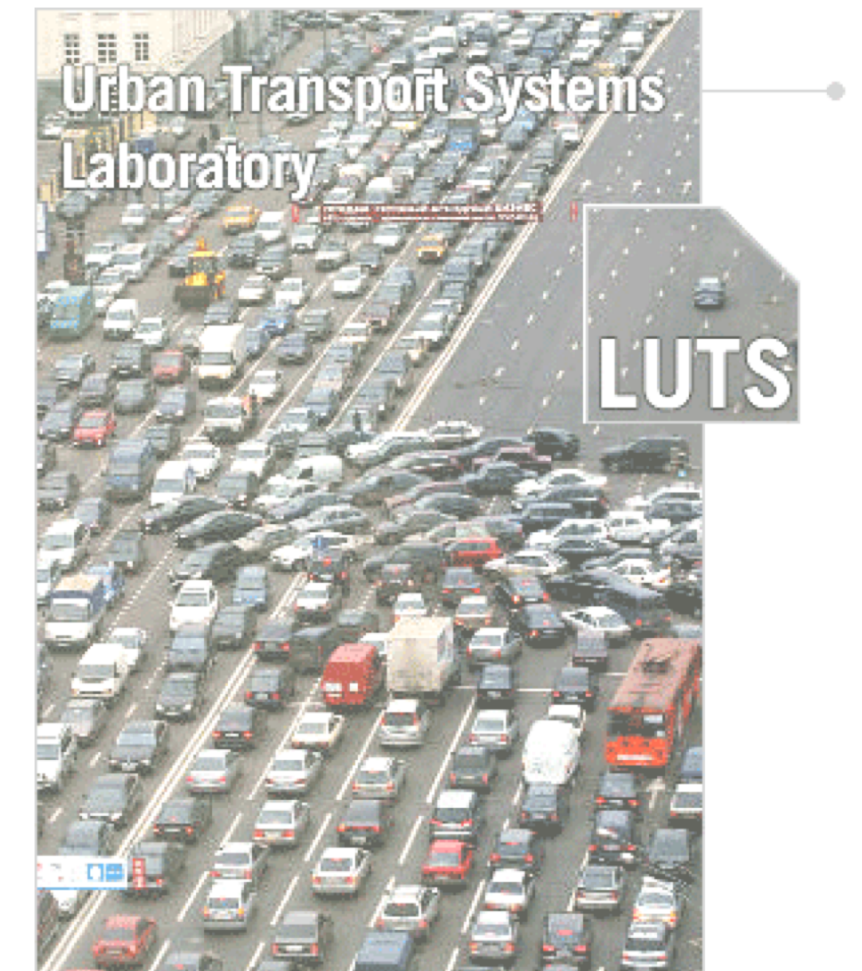


Macroscopic Fundamental Diagram: Existence, Physical Properties and Dynamic Modeling

