Networks Out of Control: Evolution & Dynamics 1 Cascades

Announcement: Project Selection

Project

Details:

- Pair project
- 4 page (2-column) report due Wednesday 23rd May.
- 15 min presentation + 5 min questions on Monday 28th May to be scheduled between 2pm-7pm.

Contact Farnood with the following by Friday 20 April:

- Who your team is (pairs)
  - In case you do not find a team, respond to the other two points and we will match you with someone
- List of 3 preferred papers, in order, on which you would like to do your project (ties get broken by when the email was sent).
- Availability to present in the 2pm-4pm time slot on Monday.

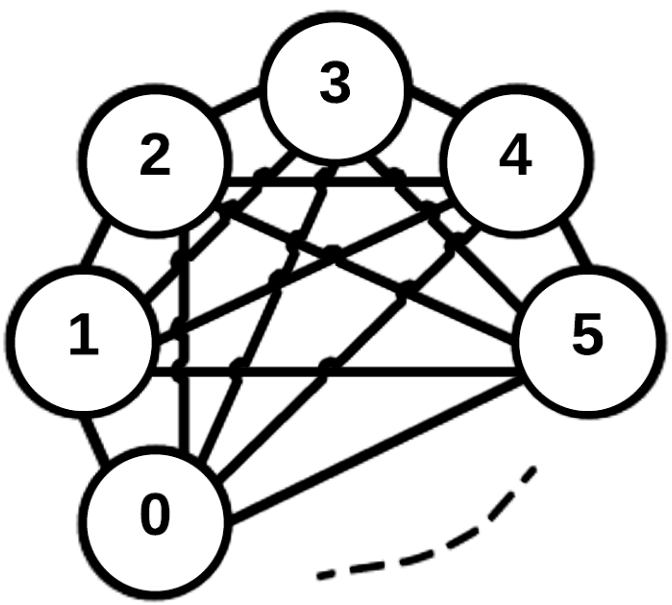
Cascades

- There is a network of people, and a behavior that spreads through the network from person to person across edges.
  - E.g., an idea, product, illness or habit spreads across the network.
  - (Already discussed this informally in the context of homophily)
- How does it spread?
  - Direct-benefit effects
  - Rational effects
- What do we study?
  - Will it spread to the entire network?
  - Are there threshold properties that determine if/when it does?

Cascades 0:
Shelling Threshold Model

Schelling Threshold Model

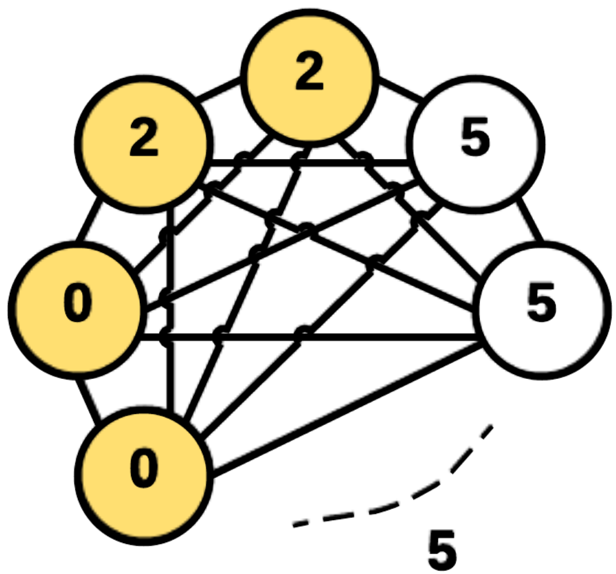
- There are n people (labeled i = 0, 1, 2, …, n-1). Each has a "willingness to riot" coefficient r_i which is *how many others* decide to riot before they join in.
- Can think of this process as occurring on the complete graph:
  - Is there a complete riot in this situation?



Schelling Threshold Model

- There are n people (labeled i = 0, 1, 2, …, n-1). Each has a "willingness to riot" coefficient r_i which is *how many others* decide to riot before they join in.

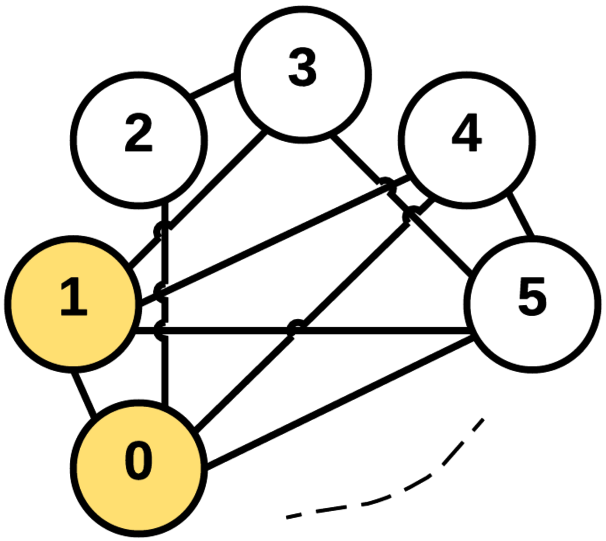
- Can think of this process as occurring on the complete graph:
  - Is there a complete riot in this situation?



- I.e., the existence of a cascade depends on the parameters.

Schelling Threshold Model

- There are n people (labeled i = 0, 1, 2, …, n-1). Each has a "willingness to riot" coefficient r_i which is *how many others* decide to riot before they join in.
- Can think of this process as occurring on arbitrary graphs:
  - Is there a complete riot in this situation?



- I.e., the existence of a cascade depends on the network.

