniproject)

actions



Aim: land between poles



Policy gradient → Next week

1. Review: Branching probabilities and policy

Policy $\pi(s,a)$

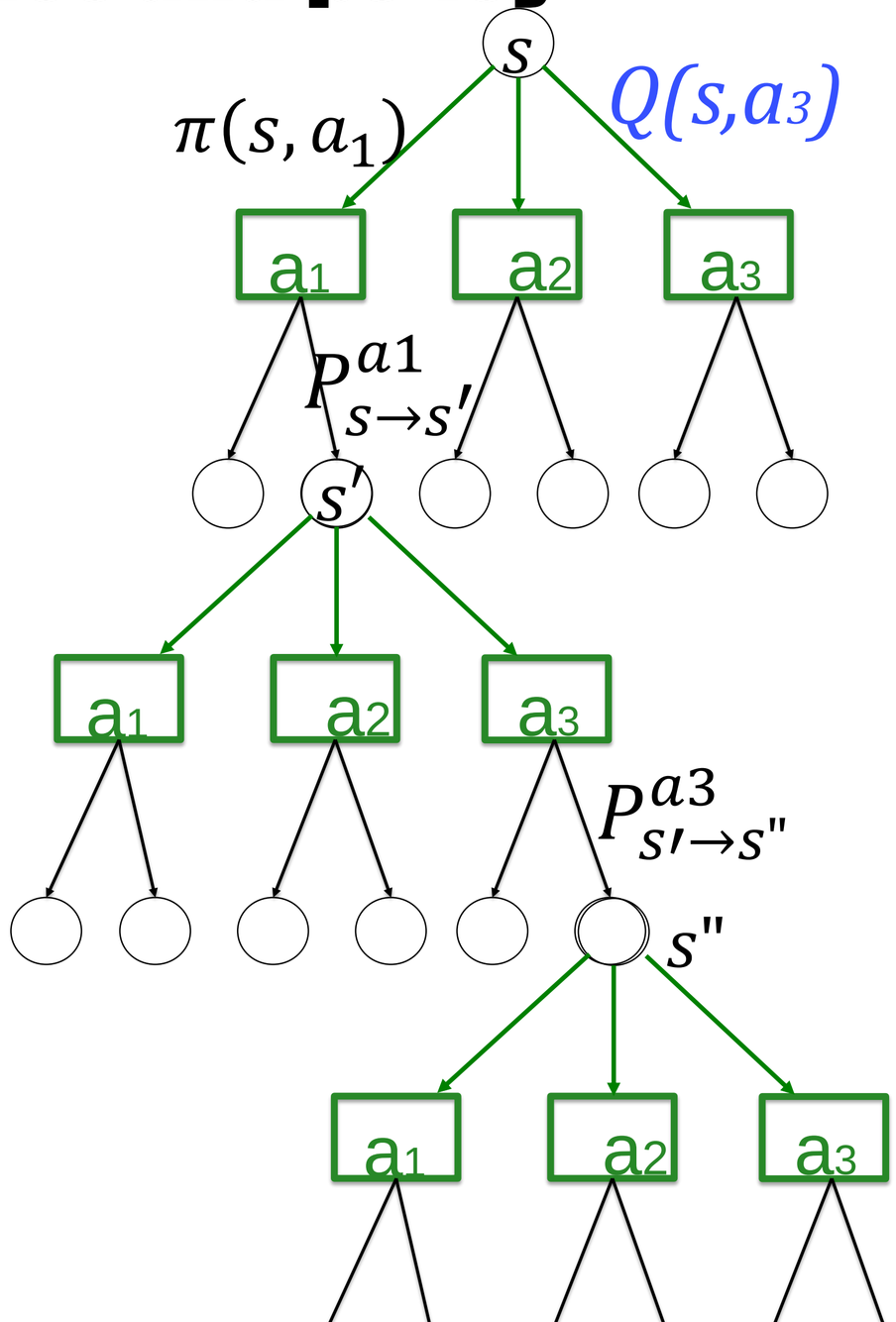
probability to choose action a in state s

$1=\sum_{a'}\pi(s,a')$

Examples of policy:
-epsilon-greedy
-softmax

Stochasticity $P^{a1}_{s\to s'}$

probability to end in state s' taking action a in state s

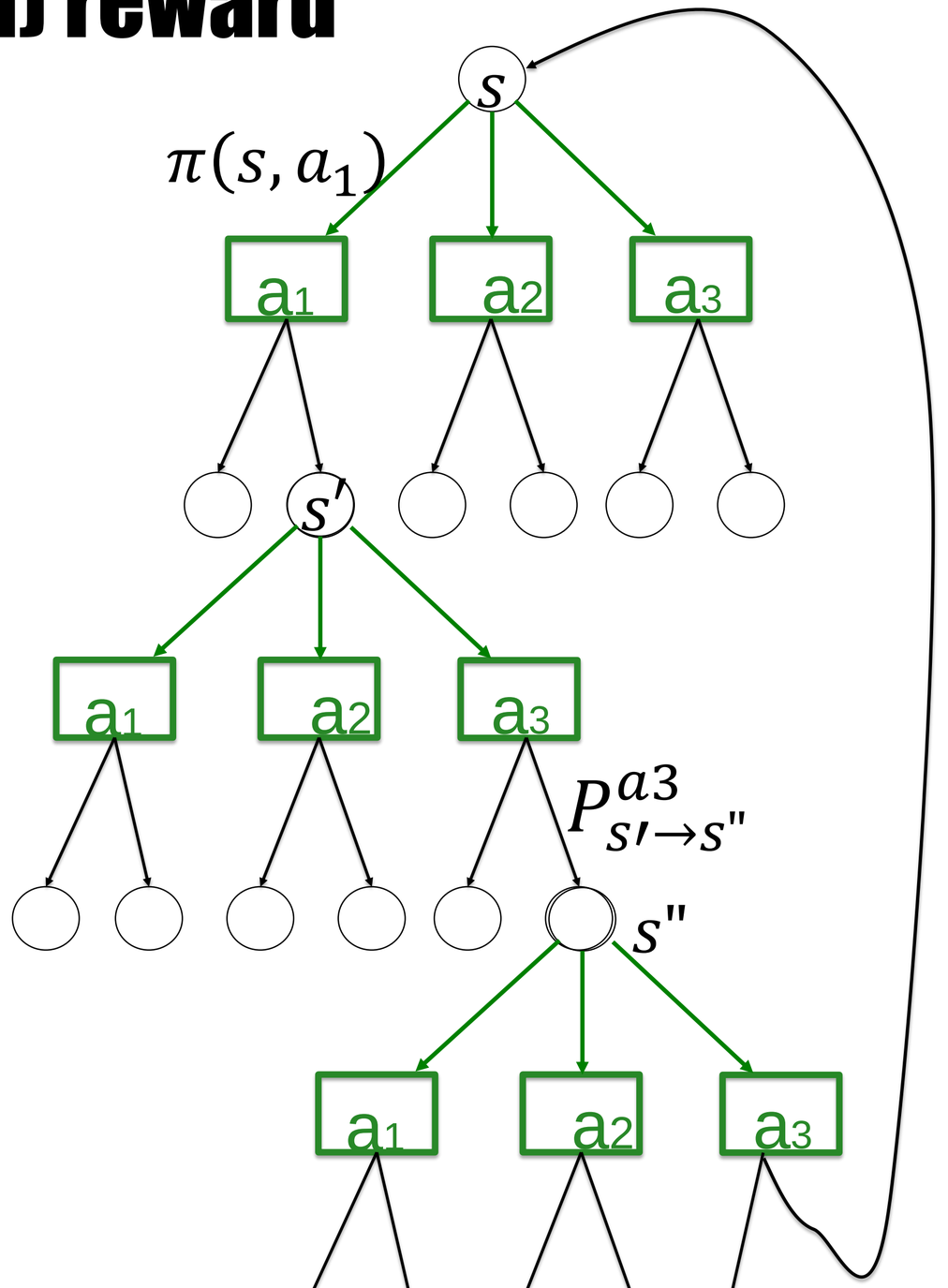


1. Review Total expected (discounted) reward

Starting in state s with action a

$Q(s,a)$ =

$$\langle r_t + \gamma\, r_{t+1} + \gamma^2 r_{t+2} + \gamma^3\, r_{t+3} + \cdots \rangle$$



Discount factor: γ <1

-important for recurrent networks!

-avoids blow-up of summation

-gives less weight to reward in **far** future

(previous slides)

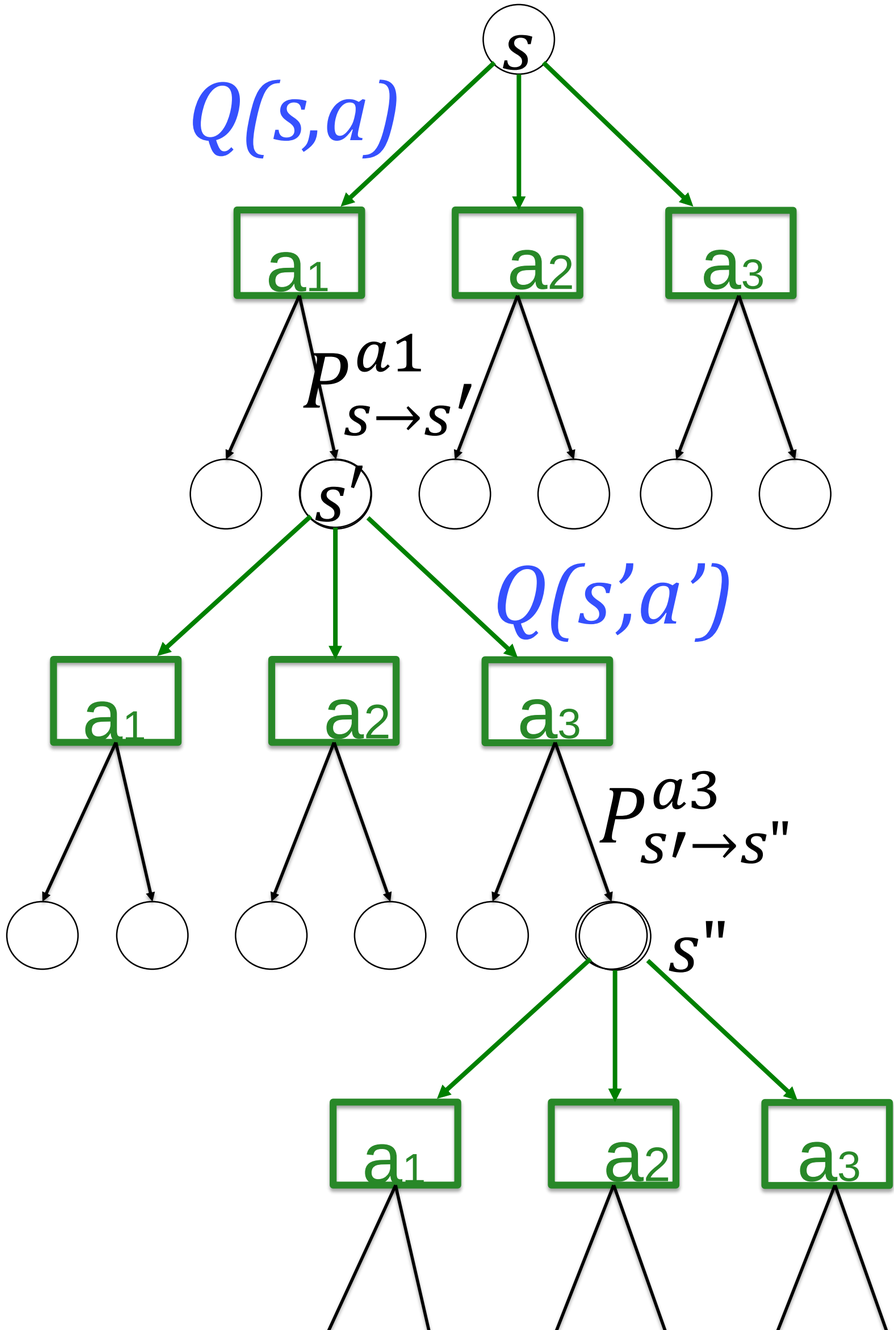
We need from previous lectures that RL works with states and actions that allow probabilistic transitions between the states.

An important quantity is the Q-value which represents the expectation of the accumulated reward (discounted with a factor gamma smaller than one).

1. Review: Bellman equation

$$Q(s,a) = \sum_{s'} P^{a}_{s \to s'} \left[R^{a}_{s \to s'} + \gamma \sum_{a'} \pi(s',a') Q(s',a') \right]$$

Bellman equation = value consistency of neighboring states



(previous slide)
The Q-value Q(s,a) further up in the graph is the expected total discounted reward – summed over all possible future actions and states.
It can be decomposed in an average over the immediate rewards, actions, and states, and the Q-values Q(s’,a’) of all possible next states.
The Bellman equation can therefore be interpreted as summarizing the consistency between the Q-values in state s, and the Q-values in neighboring states s’.
The difference between Q(s,a) and Q(s’,a’) must be explained by the immediate reward.