Intro to traffic flow modeling and ITS

Prof. Nikolas Geroliminis



MFD Dynamics of a single-reservoir system

$$n_1 = n_0 + q_0 - G(n_0)$$

$$n_2 = n_1 + q_1 - G(n_1)$$

⋮

⋮

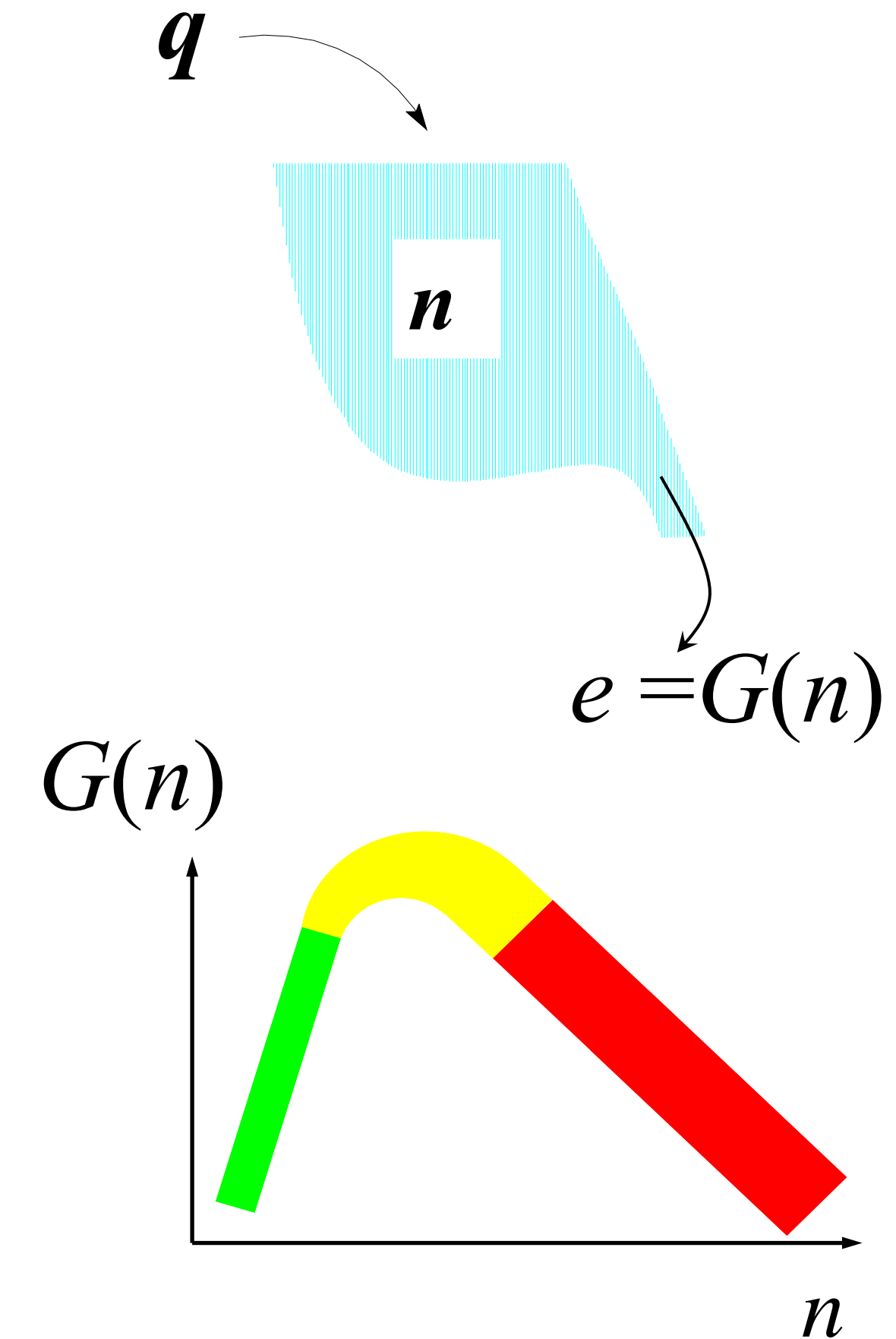
⋮

⋮

⋮

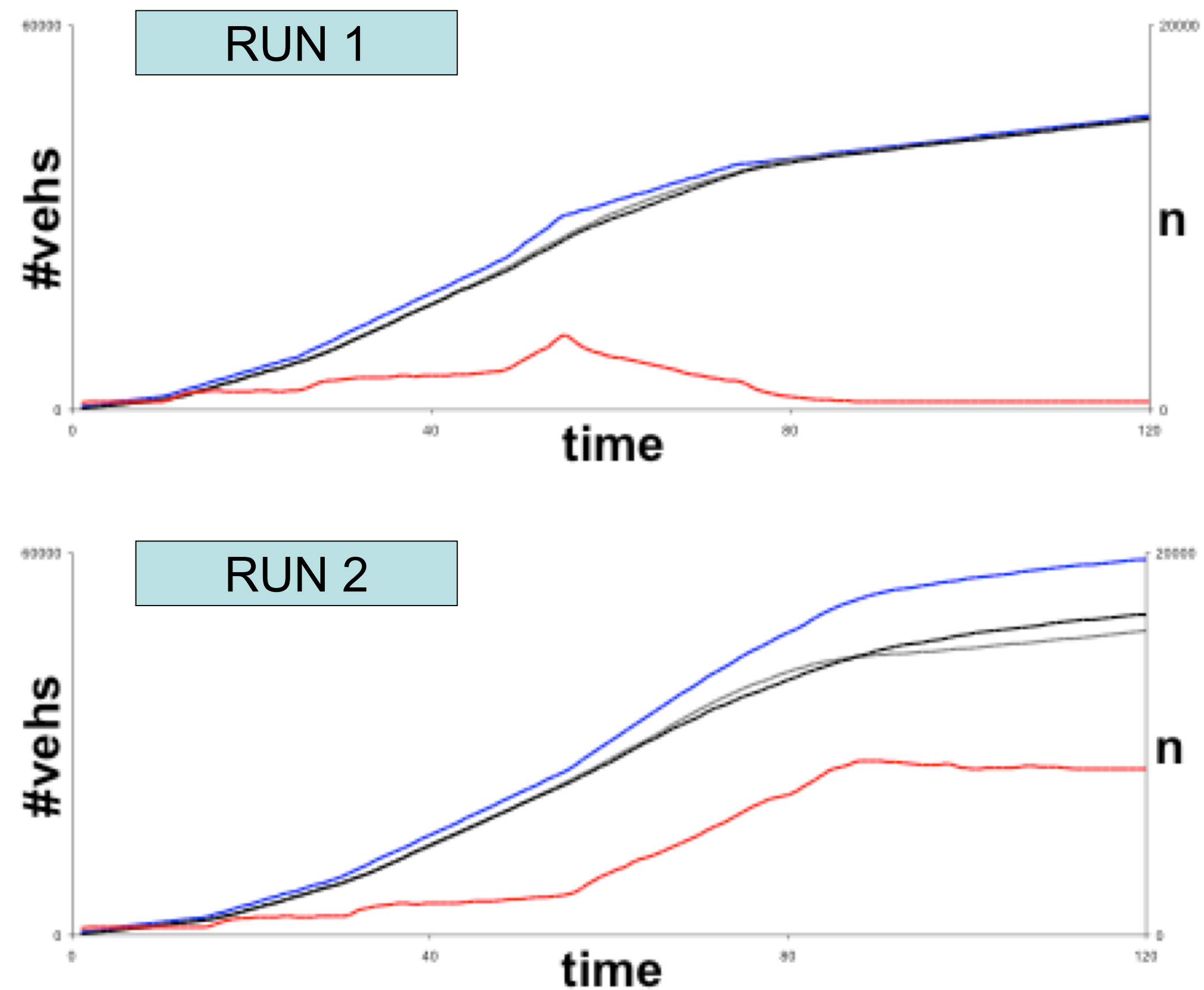
$$n_t = n_{t-1} + q_{t-1} - G(n_{t-1})$$