Cascades 1:
Direct-Benefit Effects

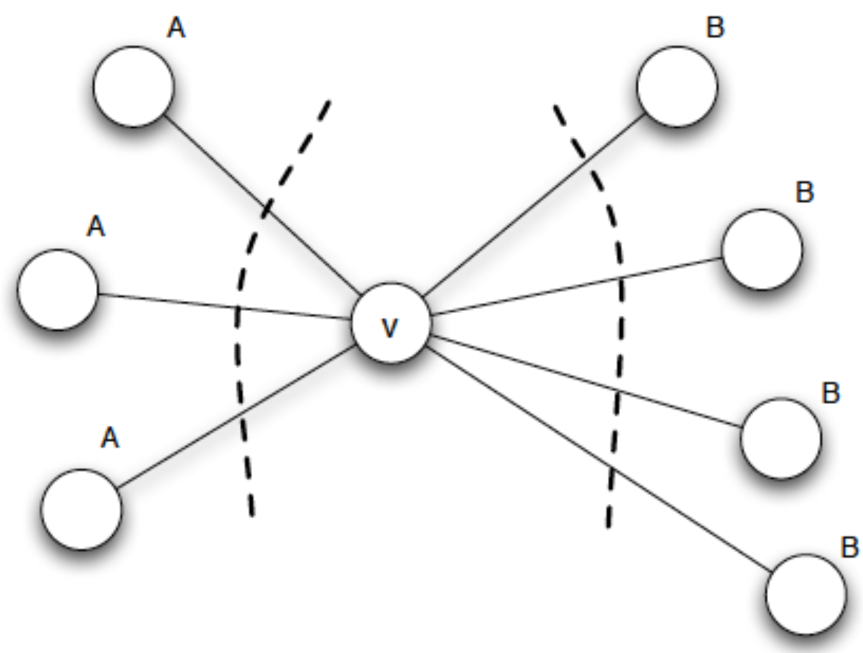
+
Crash Course in Game Theory

- **Agents:** two or more participants.
- **Actions:** options available to the agent.
- **Outcome:** global end result (function of all agent's actions).
- **Utility:** real-valued function of outcome for a given agent.

- In game theory:
  - Agents want to maximize their own utility (selfish behavior)
  - We study Nash equilibira, i.e., outcomes such that no agent can unilaterally increase their utility by changing their action.

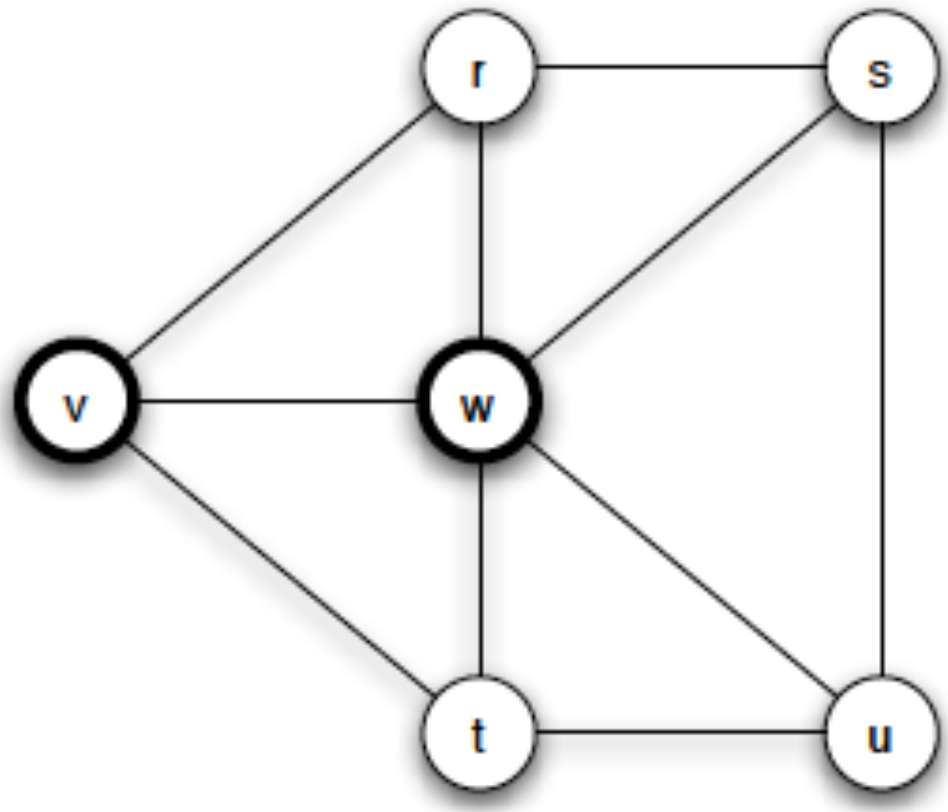
Diffusion on Networks

- There are two products, A and B
- There is a social network G, and each node (*agents*) can select to use a single product A or B (*actions*).
- Let $n_A(v)$ be the number of v's neighbors using A. Similarly $n_B(v)$
- Let a, b > 0. The *utility* for agent v is
  - If v uses A: $a\ n_A(v)$
  - If v uses B: $b\ n_B(v)$
- When does v select A instead of B?
  - $p > b / (a+b)$
- When does the whole network converge (cascade) to one option?