1. Review: SARSA algorithm

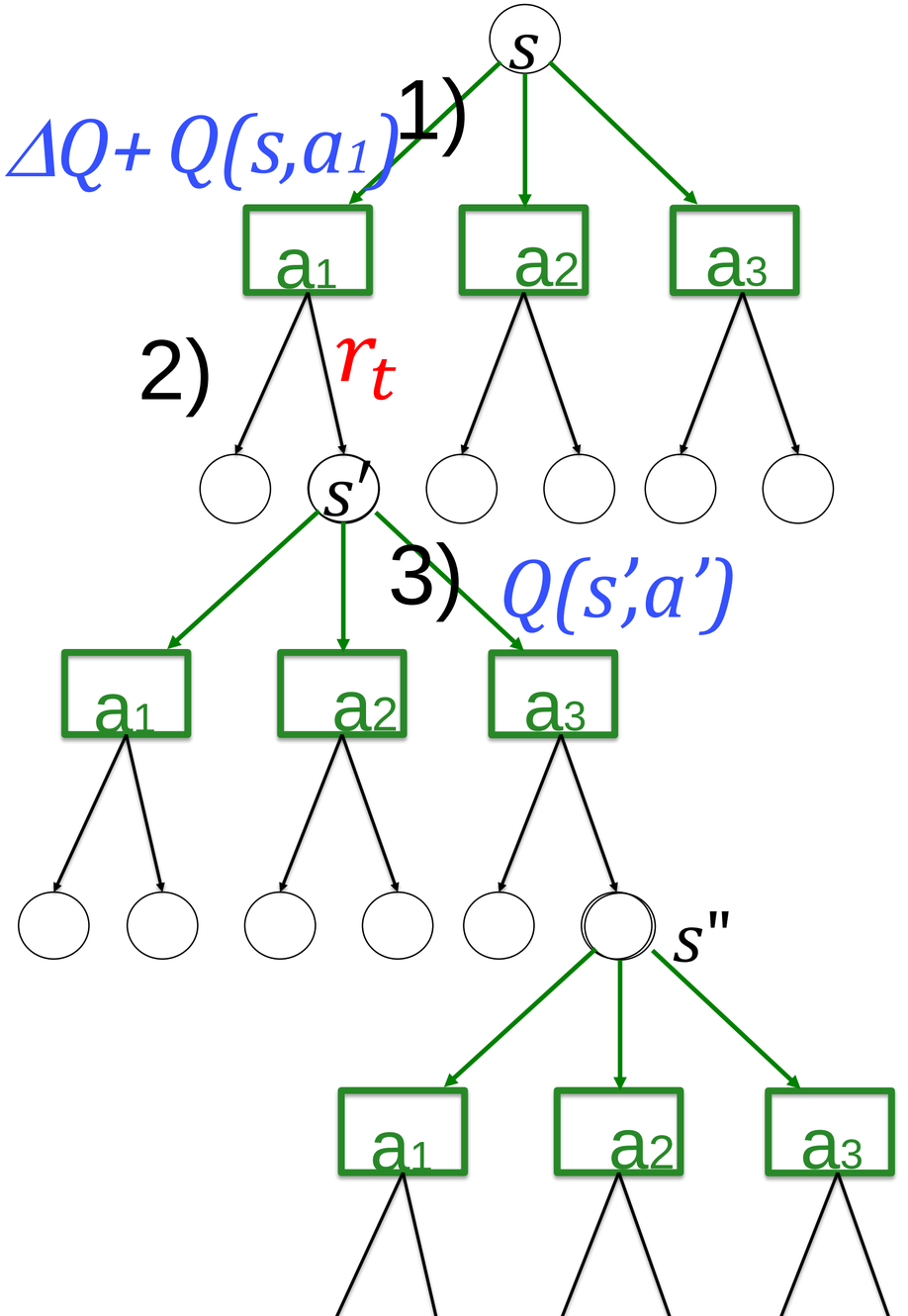
Initialise Q values
Start from initial state **s**

1) being in state **s**
   choose action **a**
   [according to policy $\pi(s,a)$]
2) Observe reward **r**
   and next state **s'**
3) Choose action **a'** in state s'
   [according to policy $\pi(s,a)$]
4) Update with SARSA update rule

$$\Delta Q(s,a) = \eta\,[r_t + \gamma Q(s',a') - Q(s,a)]$$

5) set: s ← s'; a ← a'
6) Goto 1)

Stop when all Q-values have converged



(previous slide)
The SARSA update in step 4 implements the idea that the immediate reward must account for the difference in Q-values between neighboring states.

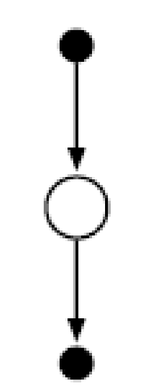
Blackboard 1: Backup diagram

(previous slide)
The backup diagram describes how many states and actions the algorithm has to keep in memory so as to enable the next update step.

1. Review: SARSA algorithm and Backup Diagram

Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Initialize $Q(s, a)$, for all $s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Choose A from S using policy derived from Q (e.g., ϵ-greedy)
  Repeat (for each step of episode):
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ϵ-greedy)
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma Q(S', A') - Q(S, A)\big]$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal



Sarsa

pick next action a' before you update

Sutton and Barto, Ch. 6.4

(previous slide)
In SARSA, we can update Q(s,a), once we have seen the next state s’ and the next action a’. In other words, the current action is a’ and we had to keep the most recent state s’ and the earlier action a in memory.

Note: one could argue that you also need to keep the earlier state s in memory because you update Q(s,a) and not Q(a); therefore you need to know s! -- But Sutton and Barto use a slightly different convention that we follow here.

The backup diagrams play a role in the following for the analysis of other algorithms.

Artificial Neural Networks: Lecture 9

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Variants of TD-learning methods and continuous space

1. Review and introduction of BackUp diagrams
2. Variations of SARSA

(previous slide)
SARSA is one example of a whole family of algorithms that all look very similar.

2. Expected SARSA

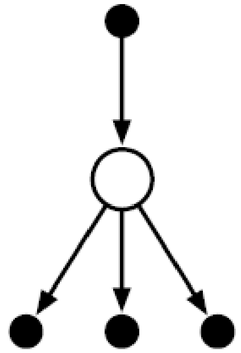
Expected SARSA for estimating $Q \approx q_*$

Initialize $Q(s,a)$, for all $s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Choose A from S using policy derived from Q (e.g., ϵ-greedy)
  Repeat (for each step of episode):
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ϵ-greedy)
    $Q(S,A) \leftarrow Q(S,A) + \alpha\left[R_{t+1} + \gamma \sum_a \pi(a|S_{t+1}) Q(S_{t+1}, a) - Q(S_t, A_t)\right]$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal

action

state

action



Expected Sarsa

Sutton and Barto, Ch. 6.6

(previous slide)
The first variant is ‘Expected SARSA’.
In standard SARSA, we pick the next action a’ and actually take it, before the updata of Q(s,a) is done.
In expected SARSA we do not yet take the next action but average over all possible next action with a weight given by the policy π.

2. Bellman equation

$$Q(s,a) = \sum_{s'} P^{a}_{s\to s'} \left[R^{a}_{s\to s'} + \gamma \sum_{a'} \pi(s',a') Q(s',a') \right]$$

Bellman equation = value consistency of neighboring states

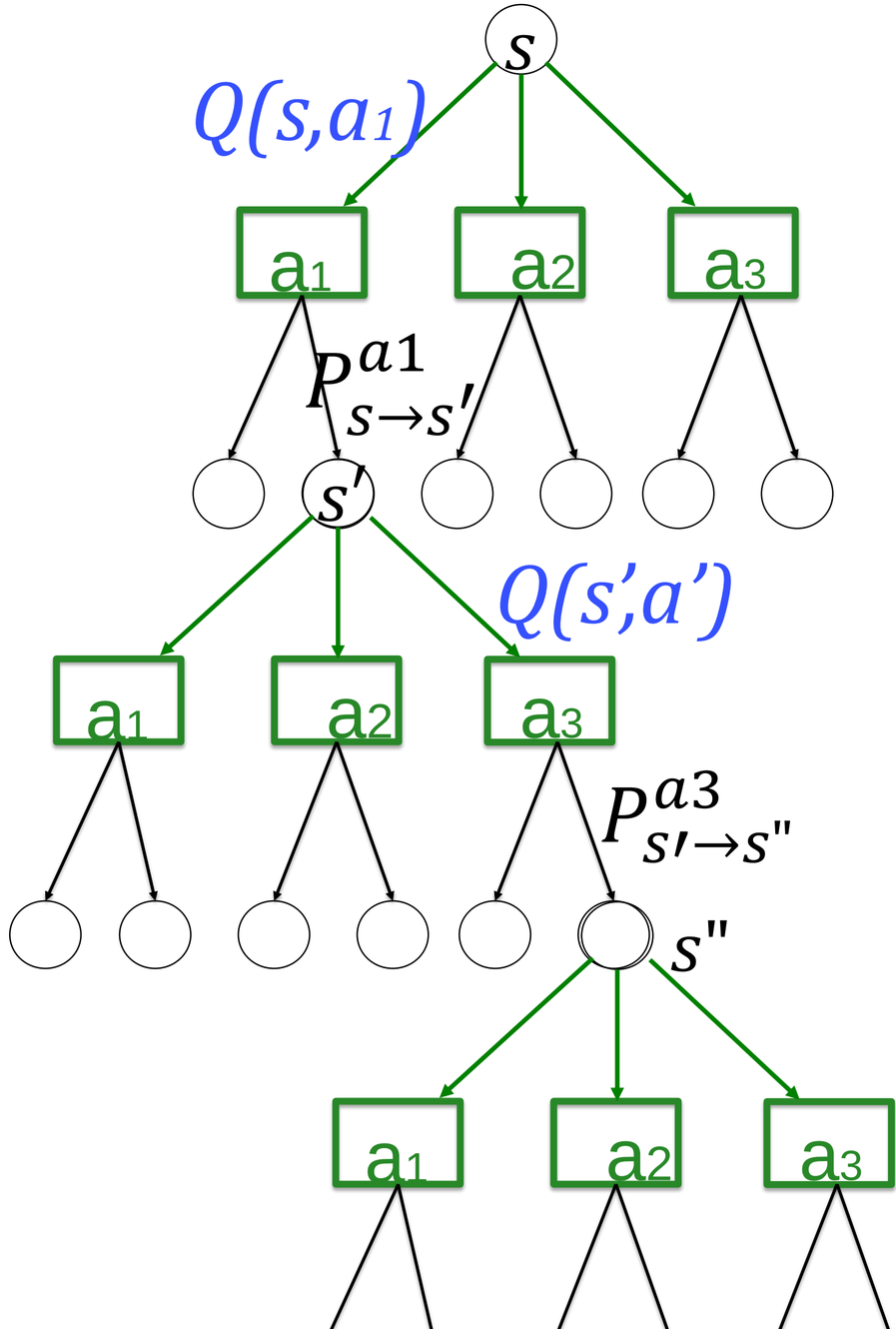
Remark:

Sometimes Bellman equation is written for greedy policy:

$$\pi(s,a) = \delta_{a,a*}$$

with action

$$a^* = \max_{a'} Q(s,a')$$

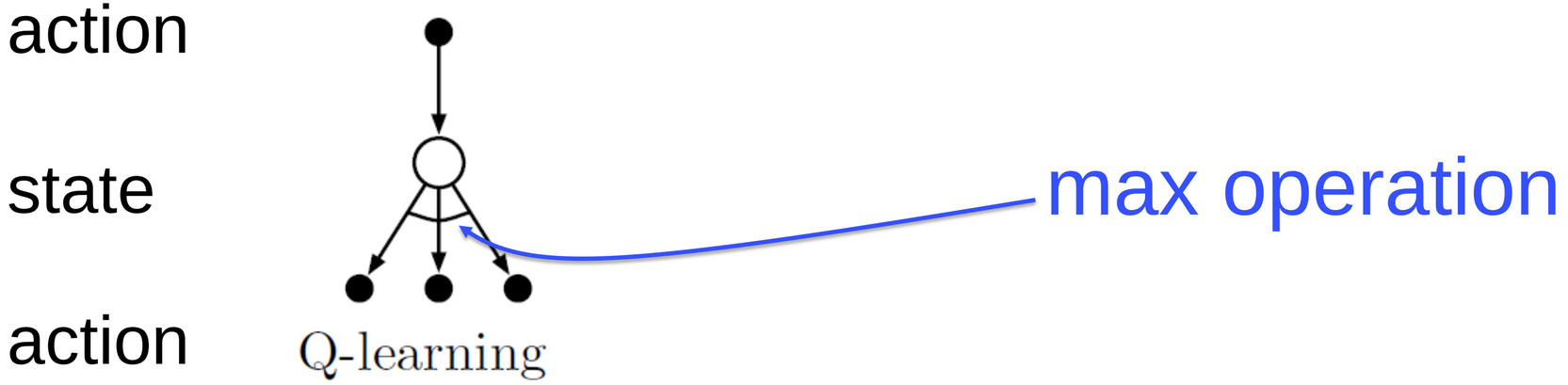


(previous slide)
The next variant is Q-learning.
Q-learning uses not an average with the current policy, but performs the averaging with the best policy, i.e., the greedy policy.
The idea is that you run a policy that includes exploration. However, since you know that after learning you will use the greedy policy so as to maximize your returns, you already update the Q-values according the greedy policy.

2. Q-Learning algorithm

Q-learning (off-policy TD control) for estimating π

Initialize $Q(s,a)$, for all $s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Repeat (for each step of episode):
    Choose A from S using policy derived from Q (e.g., ϵ-greedy)
    Take action A, observe R, S'
    $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$
    $S \leftarrow S'$
  until S is terminal



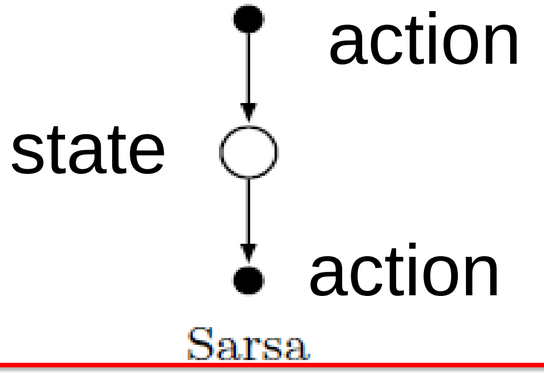
Sutton and Barto, Ch. 6.5

(previous slide)
Q-learning is called ‘off-policy’ because you update as if you used a greedy policy whereas during learning you are really running a different policy: it is as if you turn-off the current policy during the update.

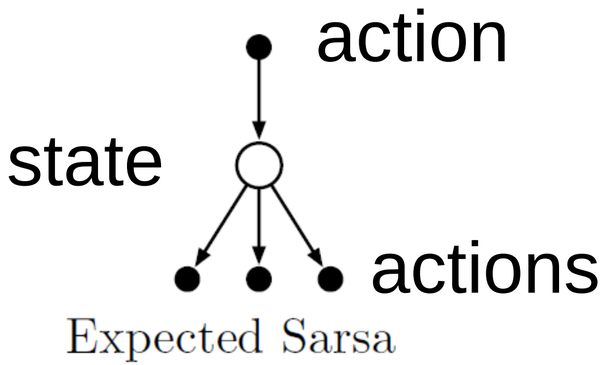
In Q-learning the update step is such that the current reward should explain the difference between Q(s,a) and the **maximum** Q(s’,a’) running over all possible actions a’. It is a TD algorithm (Temporal Difference), because neighboring states are visited one after the other. Hence neighbors are one time step away.

It does not play a role which action a’ you actually choose (according the your current policy). The max-operation is indicated in the back-up diagram by the little arc.

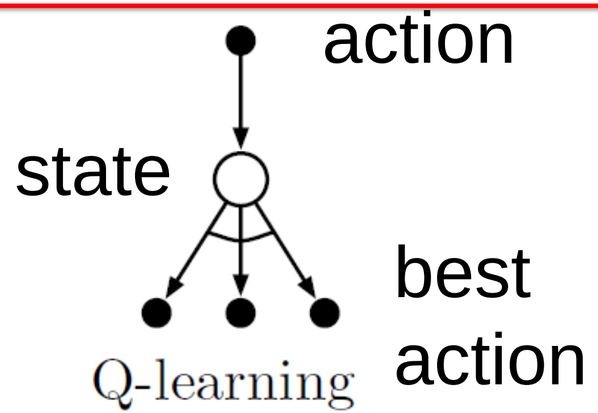
2. SARSA and related algorithms



SARSA: you actual perform **next** action,
and then you update Q(s,a)



Exp. SARSA: you look ahead and average over **potential next** actions to update Q(s,a)
and then you update Q(s,a)



Q-learning: you look ahead and **imagine greedy next** action to update Q(s,a)
(but you perform the actual next action based on your current policy)

(previous slide)
Summary of the three variations of SARSA and their back-up diagrams.