$$n_t = n_0 + \sum_{j=0}^{j=t-1} q_j - \sum_{j=0}^{j=t-1} G(n_j)$$



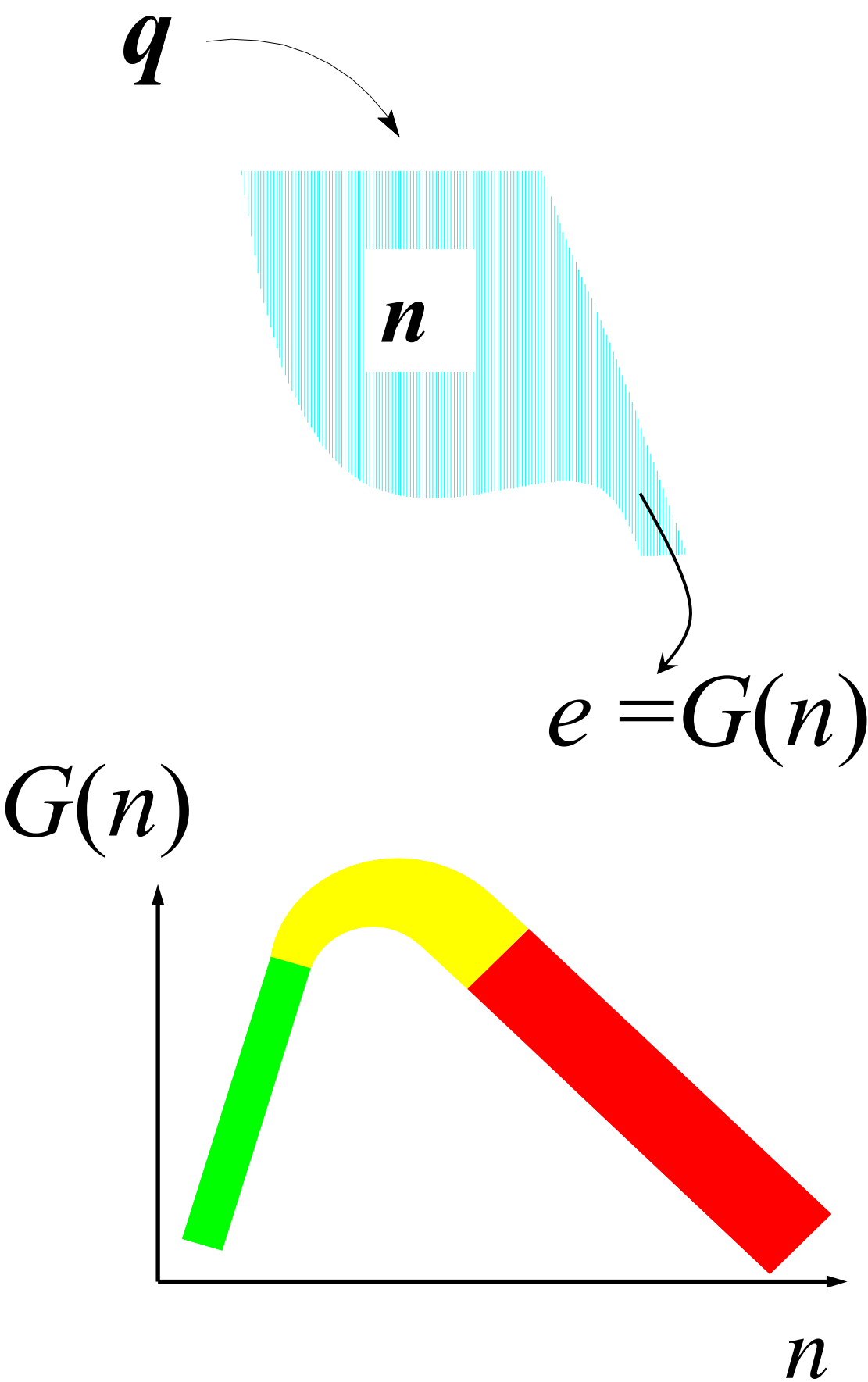
$$\frac{dn}{dt} = q(t) - G(n(t))$$

MFD Dynamics of a single-reservoir system



INPUT OUTPUT ESTIMATED OUTPUT ACCUMULATION

Geroliminis and Daganzo, 2007

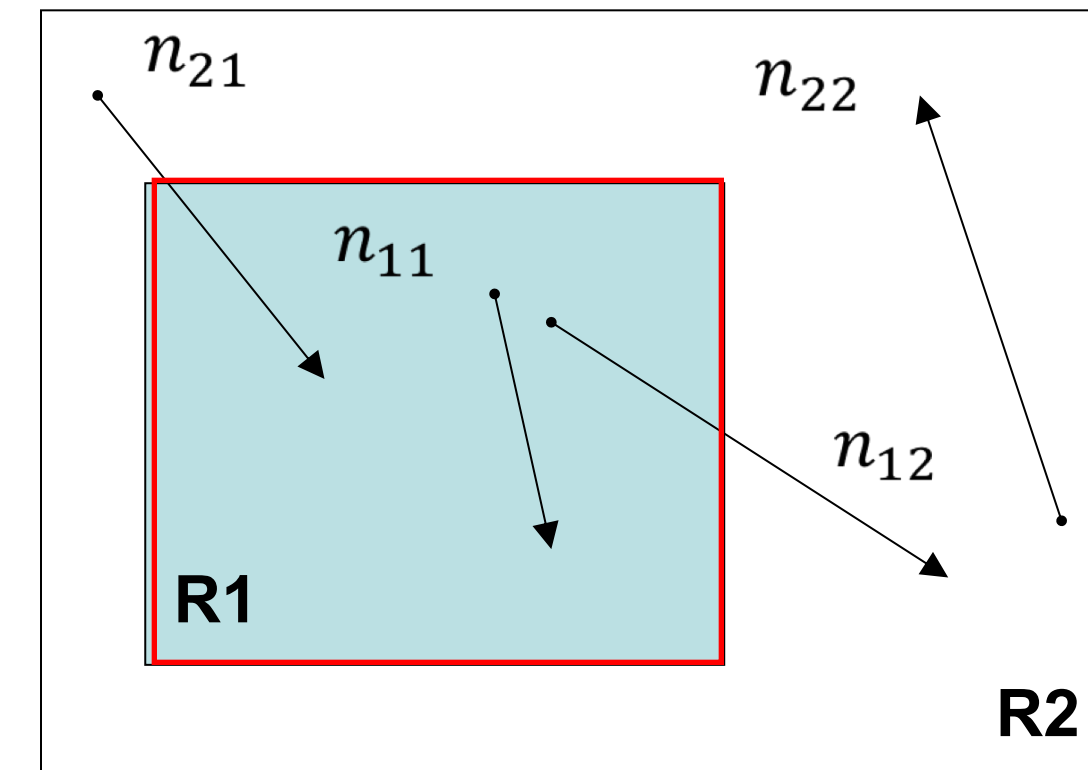


$$\frac{dn}{dt} = q(t) - G(n(t))$$

Daganzo, 2007

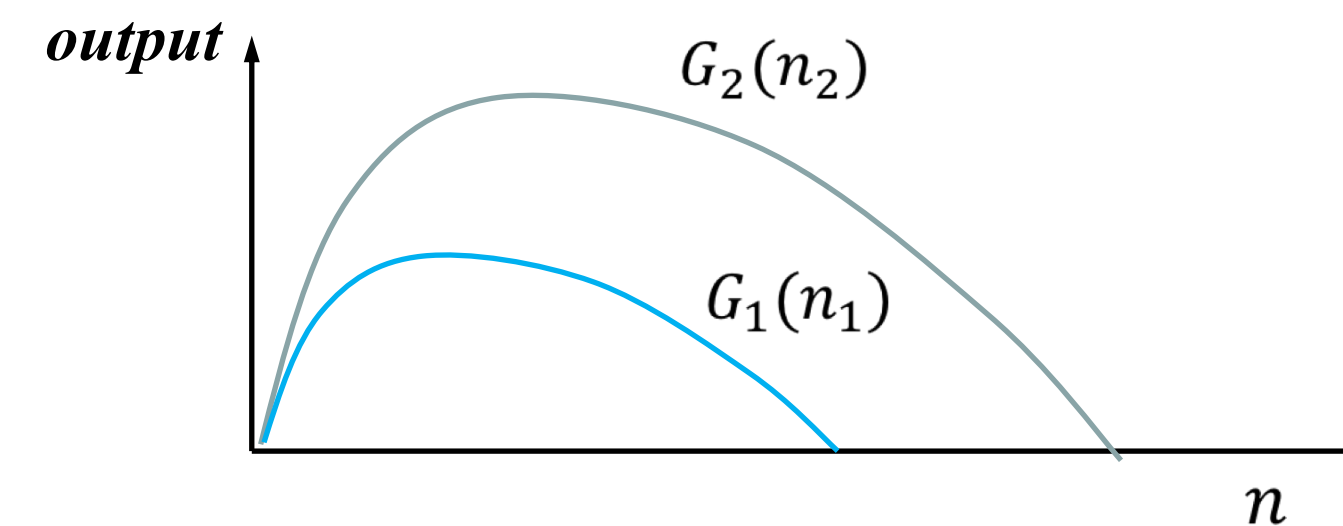
Dynamics of a two-reservoir system