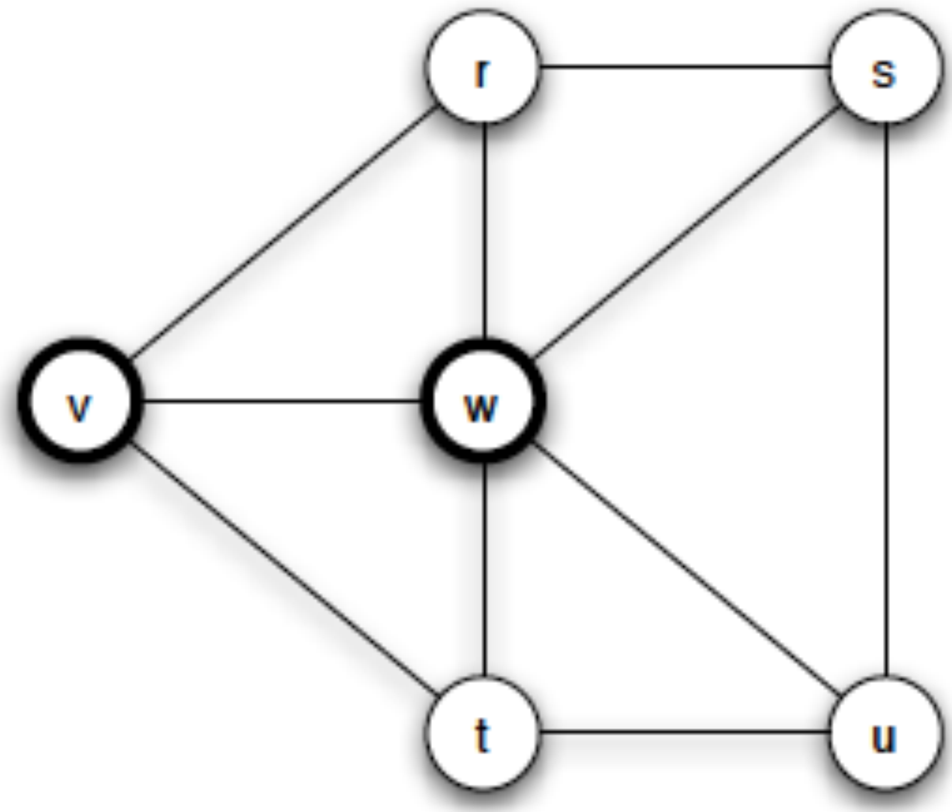
Example

- Let a = 3 and b = 2
  - So p = 2/5 is the threshold for selecting A



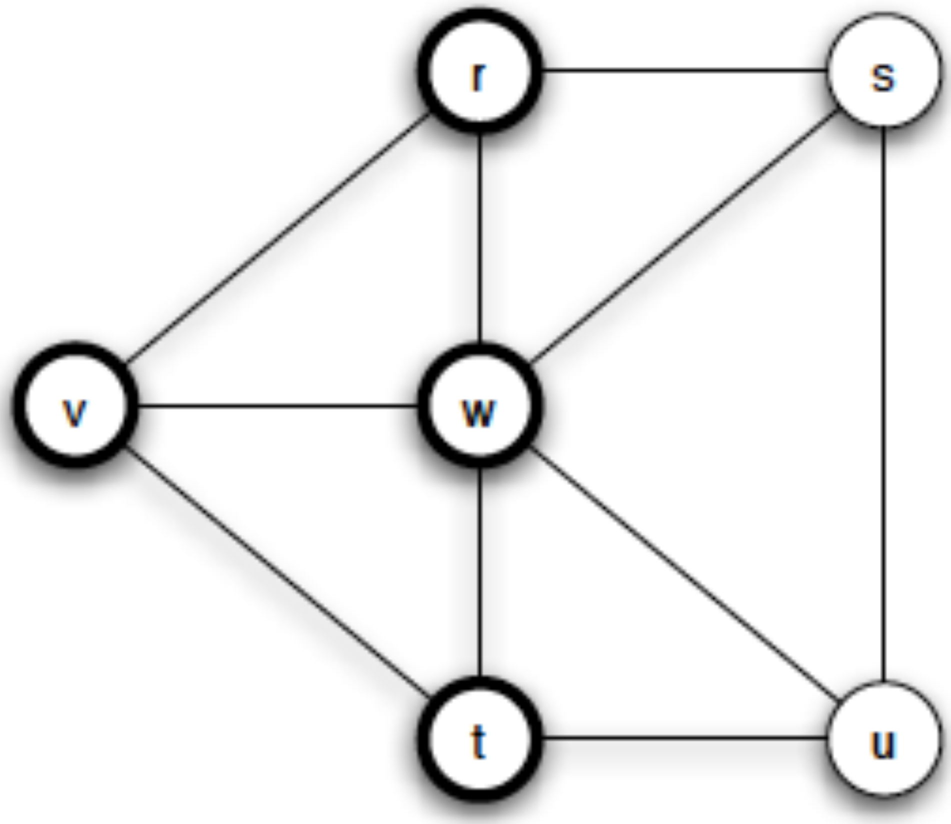
Example

- Let a = 3 and b = 2 (so p = 2/5)
- Is there a complete cascade?



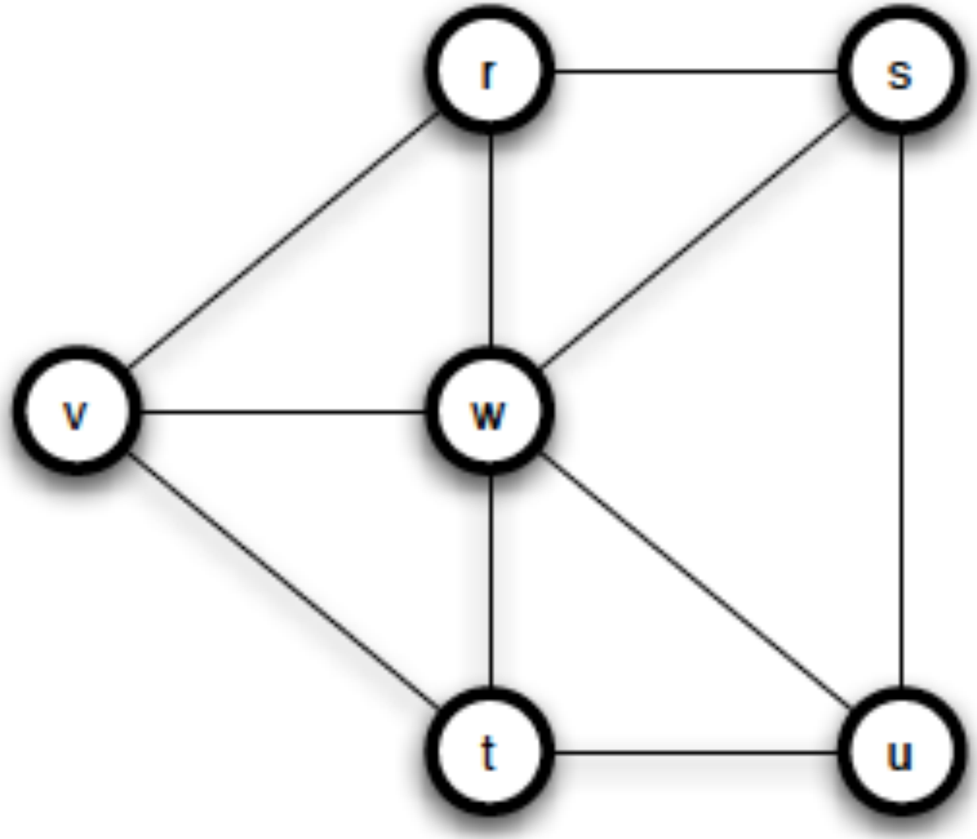
Example

- Let a = 3 and b = 2 (so p = 2/5).
- Is there a complete cascade?