Artificial Neural Networks: Lecture 9

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Variants of TD-learning methods and continuous space

1. Review
2. Variations of SARSA
3. TD – learning (Temporal Difference)

(previous slide)
We now explore other Temporal Difference algorithms

3. TD-learning as bootstrap estimation

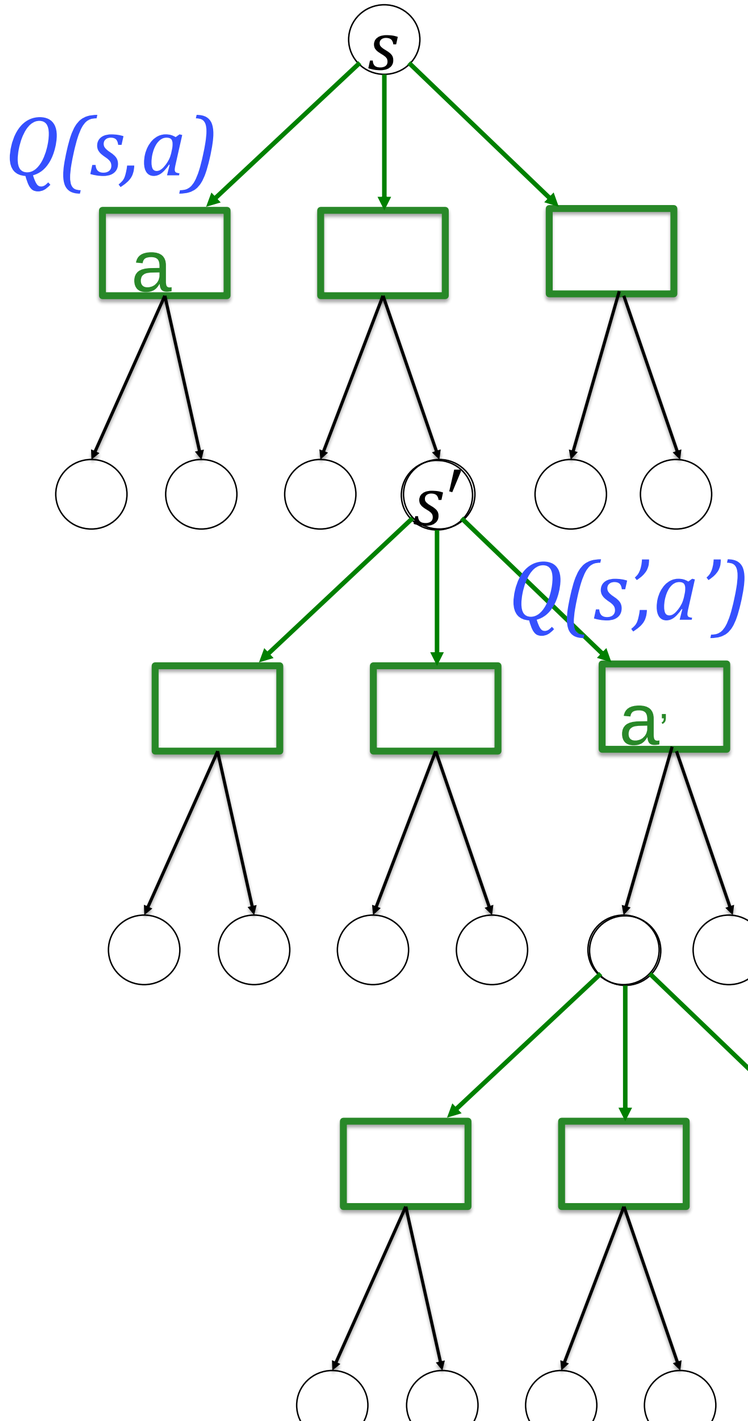
'bootstrap': summary of previous information

Temporal Difference

$$Q(s,a) = \sum_{s'} P^{a}_{s \to s'} \left[R^{a}_{s \to s'} + \gamma \sum_{a'} \pi(s',a') Q(s',a') \right]$$

Bellman equation = value consistency of neighboring states

Neighboring states → neighboring time steps



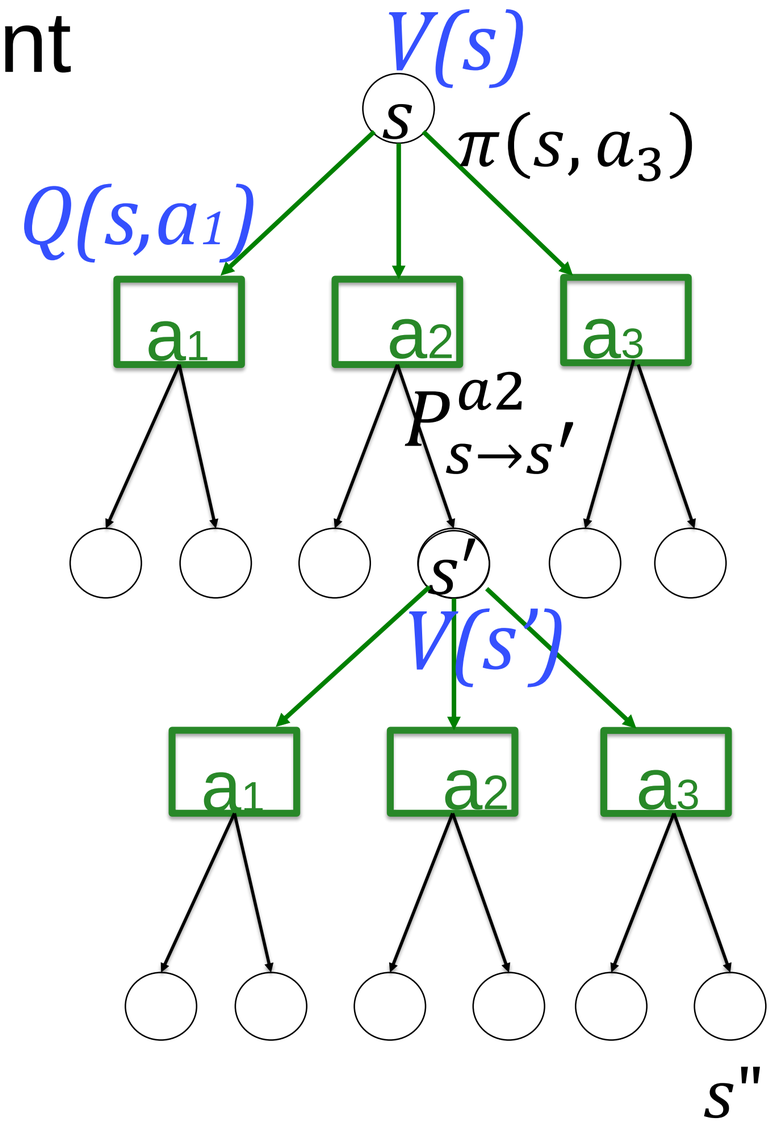
(previous slide)
1) As mentioned before:
If the agent runs through the state-action graph, neighboring states are one time step away from each other. This explains the term ‘Temporal Difference (TD)’

2) As mentioned before:
The Q-value Q(s,a) further up in the graph is the expected total discounted reward – summed over all possible future actions and states.
It can be decomposed in an average over the **immediate** rewards, actions, and states, and the Q-values Q(s’,a’) of all possible next states. Since calculation of Q(s,a) relies on (earlier) calculation of Q(s’,a’), Sutton and Barto call this a ‘bootstrap’ algorithm.

3. State-values V

Value $V(s)$ of a state s
= total (discounted) expected reward the agent gets starting from state s

$$V(s) = \sum_a \pi(s,a) Q(s,a)$$



Bellman equation for *V(s)*

$$V(s) = \sum_a \pi(s,a) \sum_{s'} P^a_{s \to s'} [\, R^a_{s \to s'} + \gamma V(s')]$$

(previous slide)
Instead of working with Q-values, we can work with V-values that describe the value of a state (as opposed to the value of a state-action pair).

While each Q-value is associated with a state-action pair, V-values are the value of a state: V-values are defined as the expected total discounted reward that the agent will collect under policy π starting at that state.

The value of a state V(s) is the average over the Q-values Q(s,a) averaged over all possible actions that start from that state. The correct weighting factor for averaging is given by the policy π(s,a).

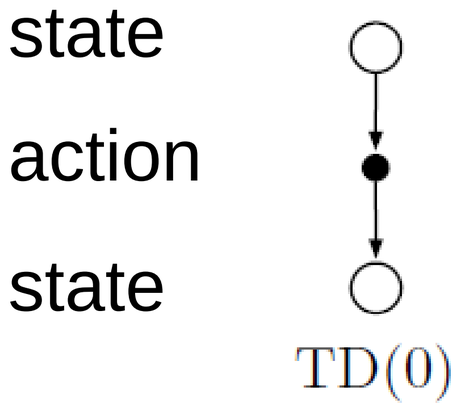
$$V(s) = \sum_{a} \pi(s,a) Q(s,a)$$

The resulting Bellman equation for V-values looks similar to that of Q-values, except that the location of the summation signs has been shifted.

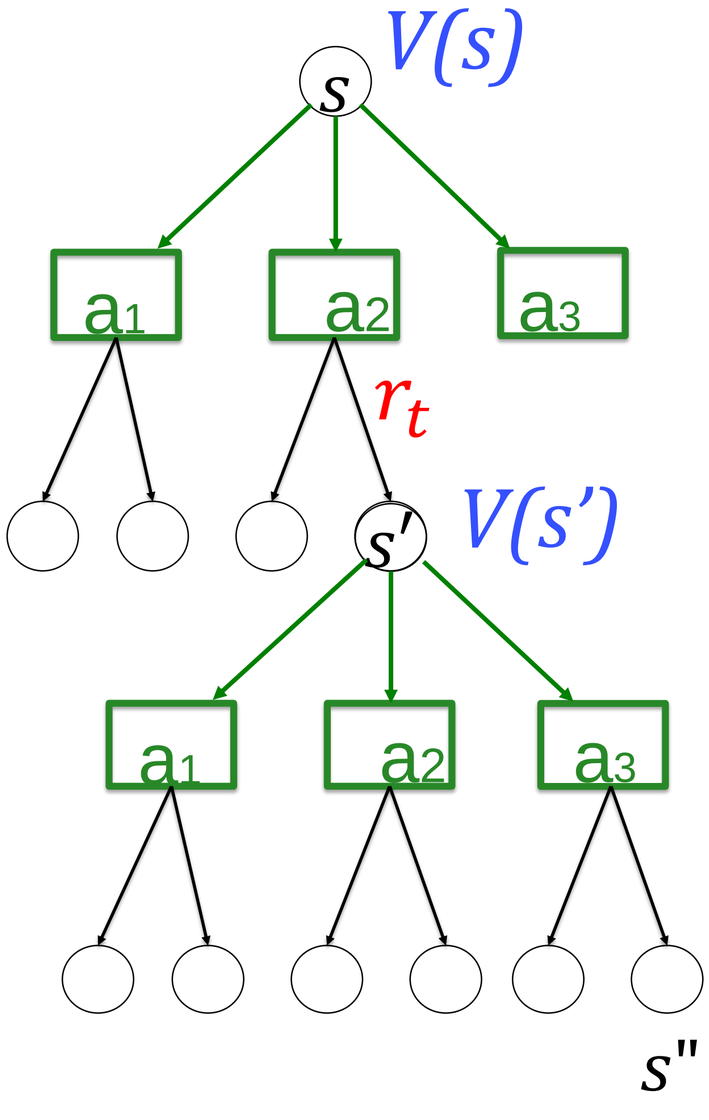
3. Standard TD-learning

Tabular TD(0) for estimating v_π

Input: the policy π to be evaluated
Initialize $V(s)$ arbitrarily (e.g., $V(s) = 0$, for all $s \in \mathcal{S}^+$)
Repeat (for each episode):
  Initialize S
  Repeat (for each step of episode):
    $A \leftarrow$ action given by π for S
    Take action A, observe R, S'
    $V(S) \leftarrow V(S) + \alpha\left[R + \gamma V(S') - V(S)\right]$
    $S \leftarrow S'$
  until S is terminal



$$\Delta V(s) = \eta\left[r_t + \gamma V(s') - V(s)\right]$$



(previous slide)
The iterative update for V-values is analogous to that of Q-values, but the back-up diagram looks different. Once the agent is in the next state s’, you can update the value V(s).
The resulting update rule is called TD learning.

Quiz: TD methods in Reinforcement Learning

[ ] SARSA is a TD method
[ ] expected SARSA is a TD method
[ ] Q-learning is a TD method
[ ] TD learning is an on-policy TD method
[ ] Q-learning is an on-policy TD method
[ ] SARSA is an on-policy TD method

(previous slide)
This quiz applies a few definitions to a few algorithms.

3. TD-learning as bootstrap estimation

$$Q(s,a) = \sum_{s'} P^{a}_{s \to s'} \left[R^{a}_{s \to s'} + \gamma \sum_{a'} \pi(s',a') Q(s',a') \right]$$

Bellman equation = value consistency of
neighboring states

Neighboring states → neighboring time steps

Temporal Difference Methods (TD methods)

- explore graph over time
- compare values (Q-values or V-values)
  at neighboring **time steps**
- 'bootstrap' estimation of values
- update after next time step, based on 'temporal difference'

(previous slide)
Summary – add your own comments. All terms should be clear by now.