- State Variables
- Dynamic Equations



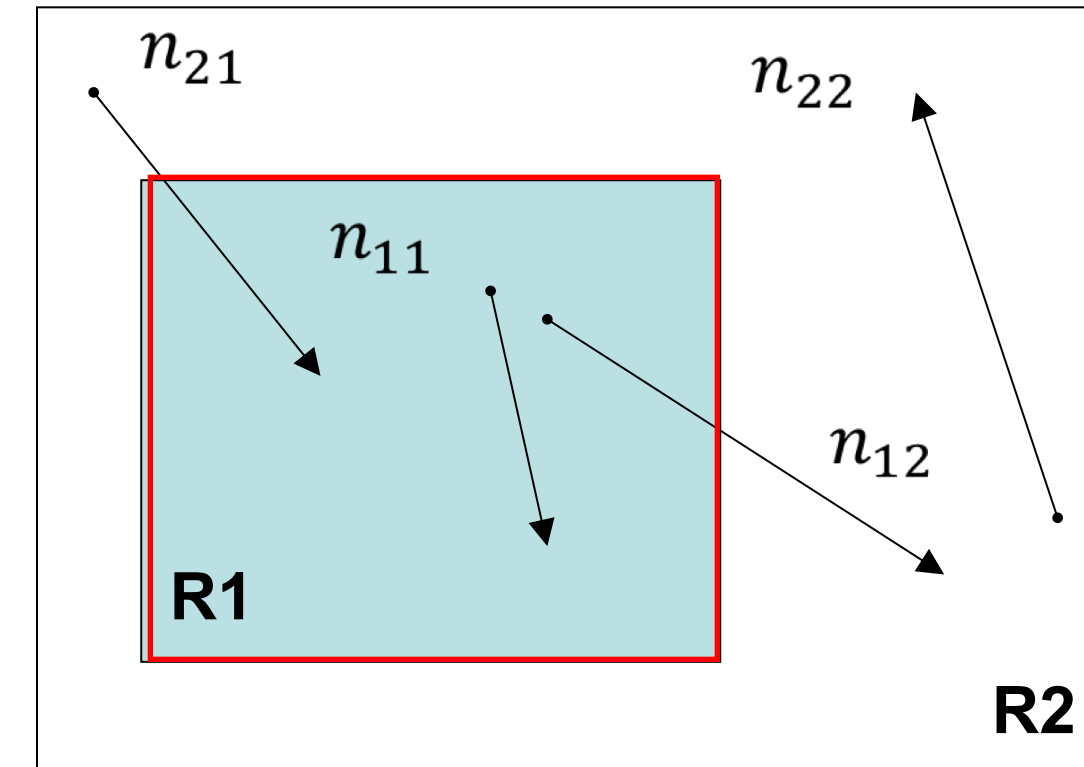
$$\frac{dn_{11}}{dt} = q_{11} + Q_{1 \rightarrow 2} - \frac{n_{11}}{n_1} * G_1(n_1)$$

$$\frac{dn_{12}}{dt} = q_{12} - Q_{1 \rightarrow 2}$$



Dynamics of a two-reservoir system

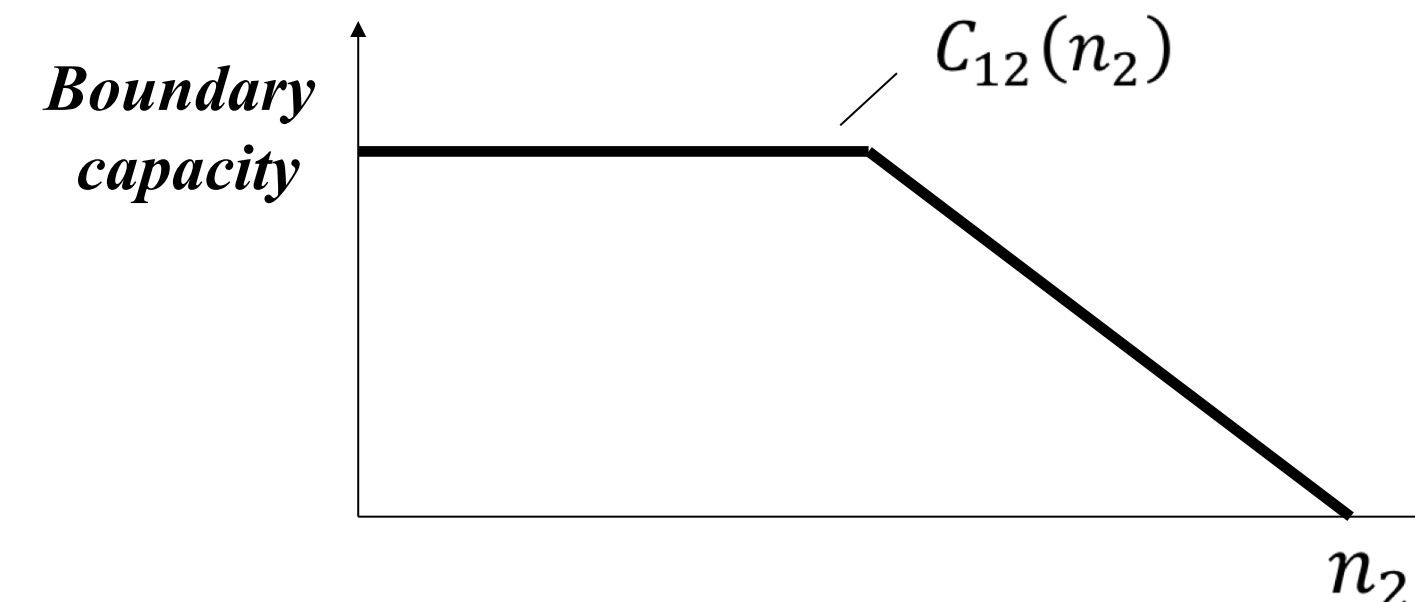
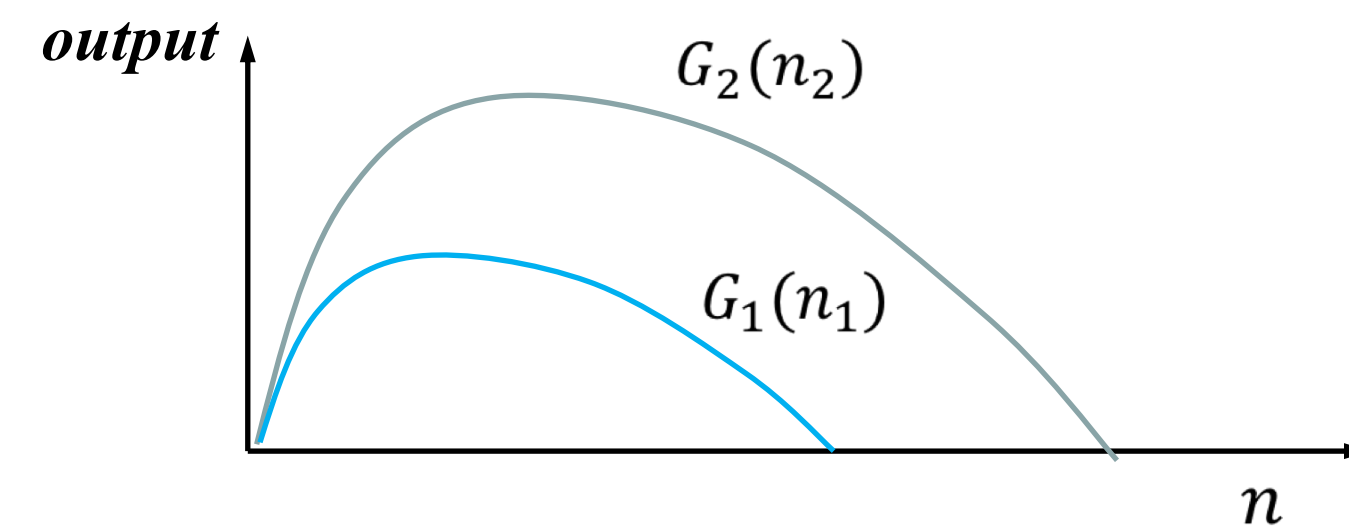
- State Variables
- Dynamic Equations
- Boundary Capacity