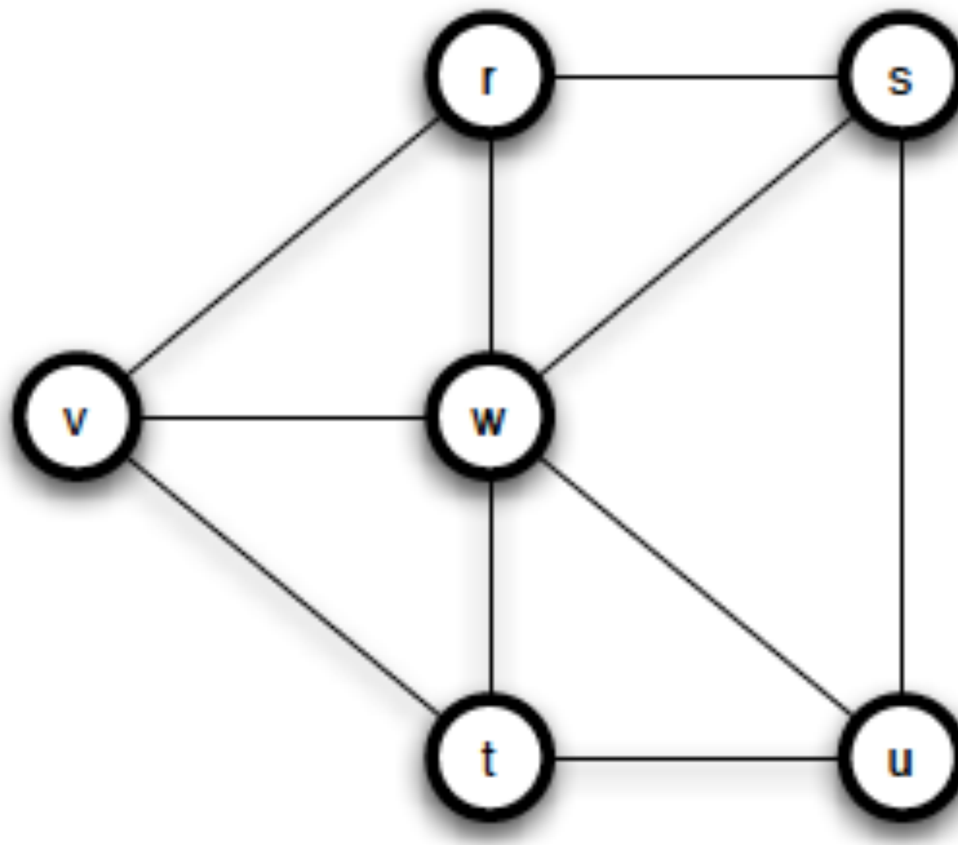
Example

- Let a = 3 and b = 2 (so p = 2/5).
- Is there a complete cascade?



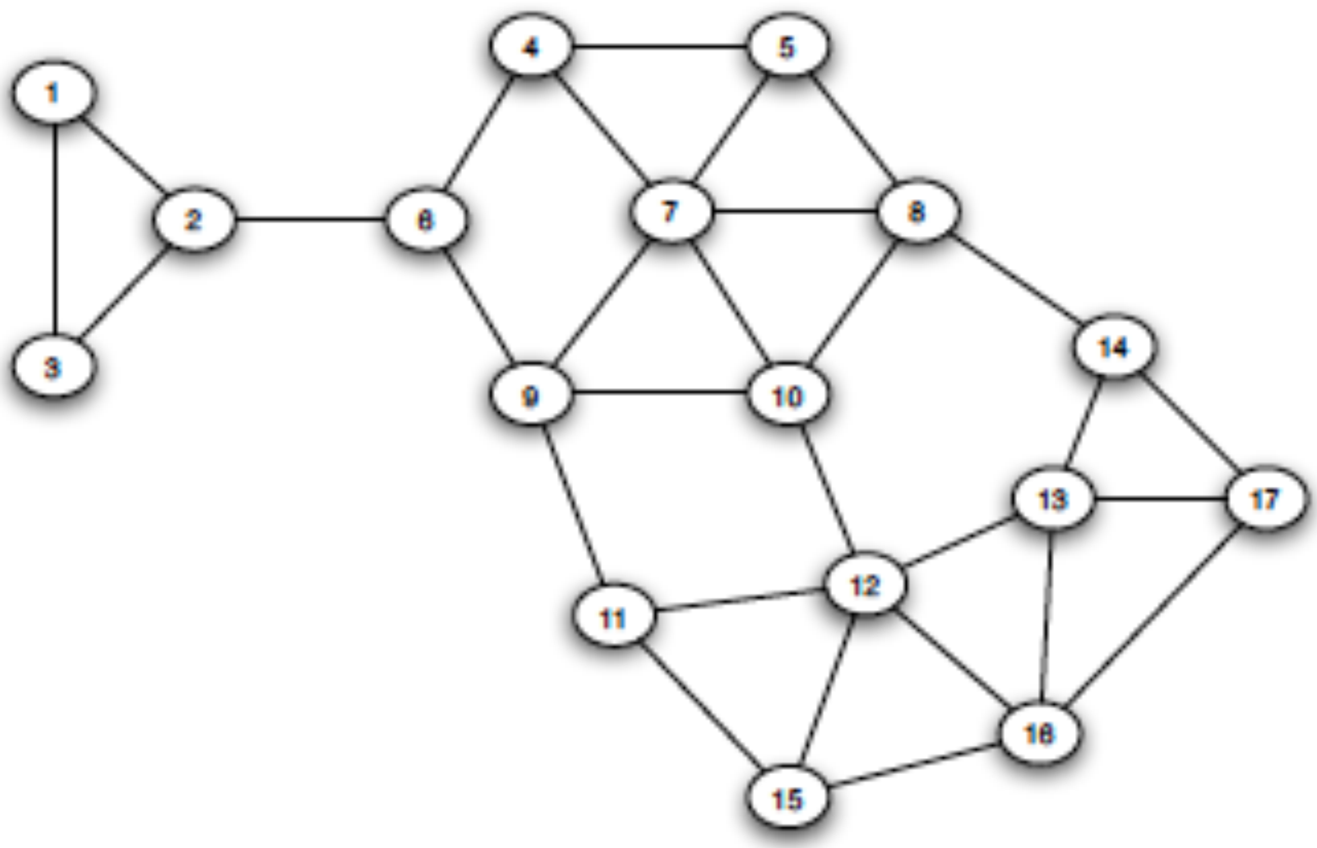
Example

- Let a = 3 and b = 2 (so p = 2/5).
- Has applications to viral marketing – which nodes should one target (and then hope they influence their neighbors)?
- Will a cascade always occur?