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Variants of TD-learning methods and continuous space

1. Review
2. Variations of SARSA
3. TD – learning (Temporal Difference)
4. Monte-Carlo methods

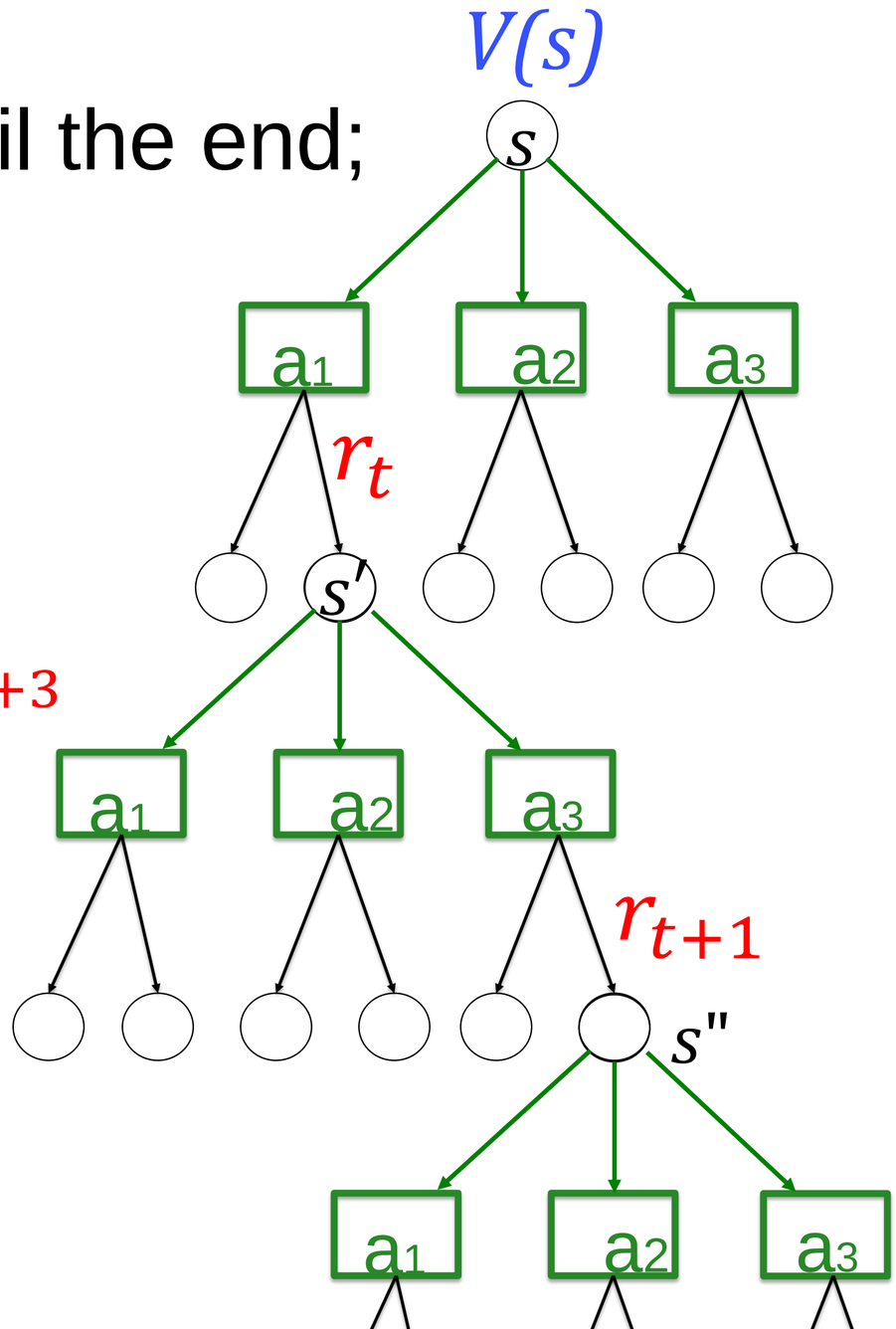
(previous slide)
Instead of using TD methods, the same state-action graph can also be explored with Monte-Carlo methods

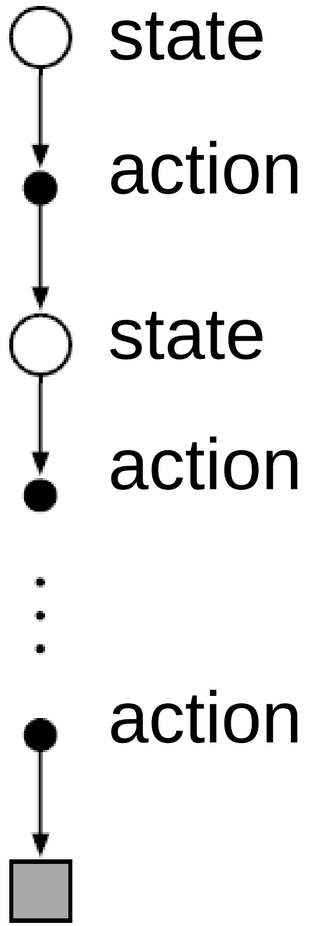
4. Monte-Carlo Estimation

play a trial (episode) until the end;

then update, using
the total accumulated
reward (='return') =

$r_t + \gamma\, r_{t+1} + \gamma^2 r_{t+2} + \gamma^3\, r_{t+3}$





end of trial

(previous slide)
1) Suppose you want to estimate the value V(s) of state s.
V(s) is the EXPECTED total discounted reward.

To estimate V(s) you start in state s, run until the end and evaluate for this single episode the return

$$Return\,(s) = r_t + \gamma\, r_{t+1} + \gamma^2 r_{t+2} + \gamma^3\, r_{t+3}$$

This is a single episode. If you start several times in s, you get a Monte-Carlo estimate of V(s).

2) You can be smart and you the SAME episode also to estimate the value V(s') of other states s'. Thus while you move along the graph, you open an estimation variable for each of the states that you visit.

Combining points 1) and 2) give the following algorithm.

4. Monte-Carlo Estimation of V-values

$$Return(s) = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \gamma^3 r_{t+3}$$

First-visit MC prediction, for estimating $V \approx v_\pi$

Initialize:

$\pi \leftarrow$ policy to be evaluated

$V \leftarrow$ an arbitrary state-value function

$Returns(s) \leftarrow$ an empty list, for all $s \in \mathcal{S}$

Repeat forever:

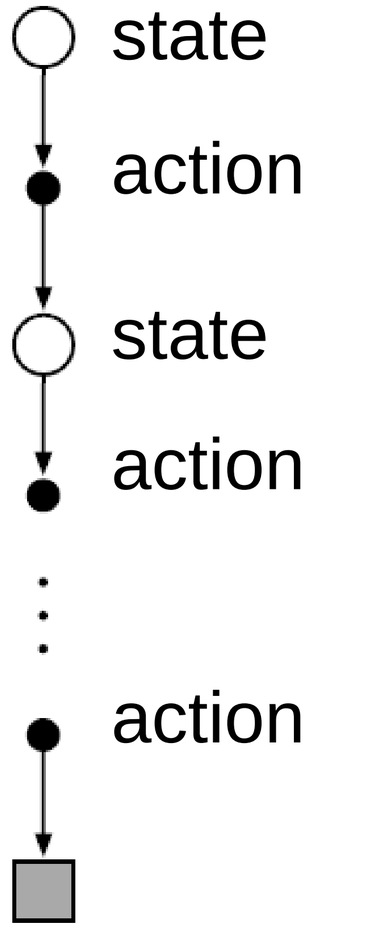
Generate an episode using π

For each state s appearing in the episode:

$G \leftarrow$ the return that follows the first occurrence of s

Append G to $Returns(s)$

$V(s) \leftarrow \text{average}(Returns(s))$



(previous slide)
In this (version of the) algorithm you first open estimators for all states.

For each state s that you encounter, you observe the (discounted) rewards that you accumulate until the end of the episode. The total accumulated discounted reward starting from s is the ‘Return(s)’

After many episode you estimate the V-values V(s) as the average over the Returns(s).

Note that the above estimations are done in parallel for all states s that you encounter on your path.

Also note that the Backup diagram is much deeper than that of Q-learning, since you always continue until the end of the trial before you can update Q-values of state-action pairs that have been encountered many steps before.

4. Monte-Carlo Estimation of Q-values (batch)

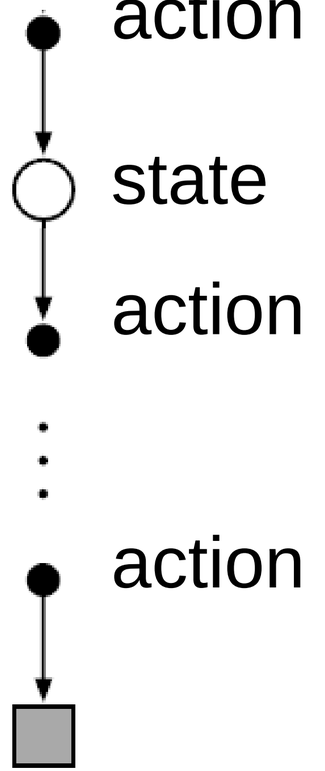
Start at a random state-action pair (s,a) (exploring starts)

$$Return(s,a) = r_t + \gamma\, r_{t+1} + \gamma^2 r_{t+2} + \gamma^3\, r_{t+3} + \ldots$$

Monte Carlo ES (Exploring Starts), for estimating $\pi \approx \pi_*$

Initialize, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$:
  $Q(s,a) \leftarrow$ arbitrary
  $\pi(s) \leftarrow$ arbitrary
  $Returns(s,a) \leftarrow$ empty list

Repeat forever:
  Choose $S_0 \in \mathcal{S}$ and $A_0 \in \mathcal{A}(S_0)$ s.t. all pairs have probability > 0
  Generate an episode starting from S_0, A_0, following π
  For each pair s,a appearing in the episode:
    $G \leftarrow$ the return that follows the first occurrence of s,a
    Append G to $Returns(s,a)$
    $Q(s,a) \leftarrow$ average$(Returns(s,a))$
  For each s in the episode:
    $\pi(s) \leftarrow \arg\max_a Q(s,a)$



$$Q(s,a) = average[Return(s,a)]$$

Note: single episode also allows to update Q(s'a') of children

(previous slide)
The Monte-Carlo estimation of Q-values is completely analogous to that of V-values, except that you need to provide estimates for all state-action pairs.

We call this a Monte-Carlo Batch algorithm because at the end of several episodes you calculate the estimated Q-Values.

Note: Standard Q-learning or standard SARSA is an online algorithm since you make updates after every step of every episode (except the first 2 steps until you have filled up the back-up memory).

Oh, so many, many variants ….

Question:

We have three variants to estimate Q-values:

1) Q-learning (online, like in SARSA, but max operator)
2) Q-learning (batch) =bootstrap=Dyn. Programming
3) Monte-Carlo (Batch)

We have played N trials.
How do the three algorithms rank?

Which one is best? → commitment:
write down 1 or 2 or 3

(previous slide)
There are many variants of algorithms – but which one is the best?

In both **batch** algorithms you have to play several episodes before you do the update. The Bellman equation approach uses the idea of ‘bootstrapping’ whereas Monte-Carlo does not. The Bellman equation can be used by dynamic programming: starting from the bottom leaves of the graph (end of episodes).

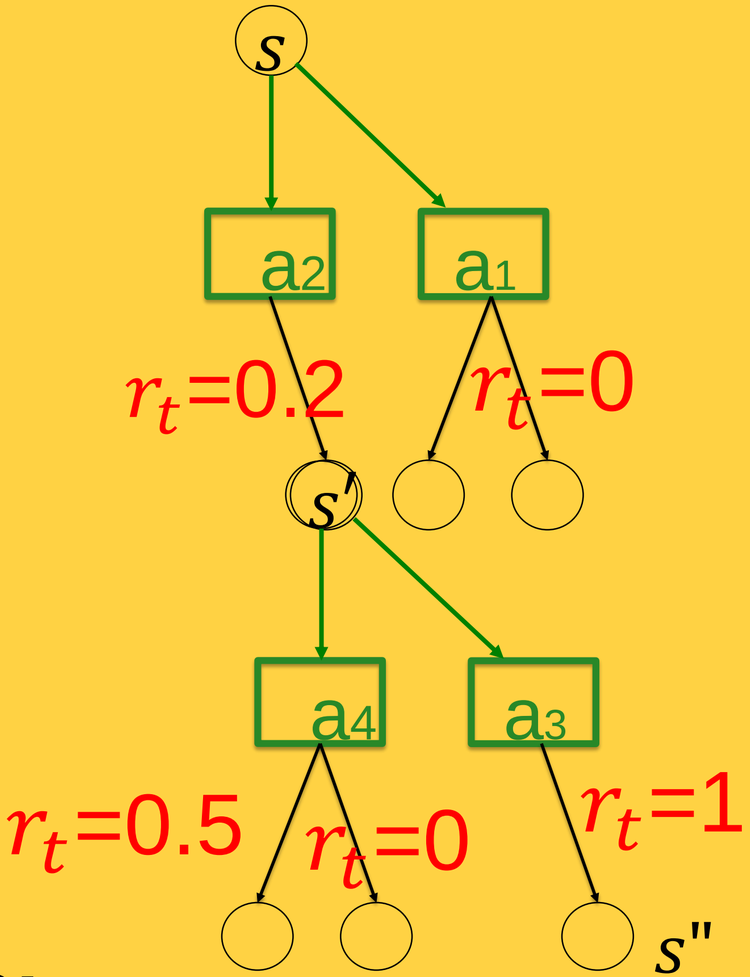
Q-learning or SARSA also use ‘bootstrapping’ since they update Q-values based on other Q-values. Q-learning has the max-operation, whereas SARSA is ‘on-policy’. Both Q-learning and SARSA are **Online**

To find out which one is best, consider the following example.

4. Monte-Carlo versus TD methods (Exercise 1, now, 8 min.)

Not discounted. 10 example episodes:

1: **s,a2**→ r=0.2,s',a4→ r=0
2: s',a3→ r=1
3: s',a4→ r=0
4: s',a3→ r=1
5: **s,a1** → r=0
6: s',a4→ r=0
7: s',a4→ r=0.5
8: s',a3→ r=1
9. **s,a2**→ r=0.2,s',a4→ r=0.5
10. **s,a1** → r=0



Batch update of Q(s,a) after all 10 trials:

(i) Monte-Carlo: average over total accumulated reward for given (a,s)

(ii) Q-learning batch (with eta =1/number of examples) = Dynamic Prog.

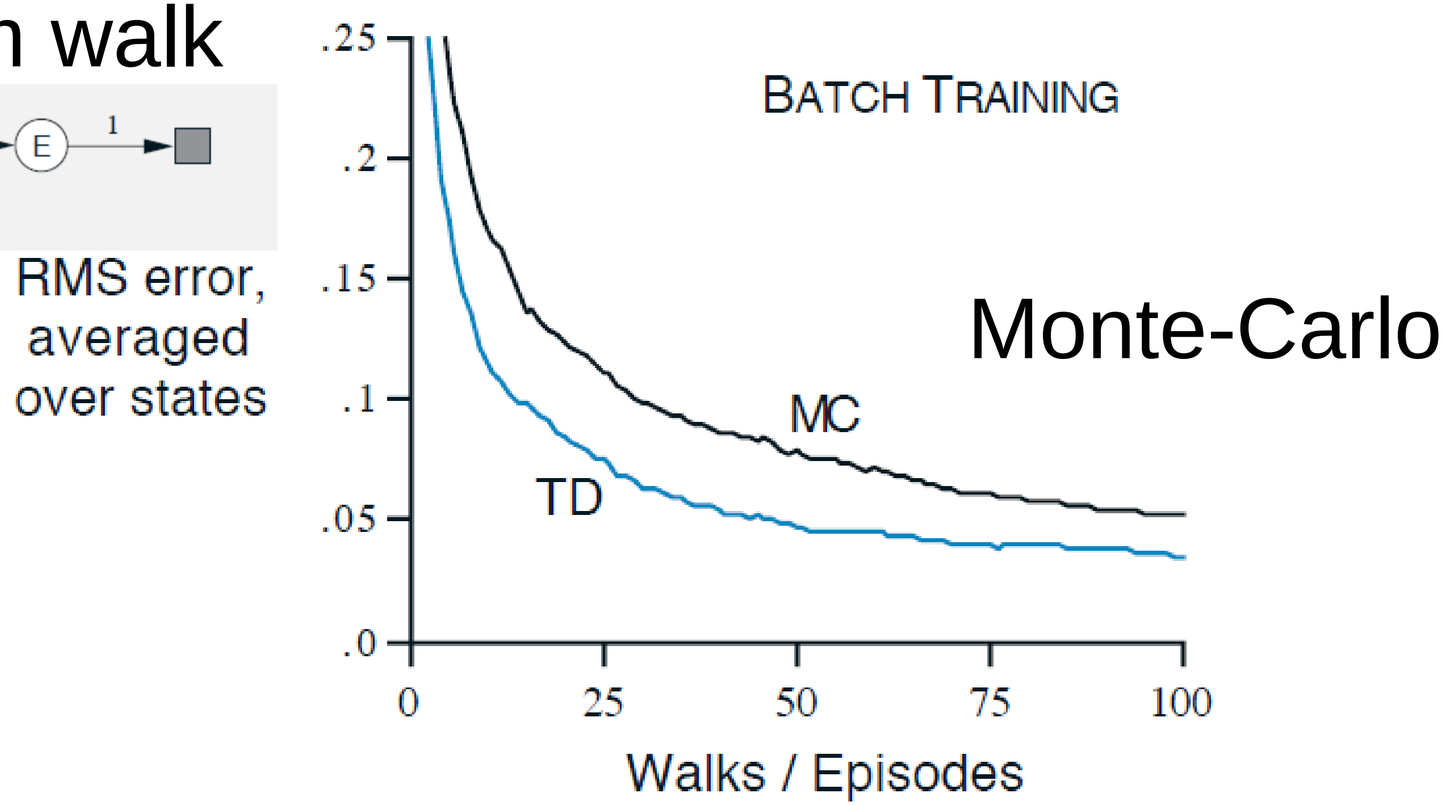
(previous slide)
Batch mode means that we update after having played all 10 trials (as opposed to normal Q-learning where you update while you run through each trial). Take the learning rate inversely proportional to the number of examples FOR THIS state-action pair.
Tip: For batch Q-learning start from the bottom of the graph.
Notes: set the discount factor to one; initialize Q-values with zero.