$$\frac{dn_{11}}{dt} = q_{11} + Q_{1 \rightarrow 2} - \frac{n_{11}}{n_1} * G_1(n_1)$$

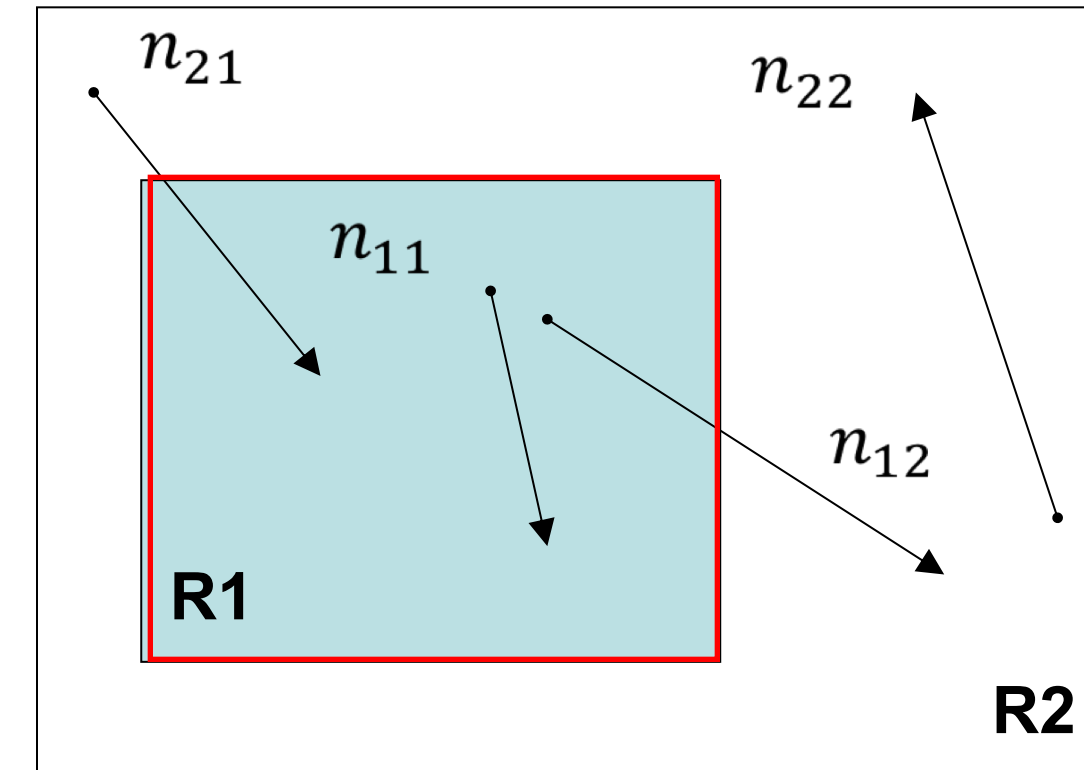
$$\frac{dn_{12}}{dt} = q_{12} - Q_{1 \rightarrow 2}$$

$$Q_{1 \rightarrow 2} = \min \left(C_{12}(n_2), \frac{n_{12}}{n_1} * G_1(n_1) \right)$$