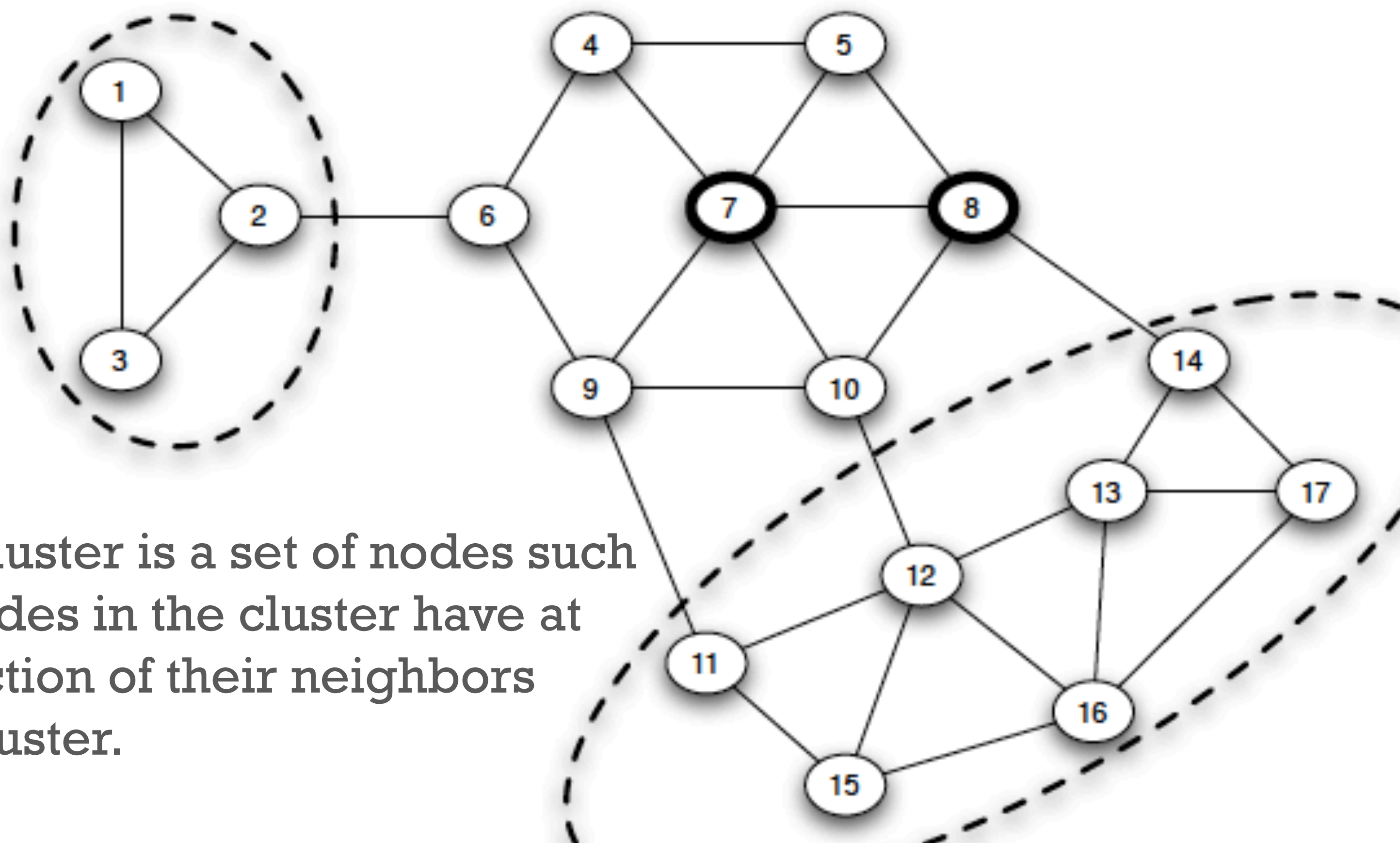
Cascades

- Will a cascade always occur?



Cascades

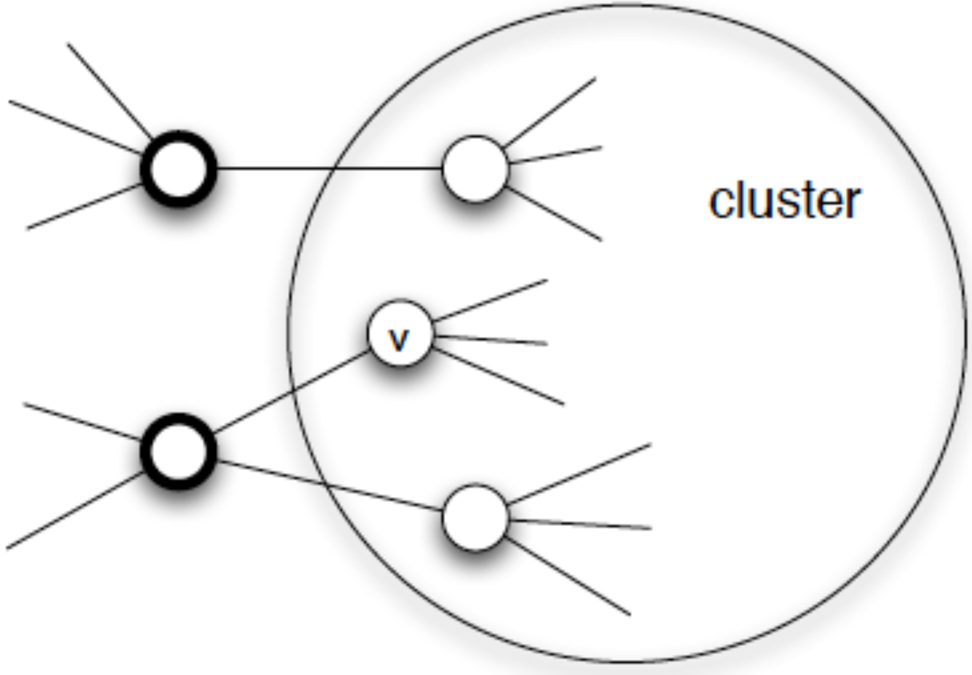
- Will a cascade always occur?
  - No – "clusters" can get in the way.