Space for your calculations.

4. Monte-Carlo versus TD methods:

Comparison in **batch mode**: We have observed N episodes, and update (once) after these N episodes.

Example: 1d random walk



Monte-Carlo

Conclusion:
TD is better than Monte Carlo

Figure 6.2: Performance of TD(0) and constant-α MC under batch training on the random walk task.■

(previous slide) All episodes start in the center state, C, then proceed either left or right by one state on each step, with equal probability (random walk). Episodes terminate either on the extreme left (reward zero) or the extreme right, (reward 1); all other rewards are zero.

Because we do not discount future rewards, the true value of each state can be calculated as, from A through E, 1/6; 2/6; 3/6; 4/6; 5/6.

The root-mean-square error (RMS) compares the estimated value with the above ‘true’ values.
We see that TD performs better than MC in this case.

Sutton and Barto, 2018

4. Monte-Carlo versus TD methods:

TD is better than Monte Carlo

The averaging step in TD methods ('bootstrap') is more efficient (compared to Monte Carlo methods) to propagate information back into the graph, since information from different starting states is combined and compressed in a Q-value or V-value.

→similar to Dynamic programming

(previous slide)
If we go back to the example: in Monte-Carlo methods you only exploit information of trials that go through the state-action pair (s,a) to evaluate Q(s,a); in TD methods (or the Bellman equation) you compare Q(s,a) with Q(s’,a’) and all trials that pass through (s’,a’) contribute to estimate Q(s’,a’) even those that have started somewhere else and have never passed through (s,a). Hence in the latter case you exploit more information.

4. Monte-Carlo Estimation of Q-values

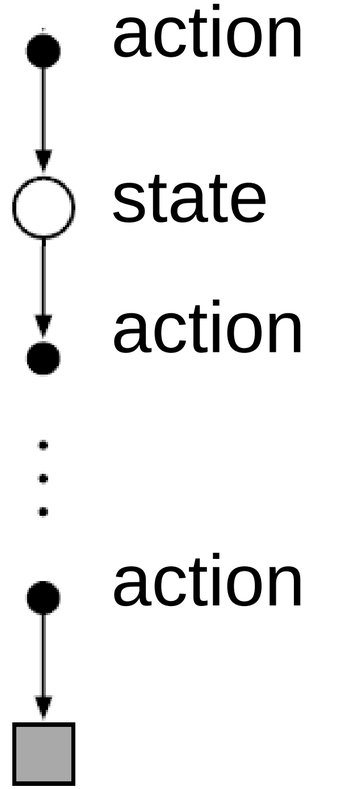
Combine epsilon-greedy policy with Monte-Carlo Q-estimates

On-policy first-visit MC control (for ε-soft policies), estimates $\pi \approx \pi_*$

Initialize, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$:
  $Q(s,a) \leftarrow$ arbitrary
  $Returns(s,a) \leftarrow$ empty list
  $\pi(a|s) \leftarrow$ an arbitrary ε-soft policy (e.g., epsilon-greedy)

Repeat forever:
  (a) Generate an episode using π
  (b) For each pair s, a appearing in the episode:
    $G \leftarrow$ the return that follows the first occurrence of s, a
    Append G to $Returns(s,a)$
    $Q(s,a) \leftarrow \text{average}(Returns(s,a))$
  (c) For each s in the episode:
    $A^* \leftarrow \arg\max_a Q(s,a)$ (with ties broken arbitrarily)
    For all $a \in \mathcal{A}(s)$:

$$\pi(a|s) \leftarrow \begin{cases} 1 - \varepsilon + \varepsilon/|\mathcal{A}(s)| & \text{if } a = A^* \\ \varepsilon/|\mathcal{A}(s)| & \text{if } a \neq A^* \end{cases}$$



(previous slide)
This algorithm combines Monte-Carlo estimates with an epsilon-greedy policy.
Note for Monte-Carlo estimates, the agent waits until the end of the episode (end of trial), before it can update the Q-values.

Similar to the earlier Monte-Carlo algorithms, the Q-values of all those state-action pairs that have been visited in that trial are updated (as opposed to an algorithm where you would only update Q(s0,a0) of the initial state and action.)

Quiz: Monte Carlo methods

We have a network with 1000 states and 4 action choices in each state. There is a single terminal state.
We do Monte-Carlo estimates of total return to estimate Q-values

Our episode starts with (s,a) that is 400 steps away from the terminal state. How many return $R(s,a)$ variables do I have to open in this episode?

[ ] one, i.e. the one for the starting configuration (s,a)
[ ] about 100 to 400
[ ] about 400 to 4000
[ ] potentially even more than 4000

(previous slide)
Your comments

Artificial Neural Networks: Lecture 9

Wulfram Gerstner

EPFL, Lausanne, Switzerland

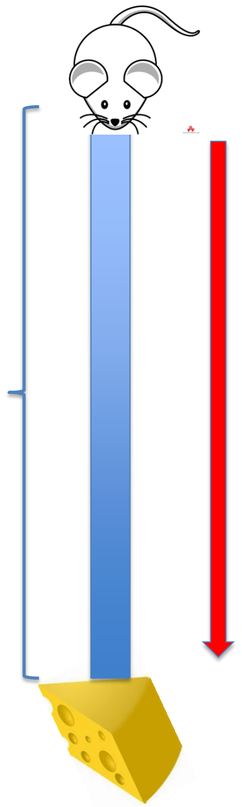
Variants of TD-learning methods and continuous space

1. Review
2. Variations of SARSA
3. TD – learning (Temporal Difference)
4. Monte-Carlo methods
5. Eligibility traces and n-step methods

(previous slide)
So far we have worked with discrete states.

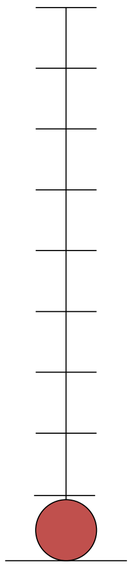
Exercise from last week: one-dimensional track

top view

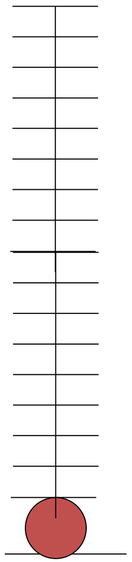


2m

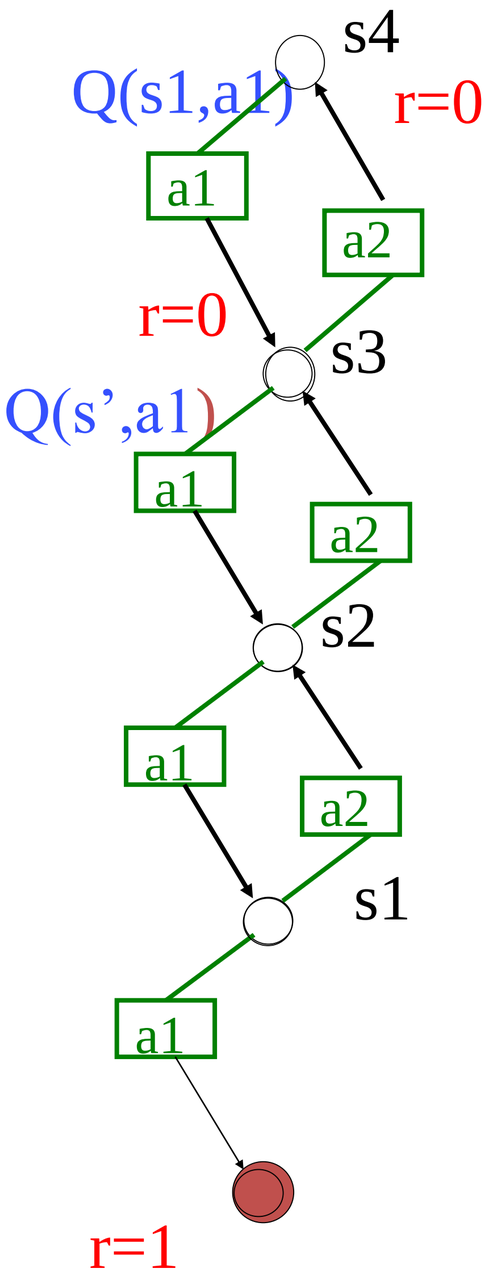
Discretize state



goal



goal



(previous slide)
However, if you think of an animal that walks along a corridor towards a piece of cheese (reward), then the natural space is continuous and any discretization is arbitrary. Why should we choose 10 states and not 20?

Once we are in the discrete space, the situation is similar to the random walk example considered earlier, except that here we are interested in an agent that adapts its policy so that it walks as quickly as possible to the reward.

Exercise from last week: one-dimensional track

- **Update of Q values in SARSA**

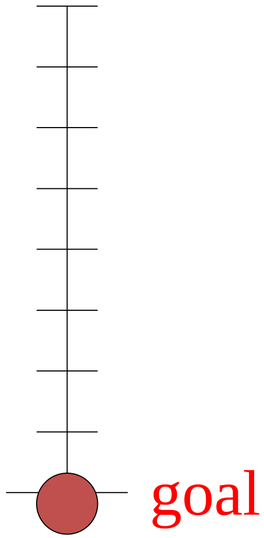
$$\Delta Q(s,a) = \eta \; [r - (Q(s,a) - Q(s',a'))]$$

- **policy for action choice:**

  Pick most often action

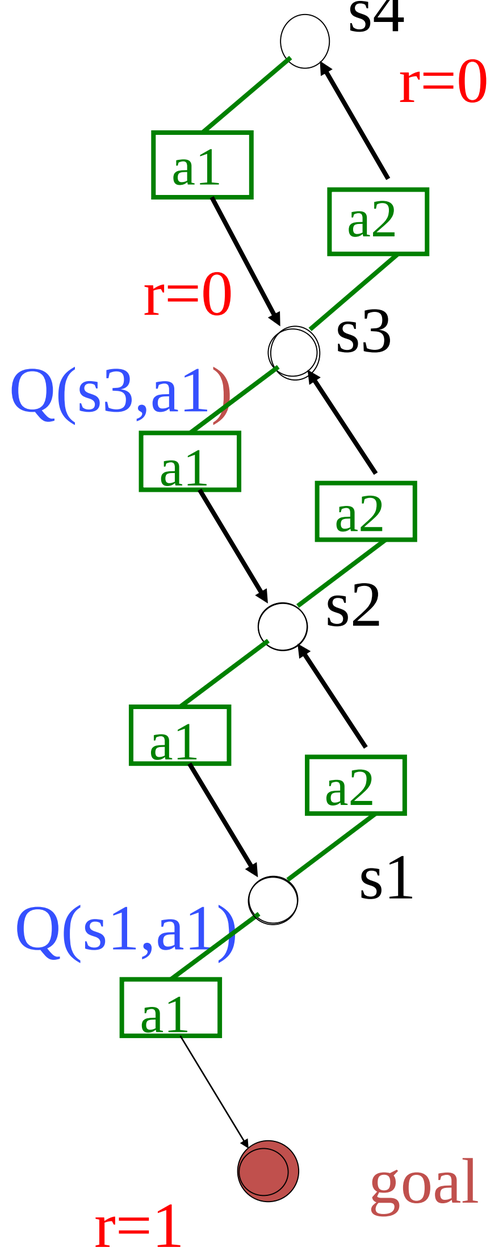
$$a_t^* = \arg\max_a Q_a(s,a)$$

Linear sequence of states.
Reward only at goal.
Actions are up or down.
Initialise Q values at 0. Start trials at top.



[ ] After 2 trials the Q-value Q(s1,a1)>0

[ ] After 2 trials the Q-value Q(s3,a1)>0



(previous slide)
Your comments. See also the solution of exercise from Lecture8-RL1.

5. Problem of TD algorithms

Problem:

-‘Flow of information’ back from target is slow

- information flows 1 step per complete trial

- 20 trials needed to get information 20 steps away from target

BUT:

- the discretization of states has been an arbitrary choice!!!

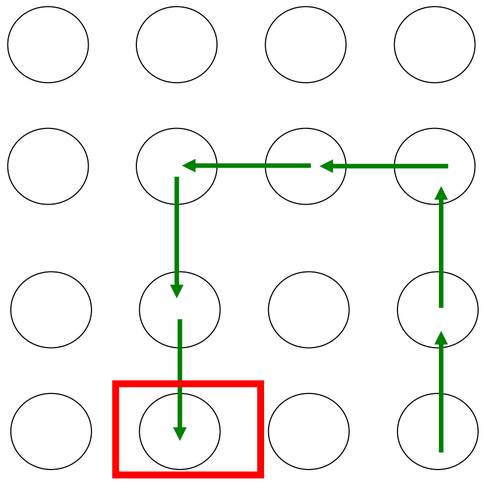
→ Something is wrong with the discrete-state SARSA algo

(previous slide)
In the SARSA algorithm and all other TD learning algorithms that we have seen so far, information about a reward at the target needs several trials before it shows up in the Q-values (or V-values) that are not close to the target.
In fact, if all Q-values are initialized at zero, it takes 10 trials before the Q-value of a state that is 10 steps away from the target is updated the first time.

So if we decide to discretize 1m of corridor into 20 states (instead of 10 states), then it will take 20 trials for the information to arrive at the start.

This is strange, because the performance of the agent (an animal!) should not depend on the discretization scheme that we have chosen.