Dynamics of a two-reservoir system

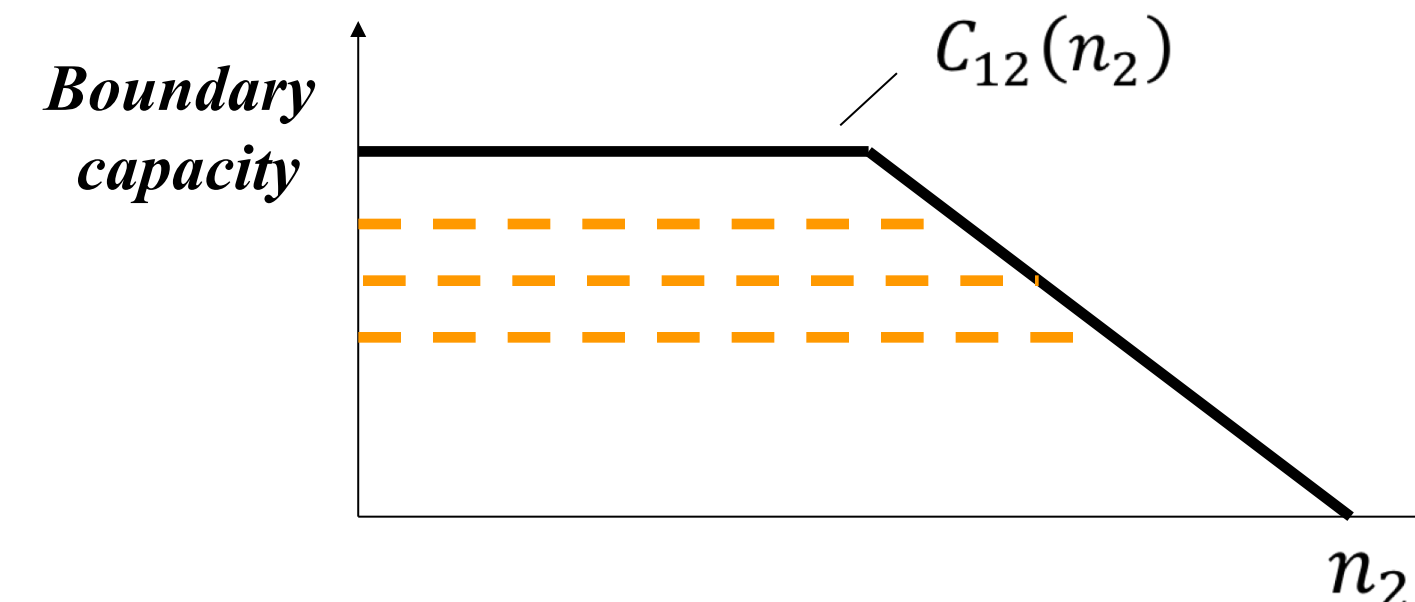
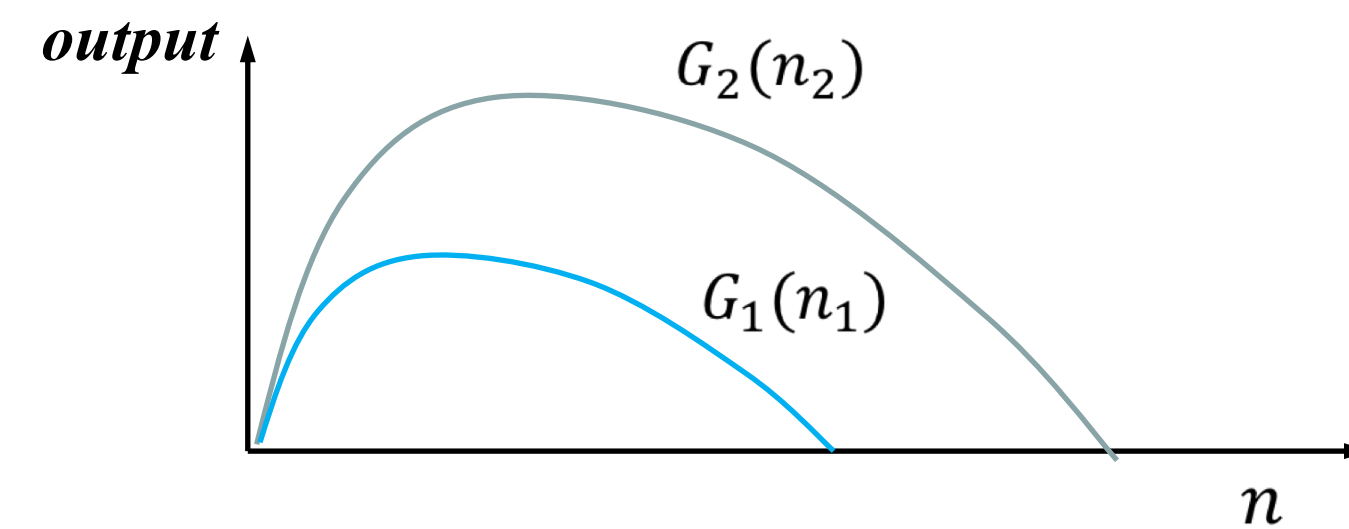
- State Variables
- Dynamic Equations
- Boundary Capacity
- Perimeter Control



$$\frac{dn_{11}}{dt} = q_{11} + Q_{1 \rightarrow 2} - \frac{n_{11}}{n_1} * G_1(n_1)$$

$$\frac{dn_{12}}{dt} = q_{12} - Q_{1 \rightarrow 2}$$

$$Q_{1 \rightarrow 2} = \min \left(C_{12}(n_2), \frac{n_{12}}{n_1} * G_1(n_1) \right)$$



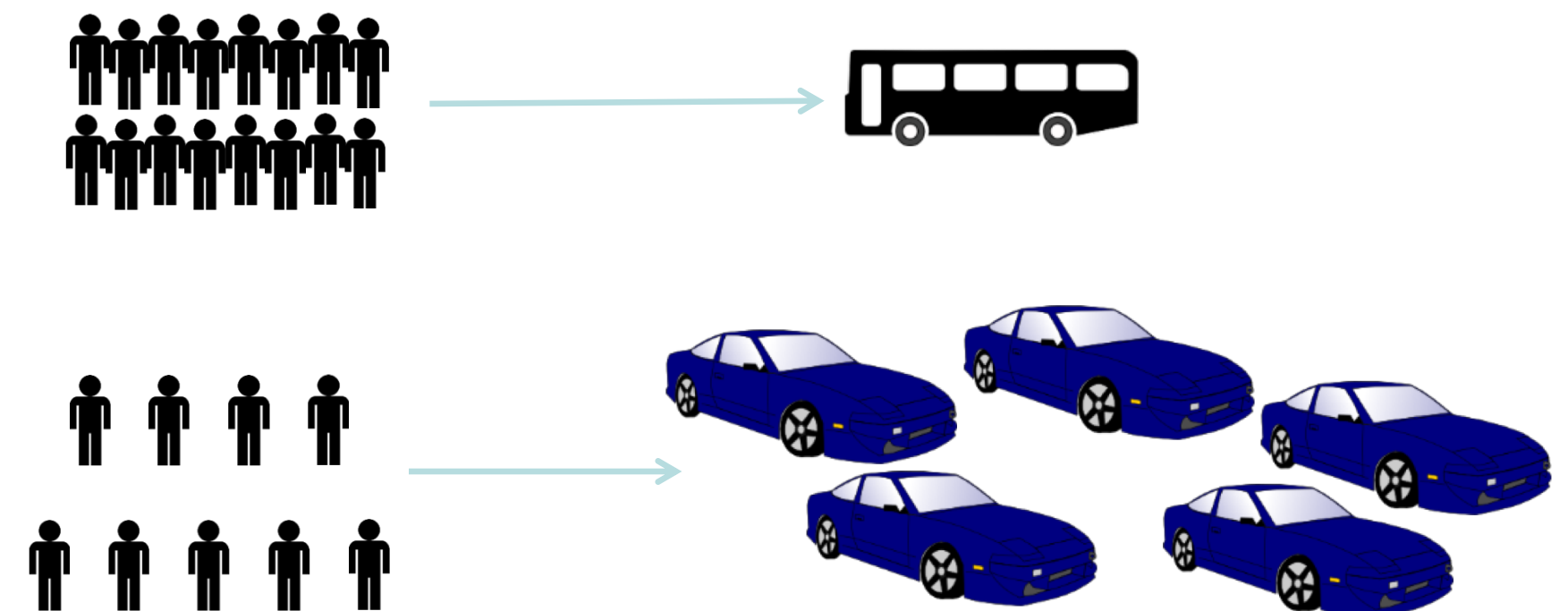
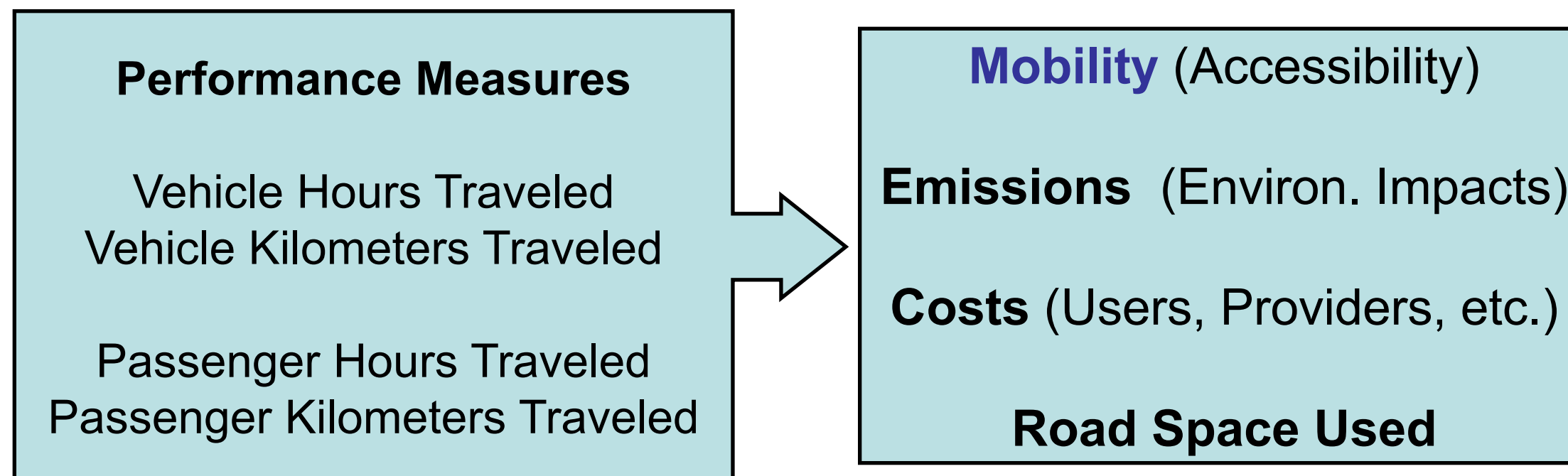
Multimodal networks