- A p-dense cluster is a set of nodes such that all of nodes in the cluster have at least a p fraction of their neighbors inside the cluster.

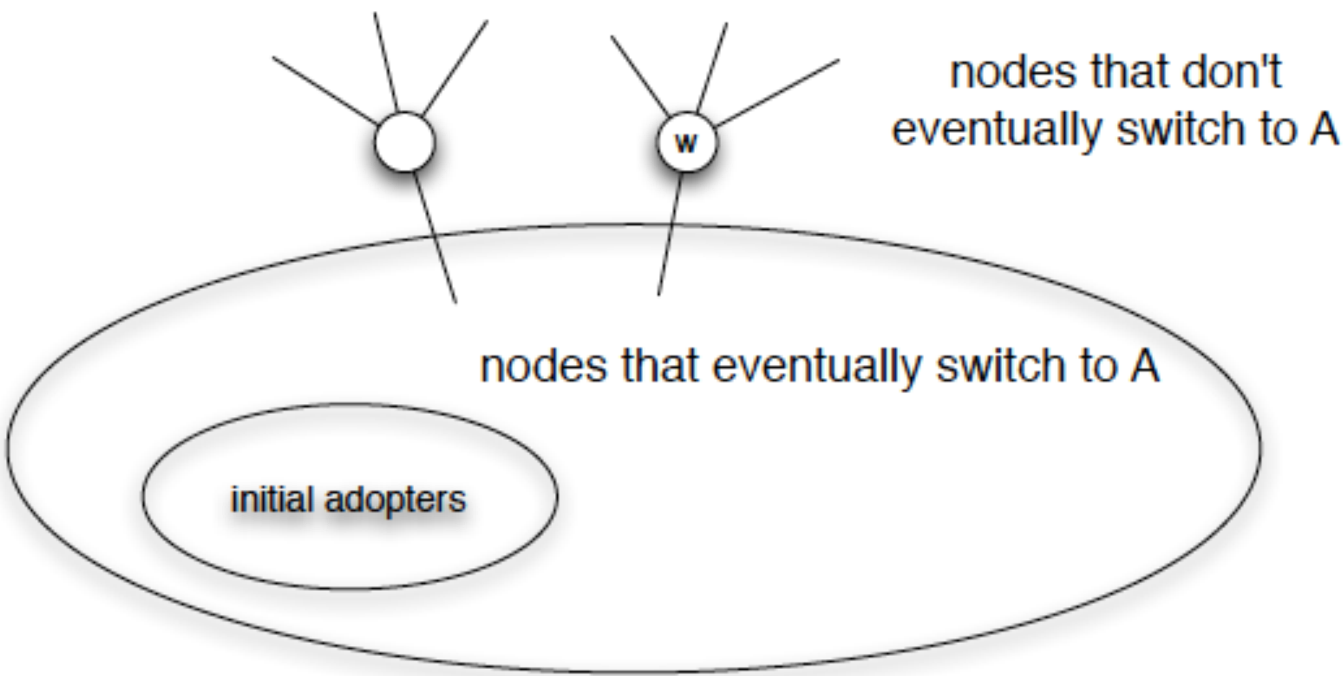
Cascades

- A p-dense cluster is a set of nodes such that all of nodes in the cluster have at least a p fraction of their neighbors inside the cluster.
- Theorem: Consider an initial set of adopters V' and a threshold p for adoption. The nodes in V \ V' contain a cluster of density greater than 1-p if and only if a complete cascade does not occur.



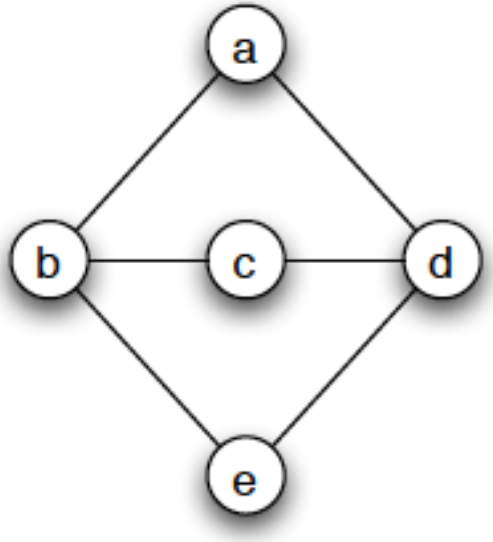
Cascades

- A p-dense cluster is a set of nodes such that all of nodes in the cluster have at least a p fraction of their neighbors inside the cluster.
- Theorem: Consider an initial set of adopters V' and a threshold p for adoption. The nodes in V \ V' contain a cluster of density greater than 1-p if and only if a complete cascade does not occur.



Small Exercise

- If you wanted to initiate a cascade, which vertex would you want to as an initial adopter?