5. Solution 1: Eligibility Traces

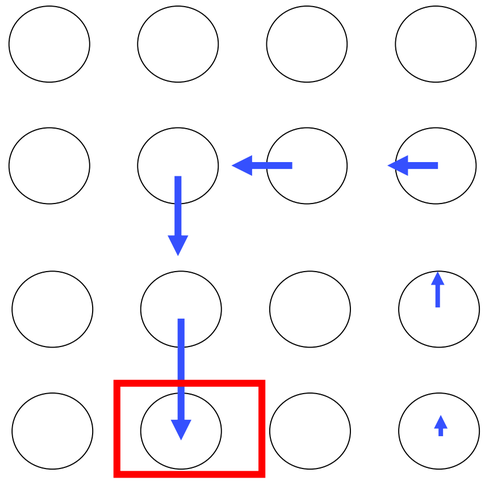


Idea:

- keep memory of previous state-action pairs
- memory decays over time
- Update an eligibility trace for state-action pair

$e(s,a) \leftarrow \lambda\, e(s,a)$ decay of **all** traces

$e(s,a) \leftarrow e(s,a) + 1$ if action a chosen in state s



- update **all** Q-values:

$\Delta Q(s,a) = \eta\ [r - (Q(s,a) - Q(s',a'))]\ e(s,a)$

⟶ SARSA(λ)

Note: lambda=0 gives standard SARSA

(previous slide)
Eligibility traces are a first solution to the above problem:

For each state-action pair we introduce a variable e(s,a), called eligibility trace. The eligibility trace is increase by one, if the corresponding state-action pair occurs. In each time step, all eligibility traces decrease by a factor λ <1.

In each time step, all Q-values Q(s,a) are update proportional to the TD error delta and proportional to the corresponding eligibility trace e(s,a).

Note: in the original SARSA algorithm we have for each state-action pair a variable Q(s,a). In the new algorithm, we have for each state-action pair two variables: Q(s,a) and e(s,a). I will sometimes call e(s,a) the ‘shadow’ variables: each the eligibility trace is the shadow of the corresponding Q-value.

5. Solution 1: Eligibility Traces

7.5 *Sarsa(λ)*

Initialize $Q(s, a)$ arbitrarily

Repeat (for each episode):

Initialize s, a and set e(s,a)=0 for all actions a and states s

Repeat (for each step of episode):

Take action a, observe r, s'

Choose a' from s' using policy derived from Q (e.g., ϵ-greedy)

$\delta \leftarrow r + \gamma Q(s', a') - Q(s, a)$

$e(s, a) \leftarrow e(s, a) + 1$

For all s, a:

$Q(s, a) \leftarrow Q(s, a) + \alpha \delta e(s, a)$

$e(s, a) \leftarrow \gamma \lambda e(s, a)$

$s \leftarrow s'; a \leftarrow a'$

until s is terminal

Figure 7.11 Tabular Sarsa(λ).

From: Reinforcement Learning,
Sutton and Barto 1998
First edition

(previous slide)
Note: in some published versions of the algorithm the decay of the eligibility traces is the product of γ and λ, and not just λ.

5. Quiz: Eligibility Traces

[ ] Eligibility traces keep information of past state-action pairs.

[ ] For each Q-value $Q(s,a)$, the algorithm keeps one eligibility trace e(s,a), i.e., if we have 200 Q-values we need 200 eligibility traces

[ ] Eligibility traces enable information to travel rapidly backwards into the graph

[ ] The update of $Q(s,a)$ is proportional to $[r-(Q(s,a)-Q(s',a'))]$

[ ] In each time step all Q-values are updated

(previous slide)
Your comments

5. Problem of TD algorithms

Problem:

-‘Flow of information’ back from target is slow

- information flows 1 step per complete trial

- 20 trials needed to get information 20 steps away from target

→ First solution: eligibility traces.

(previous slide)
Eligibility traces make the flow of information from the target back into the graph more rapid. The speed of flow is now controlled by the decay constant λ of the eligibility trace – therefore we can keep the flow constant even if the discretization changes by readjusting λ.

However, there is also a second solution, called n-step SARSA.

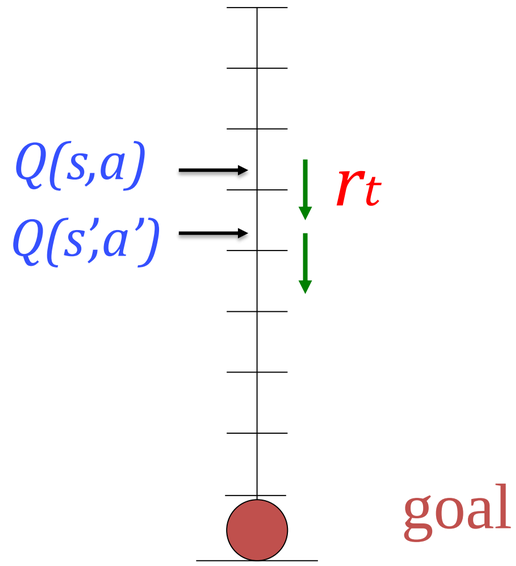
5. Solution 2: n=step SARSA

Standard SARSA

$\Delta Q(s,a) = \eta\ [r - (Q(s,a) - \gamma Q(s',a'))]$

$\Delta Q(s_t,a_t) = \eta\ [r_t - (Q(s_t,a_t) - \gamma Q(s_{t+1},a_{t+1}))]$

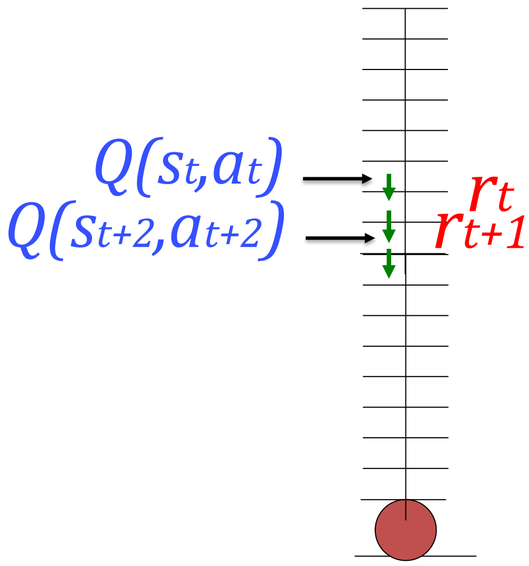
Temporal Difference (TD)



2-step SARSA

$\Delta Q(s_t,a_t) = \eta\ [r_t + \gamma r_{t+1} - (Q(s_t,a_t) - \gamma \gamma Q(s_{t+2},a_{t+2}))]$

2-step TD



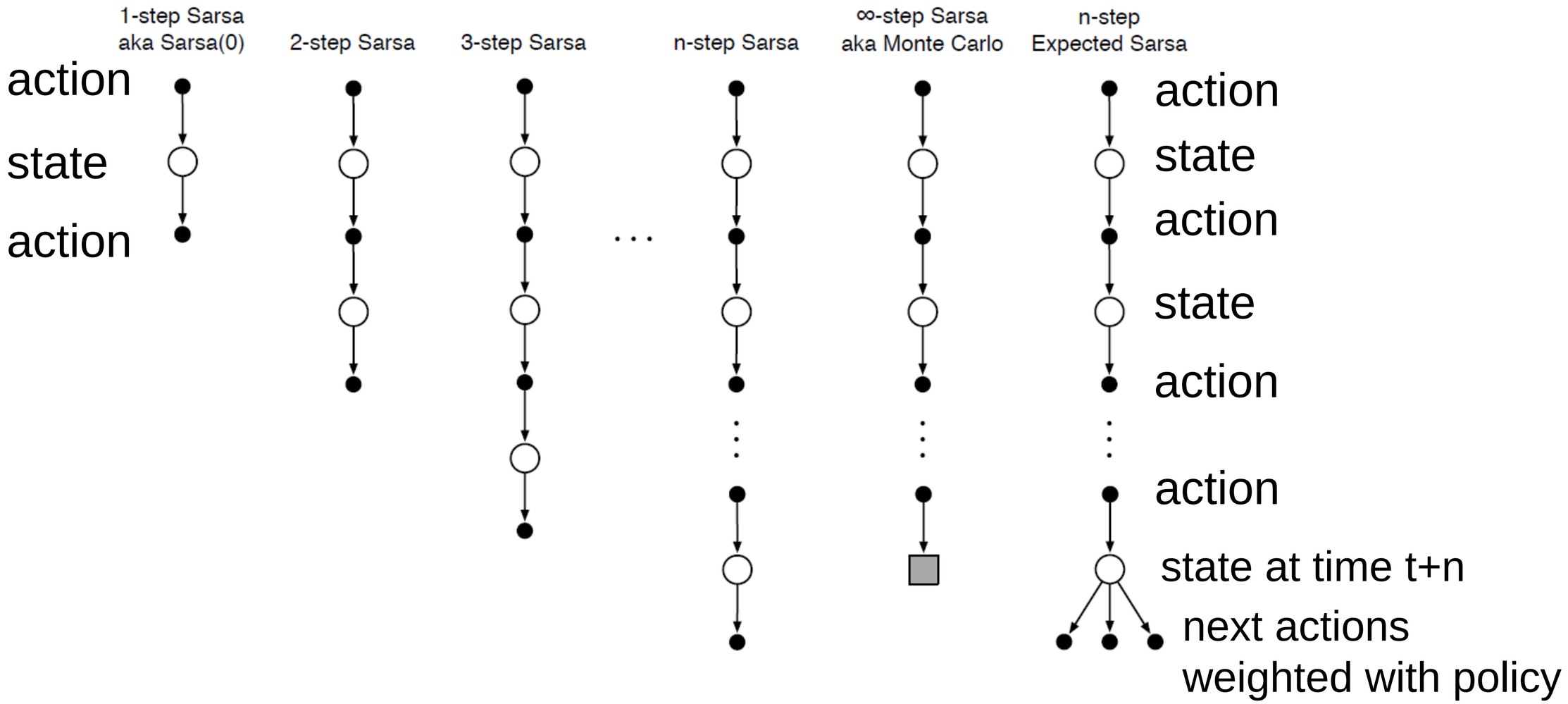
(previous slide)
Reminder:
SARSA and other standard TD methods compare the reward with **neighboring** Q-values.

In two step SARSA, we compare the two-step reward with the difference in Q-values of next-nearest neighbors.

In other words, the sum of the two rewards between s_t and s_{t+2} must be explained by the difference between the Q-values $Q(s_t,a_t)$ and (discounted) $Q(s_{t+2},a_{t+2})$.

The greek symbol γ denotes the discount factor, as before.

5. n-step SARSA and n-step expected SARSA



(previous slide)
The idea of 2-step SARSA can be extended to an arbitrary n-step SARSA.
Interestingly, if the number n of steps equals the total number of steps to the end of the trial, we are back to standard Monto-Carlo estimation.

Hence, n-step SARSA is in the middle between normal SARSA and Monte-Carlo estimation.

5. n-step SARSA algorithm

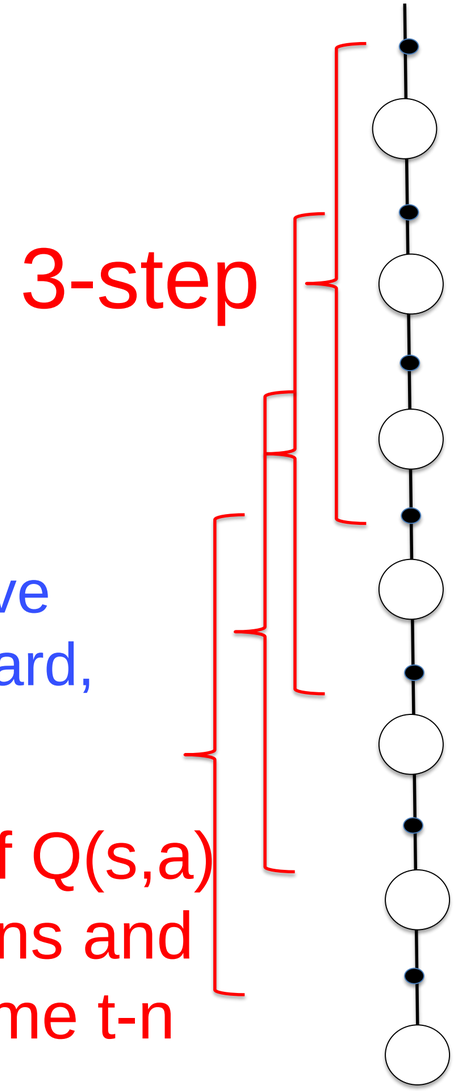
n-step Sarsa for estimating $Q \approx q_*$, or $Q \approx q_\pi$ for a given π

Initialize $Q(s,a)$ arbitrarily, for all $s \in \mathcal{S}, a \in \mathcal{A}$
Initialize π to be ε-greedy with respect to Q, or to a fixed given policy
Parameters: step size $\alpha \in (0,1]$, small $\varepsilon > 0$, a positive integer n
All store and access operations (for S_t, A_t, and R_t) can take their index mod n

Repeat (for each episode):
  Initialize and store $S_0 \neq$ terminal
  Select and store an action $A_0 \sim \pi(\cdot|S_0)$
  $T \leftarrow \infty$
  For $t = 0, 1, 2, \ldots$:
    If $t < T$, then:
      Take action A_t
      Observe and store the next reward as R_{t+1} and the next state as S_{t+1}
      If S_{t+1} is terminal, then:
        $T \leftarrow t+1$
      else:
        Select and store an action $A_{t+1} \sim \pi(\cdot|S_{t+1})$
    $\tau \leftarrow t - n + 1$ (τ is the time whose estimate is being updated)
    If $\tau \geq 0$:
    (1) $G \leftarrow \sum_{i=\tau+1}^{\min(\tau+n,T)} \gamma^{i-\tau-1} R_i$
    (2) If $\tau + n < T$, then $G \leftarrow G + \gamma^n Q(S_{\tau+n}, A_{\tau+n})$ $(G_{\tau:\tau+n})$
    (3) $Q(S_\tau, A_\tau) \leftarrow Q(S_\tau, A_\tau) + \alpha\,[G - Q(S_\tau, A_\tau)]$
      If π is being learned, then ensure that $\pi(\cdot|S_\tau)$ is ε-greedy wrt Q
  Until $\tau = T - 1$

Take action, observe next state and reward, choose next action

update of Q(s,a) with actions and state at time t-n



Sutton and Barto, Ch. 7.2

(previous slide)
The backup graph for three-step SARSA now contains 3 state-action pairs, because we need to keep more information in memory.

Note that we can update $Q(s_t, a_t)$ once we have chosen action a_{t+3} in state S_{t+3}

Lines marked (1), (2), (3).

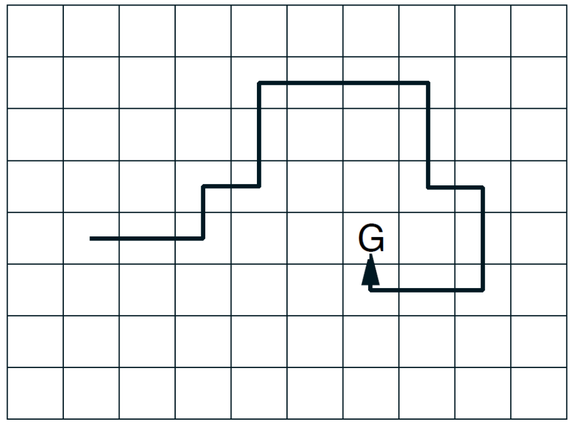
(1) G is the reward summed over n steps (with discounts for steps >1)

(2) To this G the Q-value of the *n*th state is added (unless the episode terminates before)

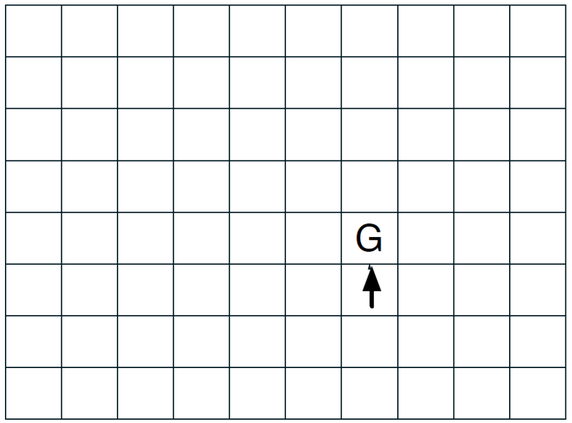
(3) The update then happens with this new G as a target and learning rate alpha.

5. Example: 10-step SARSA

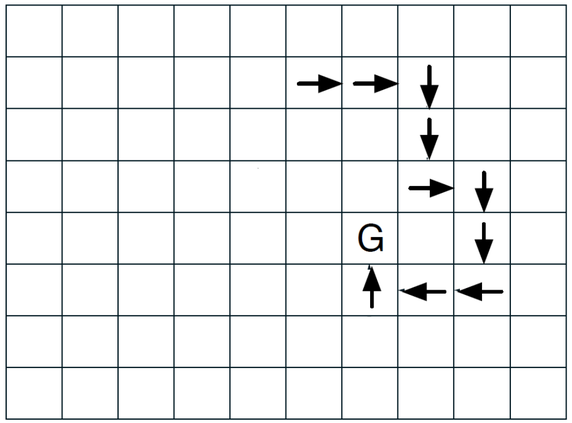
Path taken



Action values increased by one-step Sarsa



Action values increased by 10-step Sarsa



(previous slide).
The graphic suggests that the results of 10-step SARSA are very similar to an eligibility trace – which is indeed the case. therefore the two solutions (eligibility trace and n-step TD learning) are in fact closely related.

We will come back to this issue in lectures 11 and 12 on reinforcement learning.

5. Scaling Problem of TD algorithms

TD algorithms do not scale correctly if the discretization is changed

either

→ Introduce eligibility traces (temporal smoothing)

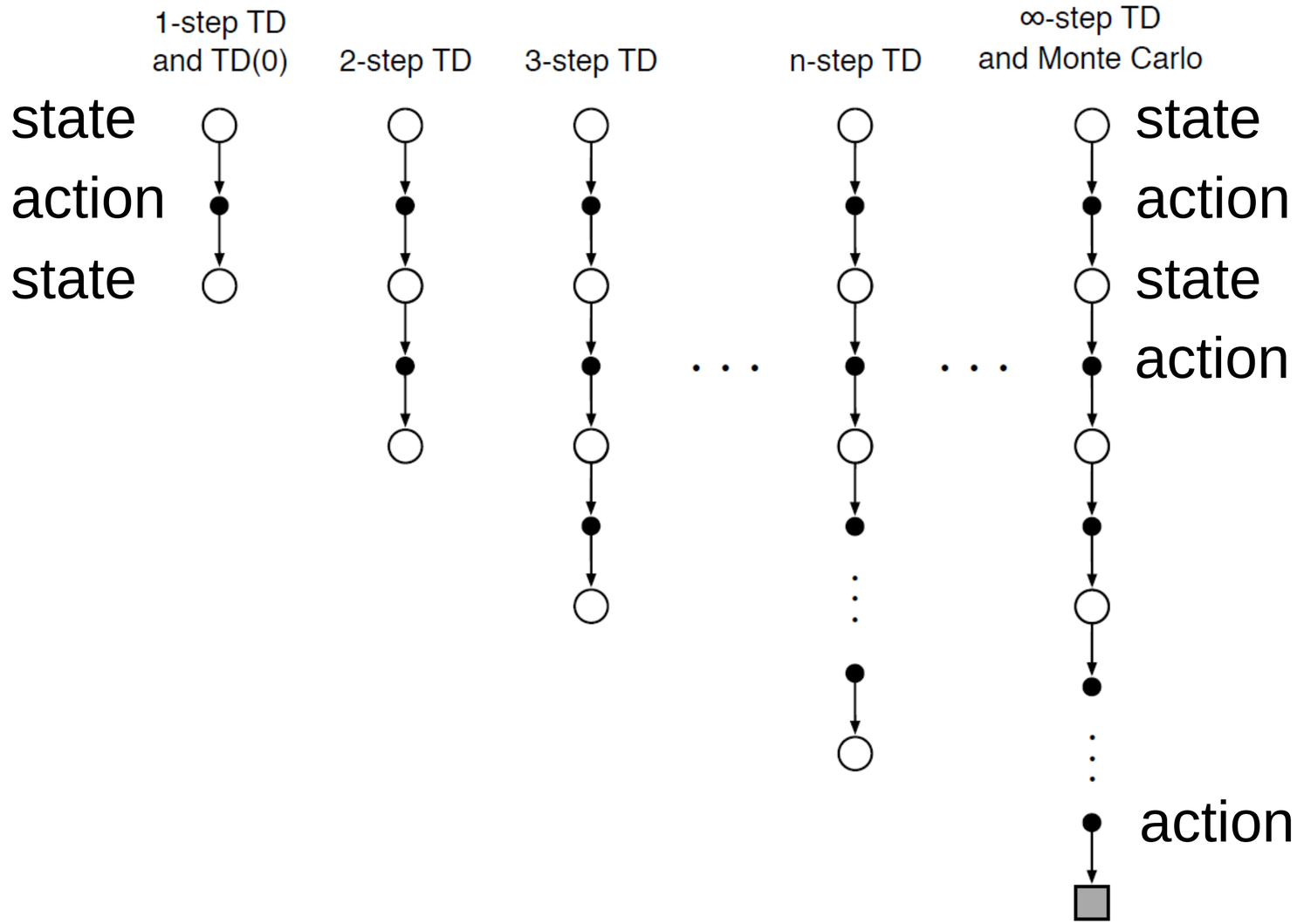
or

→ Switch from 1-step TD to n-step TD
(temporal coarse graining)

Remark: the two methods are mathematically closely related.

(previous slide)
One-step TD algorithms have problems as approximations to continuous states.
There are two solutions, eligibility trades and n-step TD algorithms.
In fact, the two solutions are closely related.

5. Detour: n=step TD methods for V-values



Sutton and Barto, Ch. 7.1

(previous slide)
Remarks regarding n-step V-value TD methods are completely analogous to those for Q-values.

5. Detour: n=step TD methods for V-values

n-step TD for estimating $V \approx v_\pi$

Initialize $V(s)$ arbitrarily, $s \in \mathcal{S}$
Parameters: step size $\alpha \in (0, 1]$, a positive integer n
All store and access operations (for S_t and R_t) can take their index mod n

Repeat (for each episode):
  Initialize and store $S_0 \neq$ terminal
  $T \leftarrow \infty$
  For $t = 0, 1, 2, \ldots$:
  | If $t < T$, then:
  | | Take an action according to $\pi(\cdot|S_t)$
  | | Observe and store the next reward as R_{t+1} and the next state as S_{t+1}
  | | If S_{t+1} is terminal, then $T \leftarrow t + 1$
  | $\tau \leftarrow t - n + 1$ (τ is the time whose state's estimate is being updated)
  | If $\tau \geq 0$:
  | | $G \leftarrow \sum_{i=\tau+1}^{\min(\tau+n,T)} \gamma^{i-\tau-1} R_i$
  | | If $\tau + n < T$, then: $G \leftarrow G + \gamma^n V(S_{\tau+n})$ $(G_{\tau:\tau+n})$
  | | $V(S_\tau) \leftarrow V(S_\tau) + \alpha\,[G - V(S_\tau)]$
  Until $\tau = T - 1$

Sutton and Barto, Ch. 7.1

(previous slide)
The essential step of the algorithm is the update in the blue ellipse where G are the discounted accumulated rewards over n step.

The algorithm looks a bit more complicated because there is a clever way of dealing with the summation over the intermediate rewards while the agent moves along the graph.

Artificial Neural Networks: Lecture 9

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Variants of TD-learning methods and continuous space

1. Review
2. Variations of SARSA
3. TD – learning (Temporal Difference)
4. Monte-Carlo methods
5. Eligibility traces and n-step methods
6. Modeling the input space

(previous slide)
Continuous input spaces have a second problem: there are many Q-values are V-values that you need to compute.

6. Problem of TD algorithms: representation of input

All algorithms so far are ‘tabular’:

Q-learning or SARSA:

→ build a table $Q(s,a)$ with entries for all states s and actions a

TD-learning of V-values:

→ build a table $V(s)$ for all states s

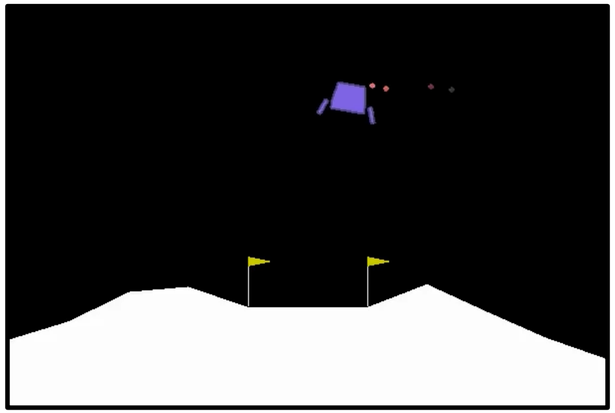
discrete states and actions

(previous slide)
Two observations:
First, in a table all entries are independent – the only relation between Q-values or V-values arises from the self-consistency condition of the Bellman equation.
Second, there are many (!) entries.

6. Problem of TD algorithms: representation of input

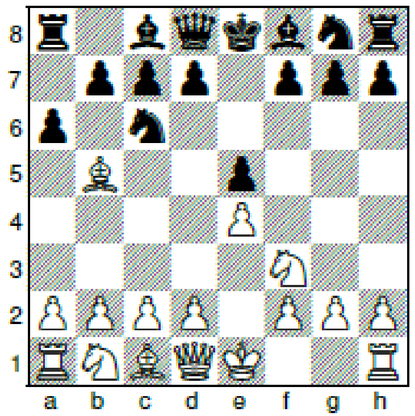
- for control problems, input space is naturally continuous

Moon lander
Aim: land between poles

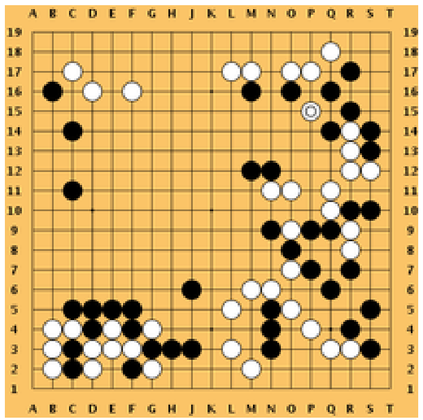


- for discrete games, the input space often too big

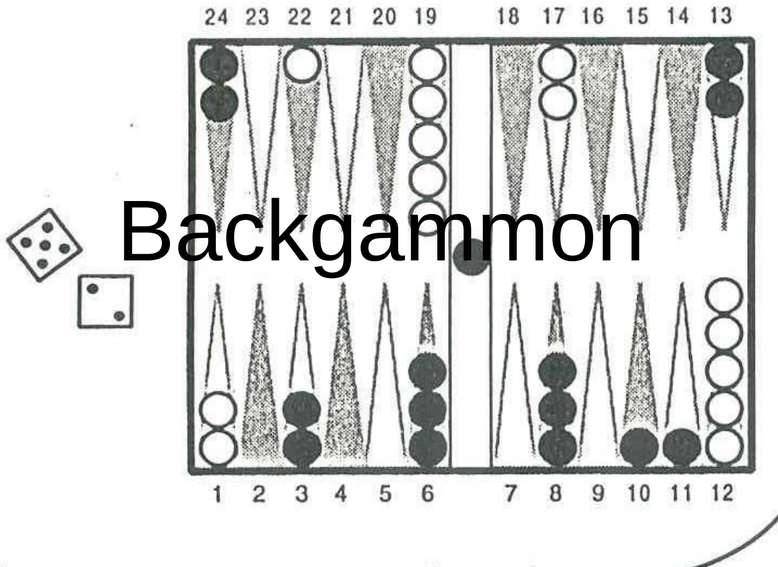
Chess



Go



Backgammon



(previous slide)
Even in cases where the natural input space is descrete, such as in games, there might simply be too many states to keep fill tables with meaningful values.