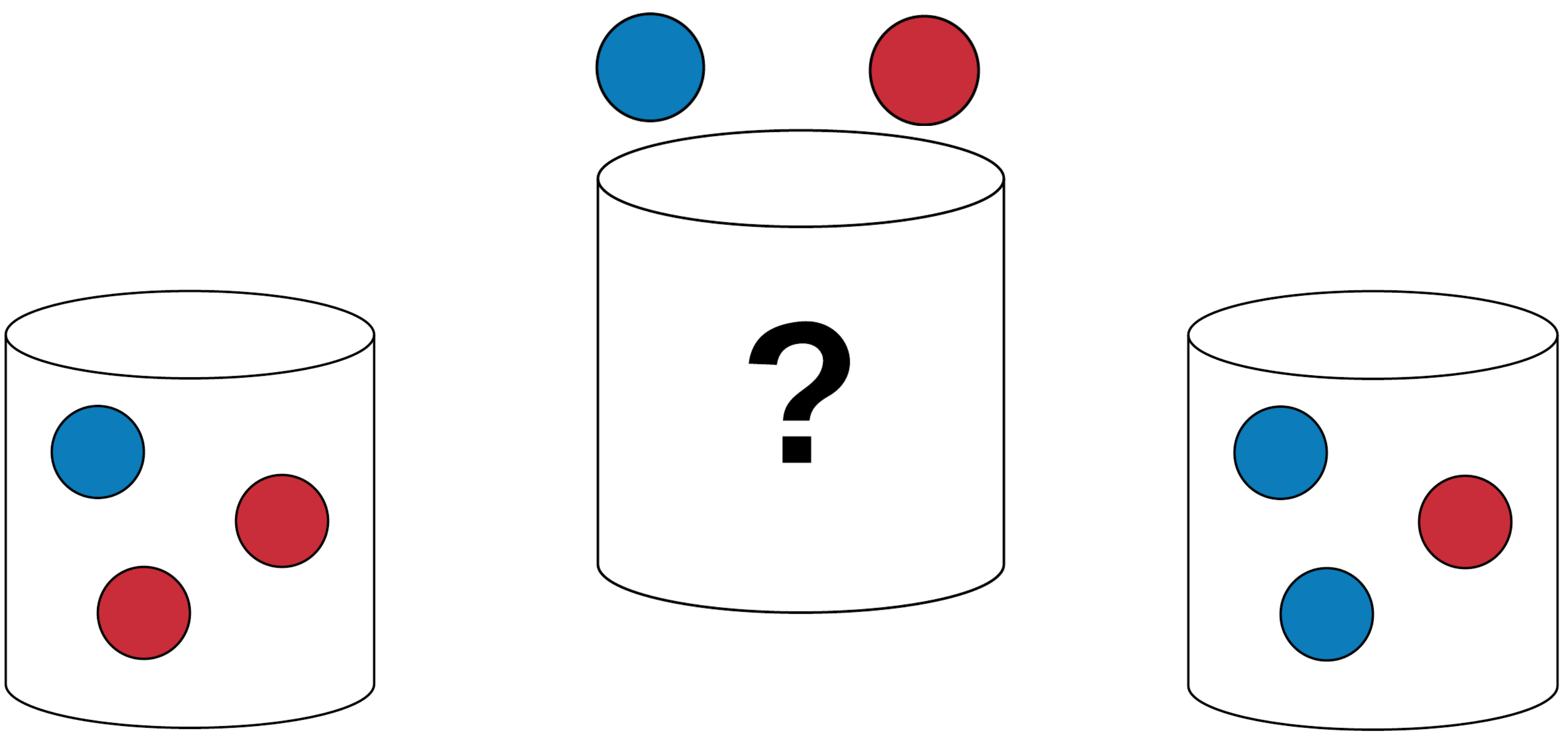
Cascades 2: Rational Effects

Information Cascades

- Two options A and B
  - You have some (private) information as to which option is better.
  - You have some public information as to which option other people think is better.

Cascades

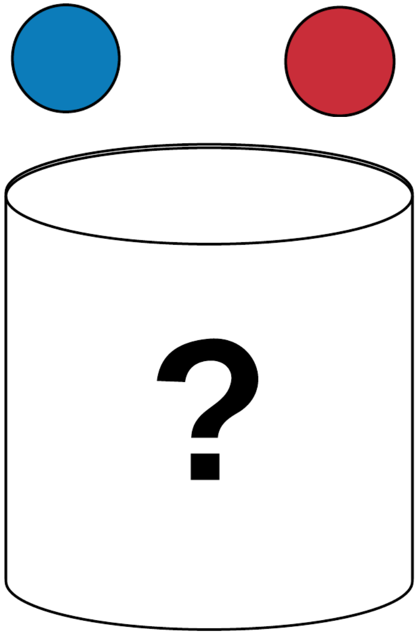
- Example:
  - A = guess red urn,   B = guess blue urn
  - Noisy signal = sample from urn
  - Public decision = guess of previous players



Cascades

- Two options A and B
  - Each player receives a private **noisy signal** that indicates which option is better.
  - Each player observes sequential public **decisions**.

B
R
B
B
B
…
B



Cascade Example

- How should players make their decisions?

$$\Pr\left[\textit{majority-blue} \mid \textit{what she has seen and heard}\right] > \frac{1}{2}$$

- In order to analyze this, we'll make heavy use of Bayes' rule:

$$\Pr\left[A \mid B\right] = \frac{\Pr\left[A \cap B\right]}{\Pr\left[B\right]}.$$

- Note:

$$\Pr\left[\textit{majority-blue}\right] = \Pr\left[\textit{majority-red}\right] = \frac{1}{2}.$$

$$\Pr\left[\textit{blue} \mid \textit{majority-blue}\right] = \Pr\left[\textit{red} \mid \textit{majority-red}\right] = \frac{2}{3}.$$

$$\Pr[A \mid B] = \frac{\Pr[A \cap B]}{\Pr[B]}.$$

$$\Pr[\textit{majority-blue}] = \Pr[\textit{majority-red}] = \frac{1}{2}.$$

$$\Pr[\textit{blue} \mid \textit{majority-blue}] = \Pr[\textit{red} \mid \textit{majority-red}] = \frac{2}{3}.$$

Cascade Example

- Consider the first player:
  - Assume they saw blue (the argument for red is symmetric).

$$\Pr[\textit{majority-blue} \mid \textit{blue}] =$$

$$\Pr[\textit{blue}]$$

$$\Pr[\textit{majority-blue} \mid \textit{blue}] = \frac{1/3}{1/2} = \frac{2}{3}.$$

Cascade Example

$$\Pr[A \mid B] = \frac{\Pr[A \cap B]}{\Pr[B]}.$$

$$\Pr[\textit{majority-blue}] = \Pr[\textit{majority-red}] = \frac{1}{2}.$$

$$\Pr[\textit{blue} \mid \textit{majority-blue}] = \Pr[\textit{red} \mid \textit{majority-red}] = \frac{2}{3}.$$

- Consider the second player:
  - If the first player saw blue and they also see blue:
    - Clearly P[majority-blue] > ½ so they say blue
  - If the first player saw blue and they see red:
    - Won't do this formally, but with Bayes' rule can prove that P[majority-blue] = P[majority-red] = 1/2
    - Tiebreak – (let's assume they stick with color they see).