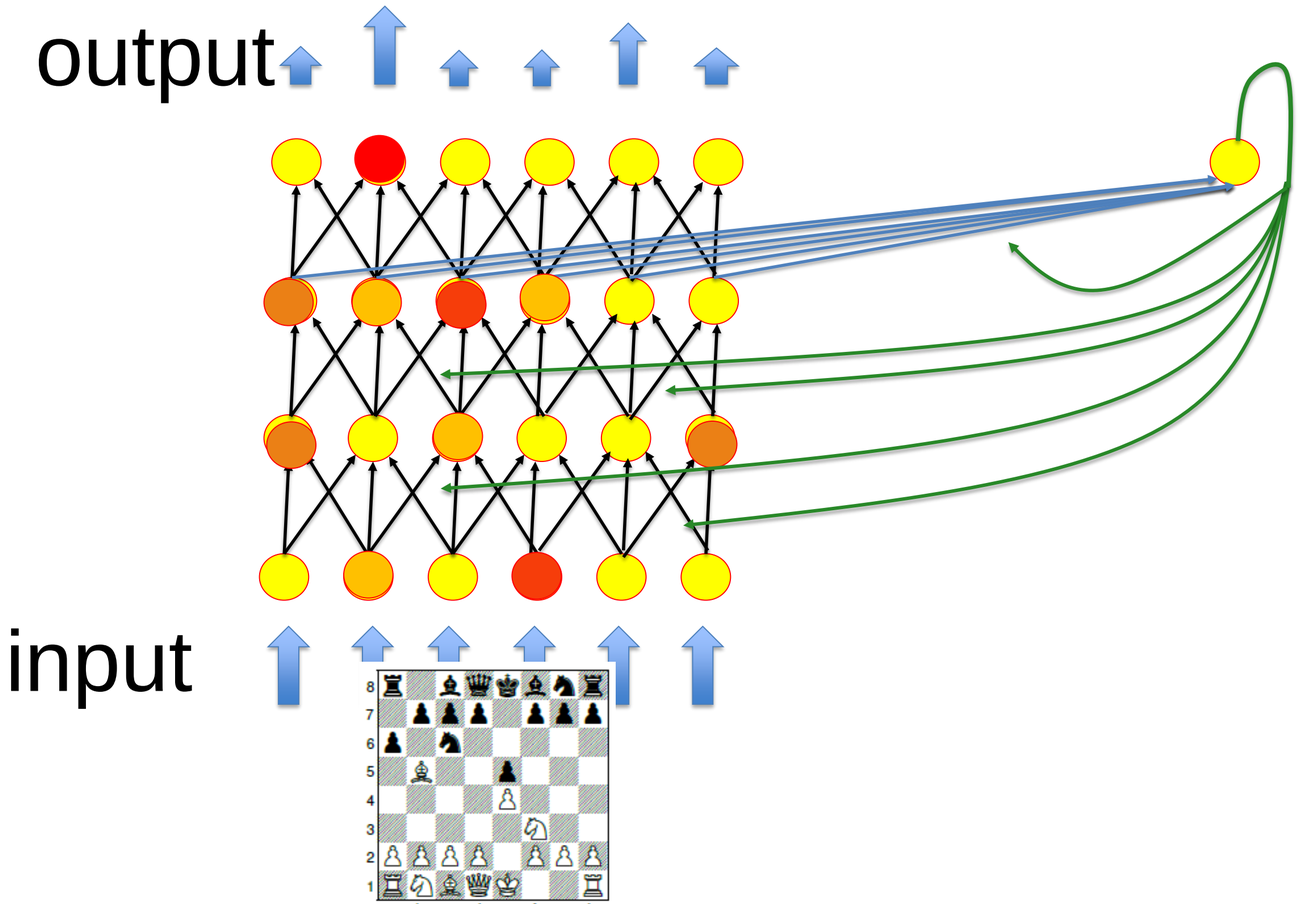
6. Solution: Neural Network to represent input configuration

Schematically (theory will follow):

action:

Advance king

2^e output for V-value for current situation:



Note: alternatively, action outputs could present Q-values

learning:

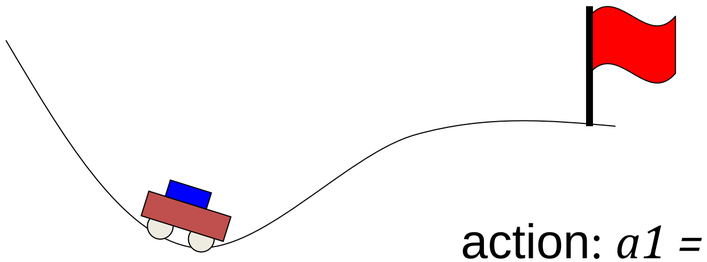
- change connections

aim:

- Predict value of position
- Choose next action to win

(previous slide)
The basic idea that we will explore this week and the following weeks is that the mapping from the input states to actions; or from the input states to value functions can be represented by a model with parameters, typically a neural network with adjustable weights.

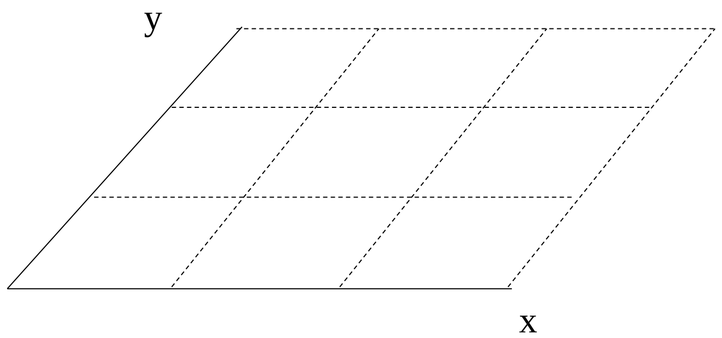
6. Solution: Continuous input representation



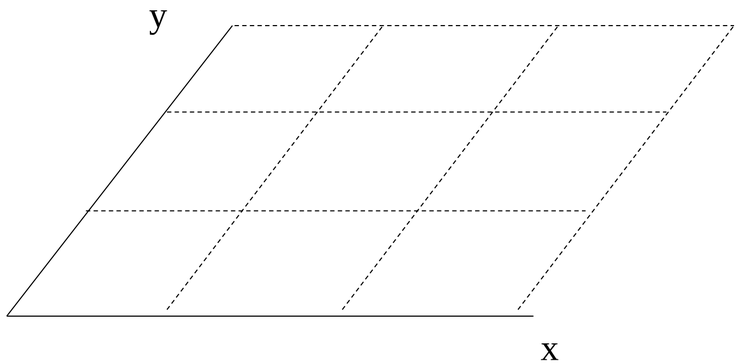
Example: Mountain Car

action: *a1 = right*
a2 = left

for action *a1*

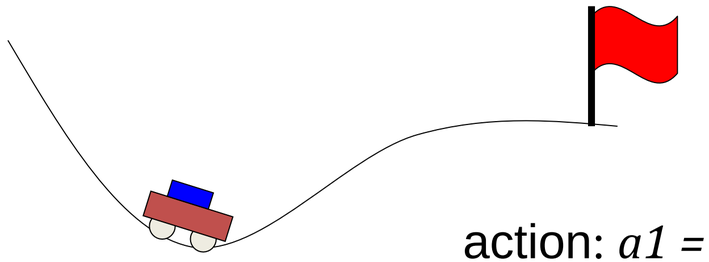


for action *a2*



(previous slide)
In the mountain car task, the input space is two dimensionals: the position x and the speed.
Suppose both dimensions are discretized into 3 values. The Q-values therefore have 9 entries for action a1 (force to the left) and 9 further entries for action a2 (force to the right).

6. Solution: Continuous input representation

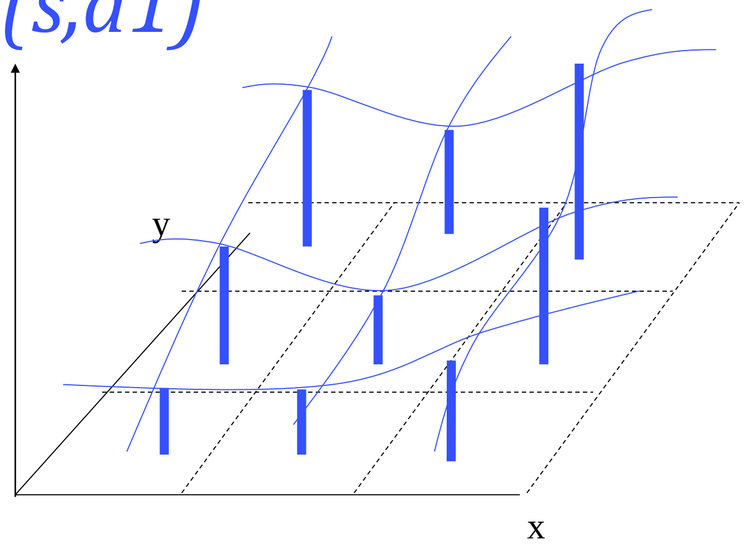


Example: Mountain Car

action: *a1 = right*
a2 = left

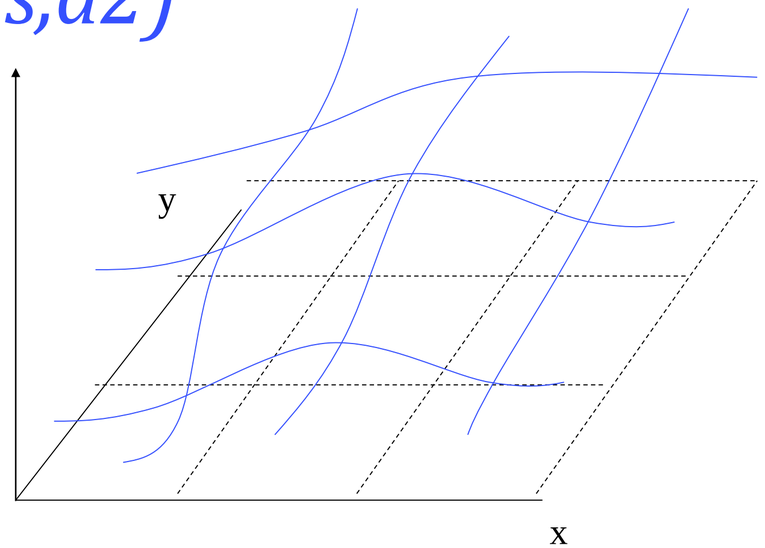
for action *a1*

$Q(s,a1)$



for action *a2*

$Q(s,a2)$



Blackboard 2: Radial Basis functions

(previous slide)
Instead of considering 9 separate table entries of Q-values Q(s,a1) for action a1, we can also think of a smooth function on the two-dimensional input space that represents Q(s,a1) as a function of s.
Similarly, Q(s,a2) is a smooth function of s, but for action a2.

A first advantage is, that the question of discretization of the input space has now disappeared, since we can model the Q-values as a function of the continuous state variable s=(x,y).

The question arises how to model such Q-value functions.
One possibility is to use a combination of basis functions ϕ
so as to describe the Q-value

$$Q(s,a) = \sum_j w_{aj} \Phi(s - s_j)$$

where the weights between basis function j and action a are denoted by w_{aj}

Blackboard: Radial Basis functions

(previous slide)
Your notes.

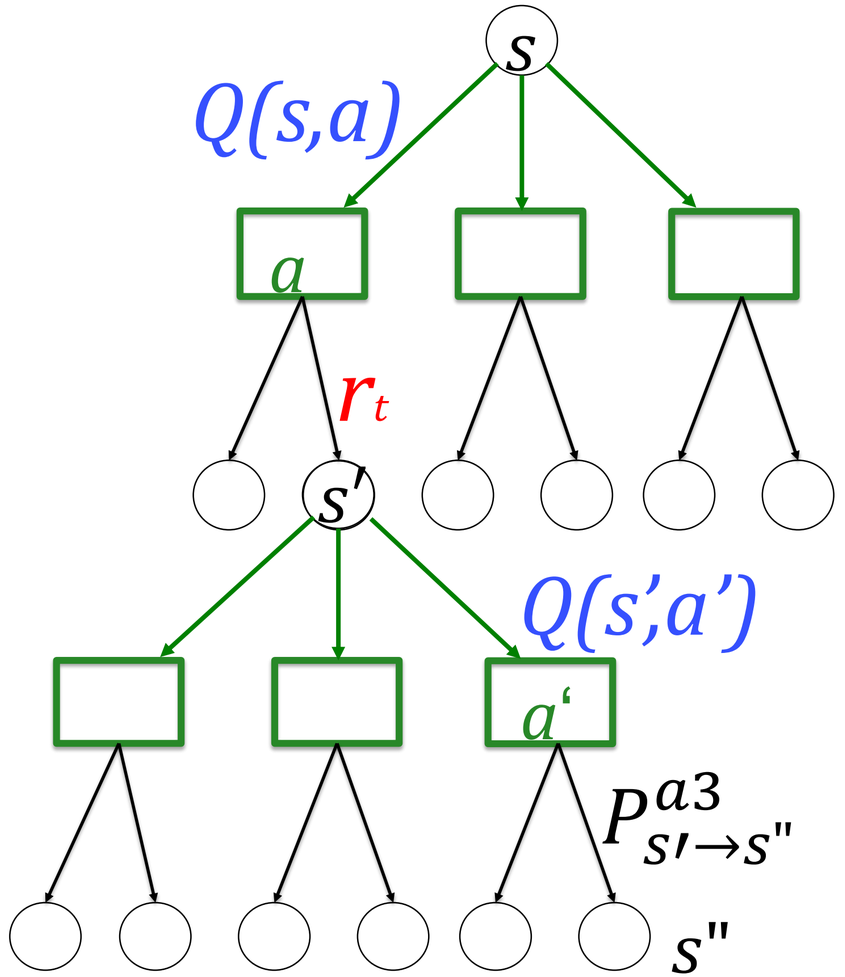
6. From Bellman equation to Error function.

Discrete time steps: s,a → s',a'

$$Q(s,a) = \sum_{s'} P^{a}_{s\to s'}\left[R^{a}_{s\to s'} + \gamma \sum_{a'} \pi(s',a') Q(s',a') \right]$$

Blackboard 3: Error function

$$Q(s,a) = \overbrace{r + \gamma\, Q(s',a')}^{\text{target}}$$



Error function (local consistency condition)

$$E(\boldsymbol{w}) = \frac{1}{2}\left[\overbrace{r_t + \gamma Q(S', a'|\boldsymbol{w})}^{\text{target}} - Q(S, a|\boldsymbol{w})\right]^2$$

take gradient w.r.t. this $\boldsymbol{w}$

(previous slide)
During the discussion of the Bellman equation and SARSA, we stated repeatedly that, if we neglect the discount factor, the difference between Q-values in neighboring time steps must be explained by the reward.
If we include the discount factor, the above statement reduces to

$$Q(s,a) = r + \gamma\, Q(s',a')$$

Where the equality sign has to be interpreted as ‘should ideally on average be close to’ and the right hand side is the ‘target of learning’

Therefore we can construct an error function *E* that measures how close we are to such an ideal case. The squared error function that implements this ideal is noted at the bottom of the slide.
Since the ‘target of learning’ should be considered as momentarily fixed, we optimize the error function by taking the derivative of *E* with respect to w but ignore that the target also depends on w. We will explore this further in the next two weeks.

Summary: Many Variations of a few ideas in TD learning

Learning outcomes and Conclusions

- TD – learning (Temporal Difference)

  → work with V-values, rather than Q-values

- **Variations of SARSA**

  - off-policy Q-learning (greedy update)
  - Monte-Carlo
  - n-step Bellman equation

- **Eligibility traces**

  - allows rescaling of states, smoothes over time
  - similar to n-step SARSA

- **Continuous space**

  → use neural network to model and generalize

Basis of all:
iterative solution of Bellman equation

(previous slide)
Today we have seen a large variety of TD algorithms. All of these can be understood as iterative solutions of the Bellman equation.
The Bellman equation can be formulated with V-values or with Q-values. Bellman equations normally formulate a self-consistency condition over one step (nearest neighbors), but can be extended to n steps.
Monte Carlo methods do not exploit the ‘bootstrapping’ aspect of the Bellman equation since they do not rely on a self-consistency condition.
An n-step SARSA is somewhere intermediate between normal SARSA and Monte-Carlo.

Discretization of continuous spaces poses several problems.
The first problem is that a rescaling becomes necessary after a change of discretization scheme. This problem is solved by eligibility traces as well as by the n-step TD methods
The second problem is that a tabular scheme brakes down for fine discretizations. It is solved by a neural network where we learn the weights. Such a neural network enables generalization by forcing a ‘smooth’ V-value or Q-value.

The END