+

Cascade Example

$$\Pr[A \mid B] = \frac{\Pr[A \cap B]}{\Pr[B]}.$$

$$\Pr[\textit{majority-blue}] = \Pr[\textit{majority-red}] = \frac{1}{2}.$$

$$\Pr[\textit{blue} \mid \textit{majority-blue}] = \Pr[\textit{red} \mid \textit{majority-red}] = \frac{2}{3}.$$

- Consider the third player:
  - If the first two see one red and one blue
    - Reduces to the first-player setting because first two samples were not informative.
  - If all three see blue
    - Clearly P[majority-blue] > ½ , and they say blue.
  - If the first two see blue and they see red…

Cascade Example

$$\Pr[A \mid B] = \frac{\Pr[A \cap B]}{\Pr[B]}.$$

$$\Pr[\textit{majority-blue}] = \Pr[\textit{majority-red}] = \frac{1}{2}.$$

$$\Pr[\textit{blue} \mid \textit{majority-blue}] = \Pr[\textit{red} \mid \textit{majority-red}] = \frac{2}{3}.$$

- Consider the third player:
  - Assume the first two see blue, but they see red.

$$\Pr[\textit{majority-blue} \mid \textit{blue},\ \textit{blue},\ \textit{red}]$$

$$\Pr[\textit{blue},\ \textit{blue},\ \textit{red} \mid \textit{majority-blue}]$$

$$\Pr[\textit{blue},\ \textit{blue},\ \textit{red}]$$

$$\Pr[\textit{majority-blue} \mid \textit{blue},\ \textit{blue},\ \textit{red}] = \frac{\frac{4}{27} \cdot \frac{1}{2}}{\frac{1}{9}} = \frac{2}{3}.$$

Cascade Example

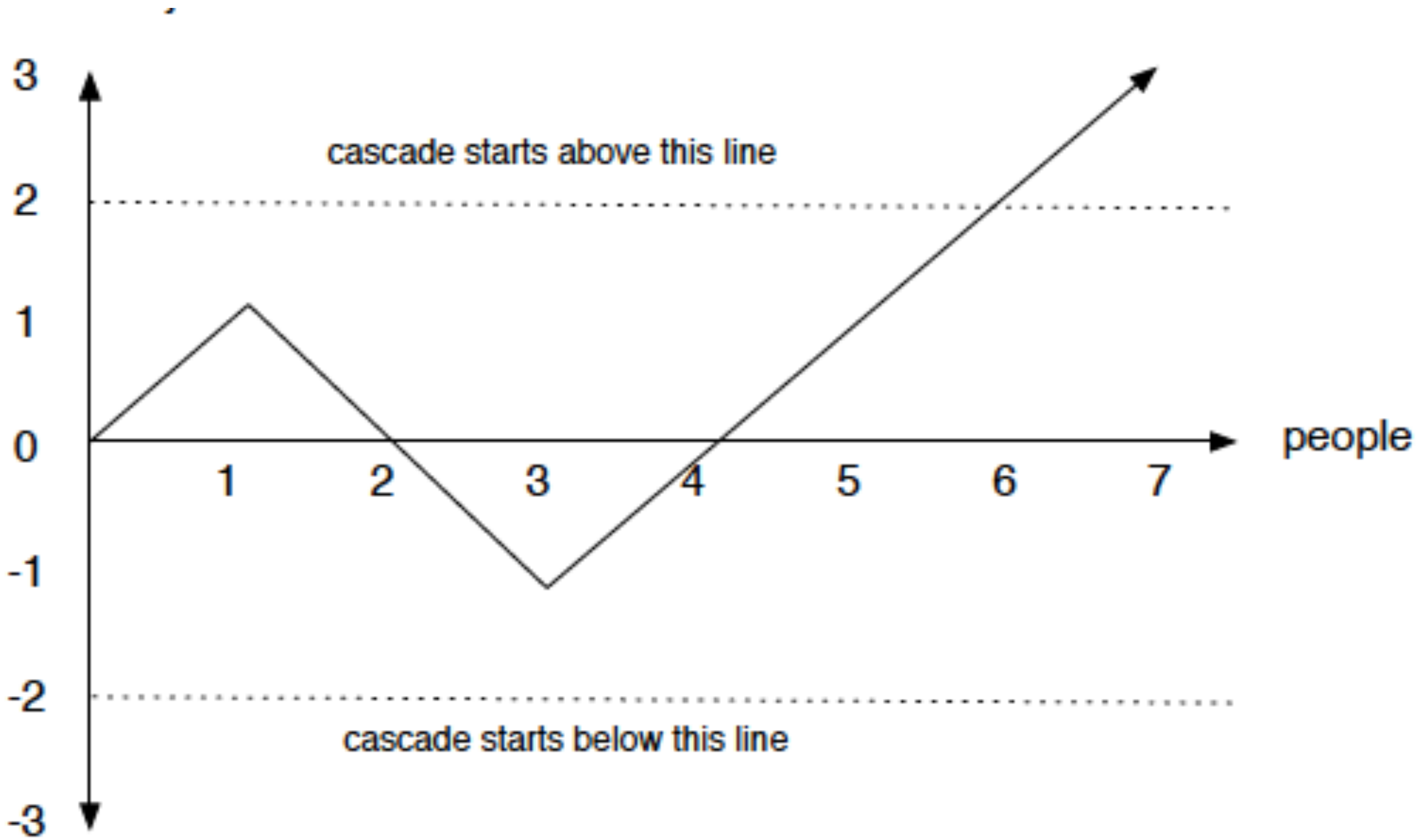
$$\Pr[A \mid B] = \frac{\Pr[A \cap B]}{\Pr[B]}.$$

$$\Pr[\mathit{majority\text{-}blue}] = \Pr[\mathit{majority\text{-}red}] = \frac{1}{2}.$$

$$\Pr[\mathit{blue} \mid \mathit{majority\text{-}blue}] = \Pr[\mathit{red} \mid \mathit{majority\text{-}red}] = \frac{2}{3}.$$

- Consider the fourth player:
  - If the first thre players say:
    - blue blue x
    - blue red x
  - Reduces to the three-player setting!

Cascade Example



Information Cascades

- Generally, cascades
  - Can lead to sub-optimal outcomes (the crowd may not be wise)
  - Can be based on very little information
  - Are fragile
- Main bottleneck to "wisdom" -- decisions NOT independent.
  - Also known as "herding"
- Can think of this process as occurring on a graph.