- In urban networks, buses share the same network with the other vehicles.
- Conflicts in multi-modal urban traffic systems:
- Bus stops affect the system like red signals in a single lane (instead of blocking all lanes).

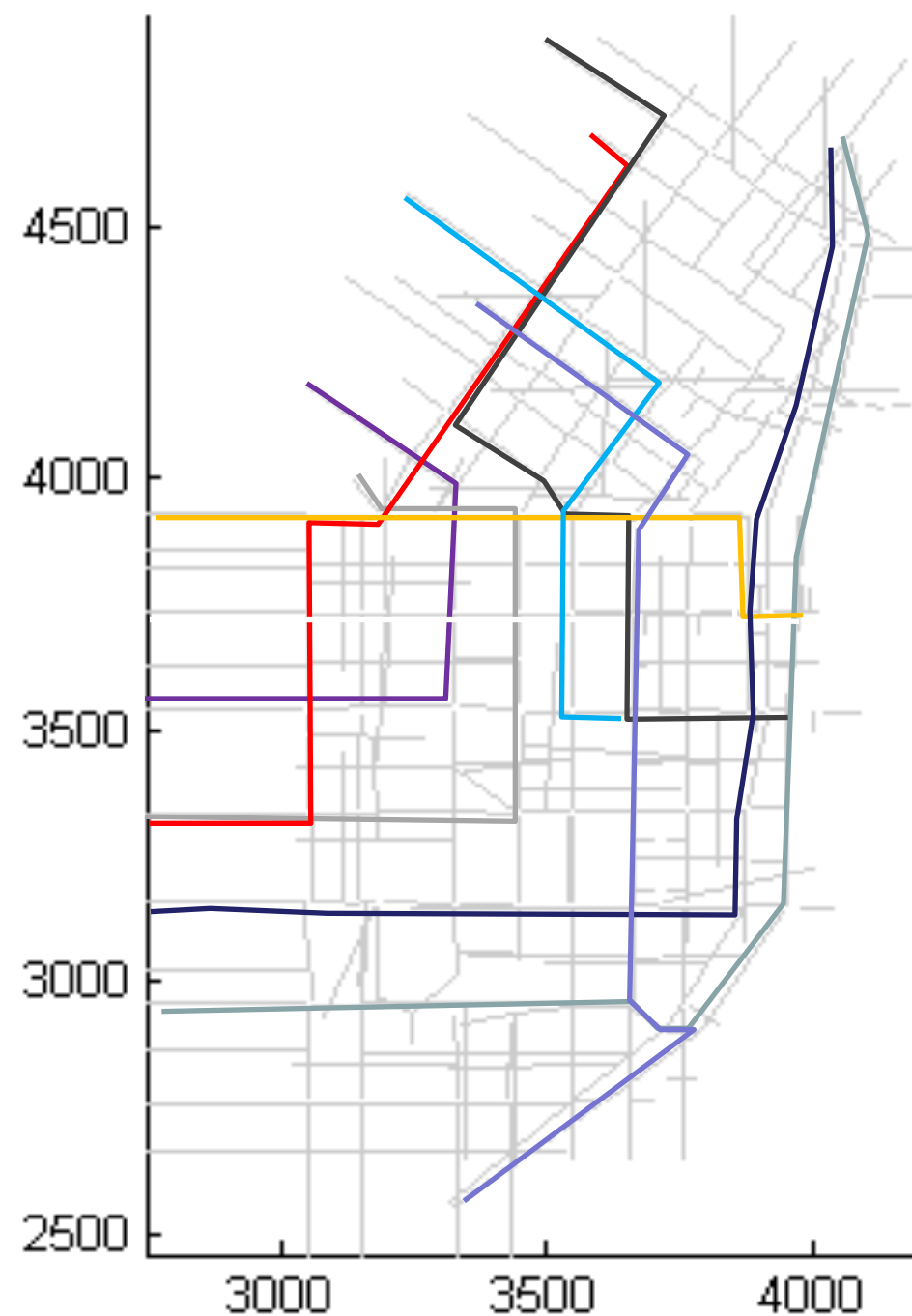
bus frequency

flow of vehicles

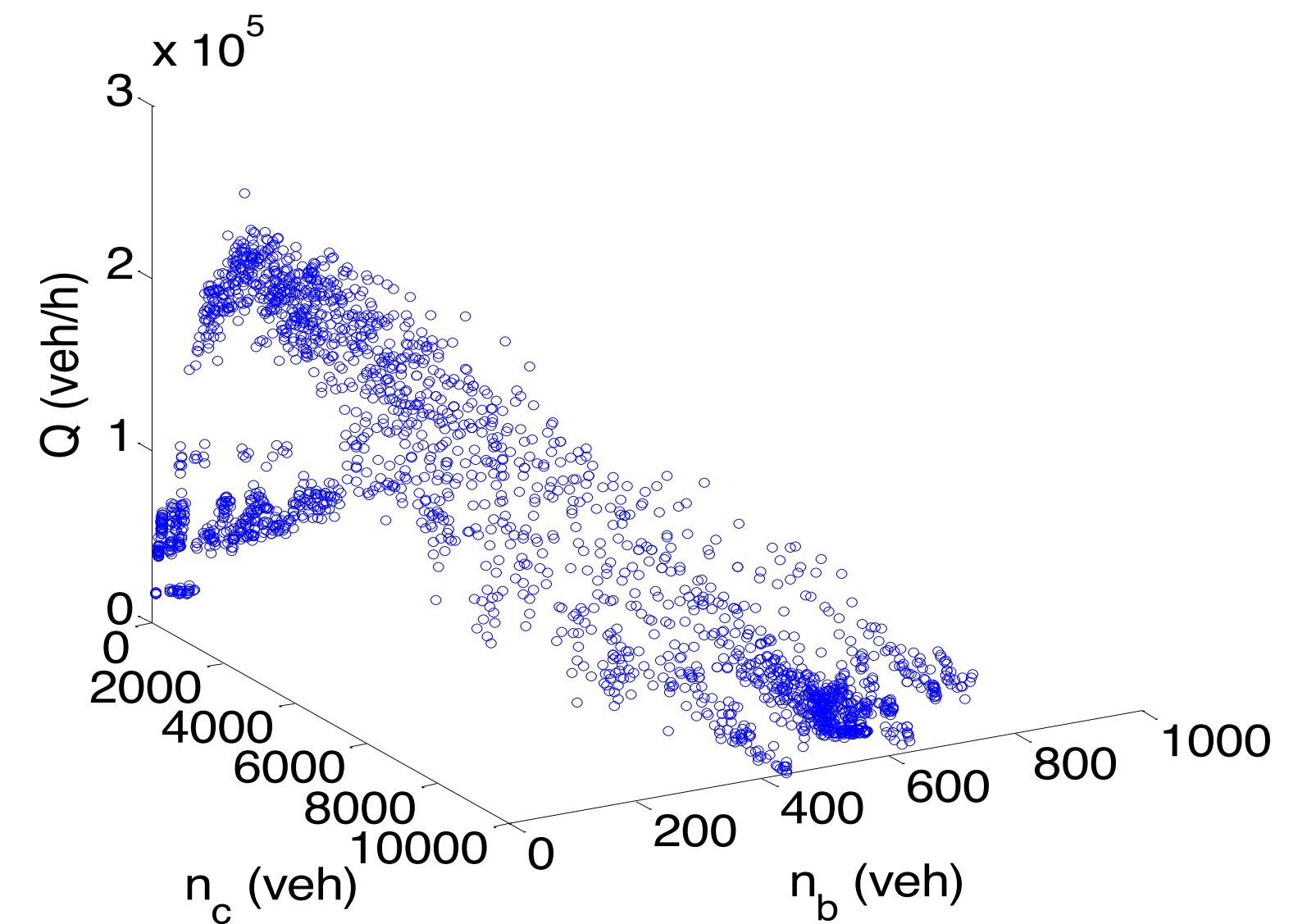
flow of passengers



3-dimensional MFD for bus-car systems

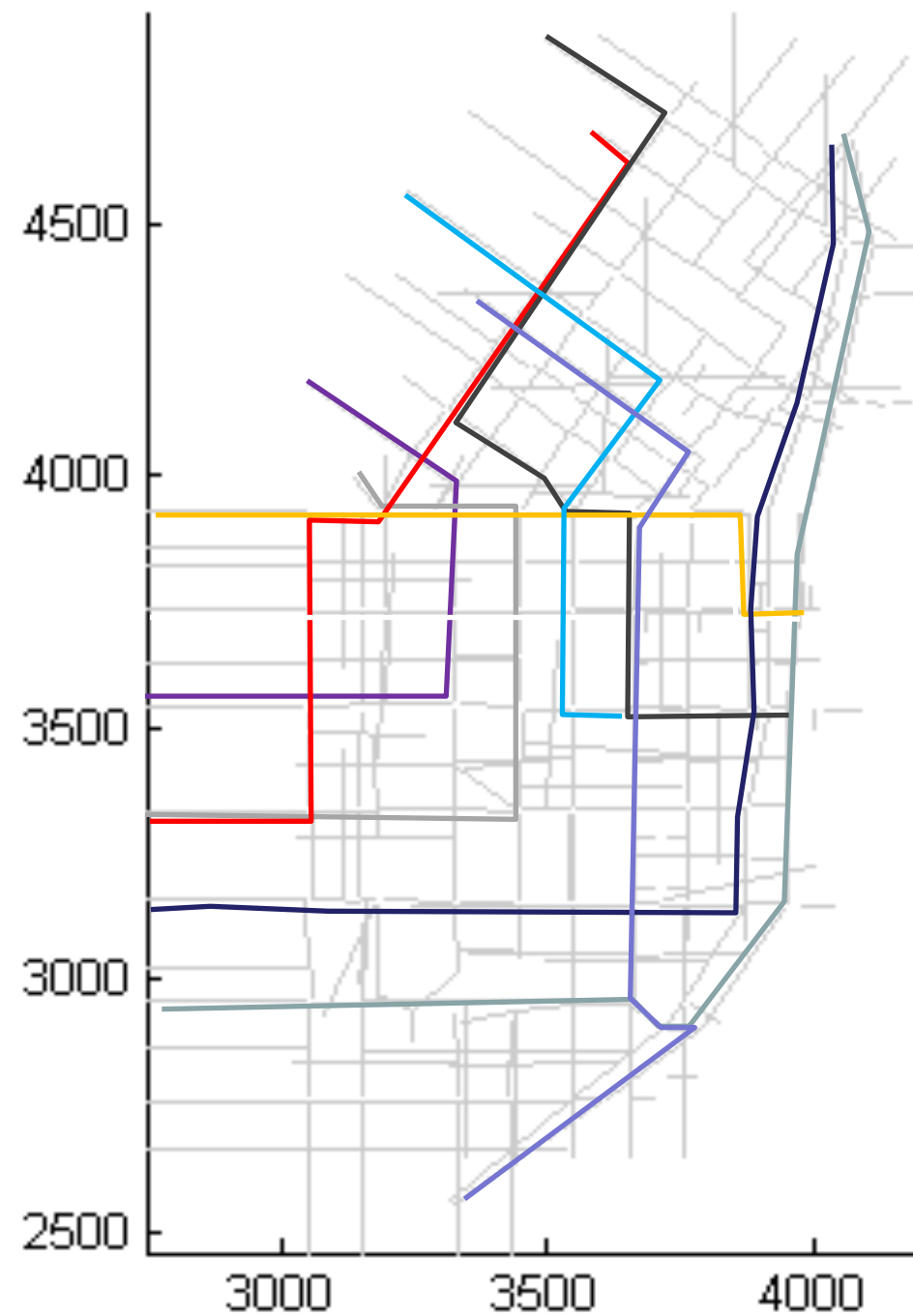


$$Q = Q_c + Q_b = n_c * v_c + n_b * v_b$$



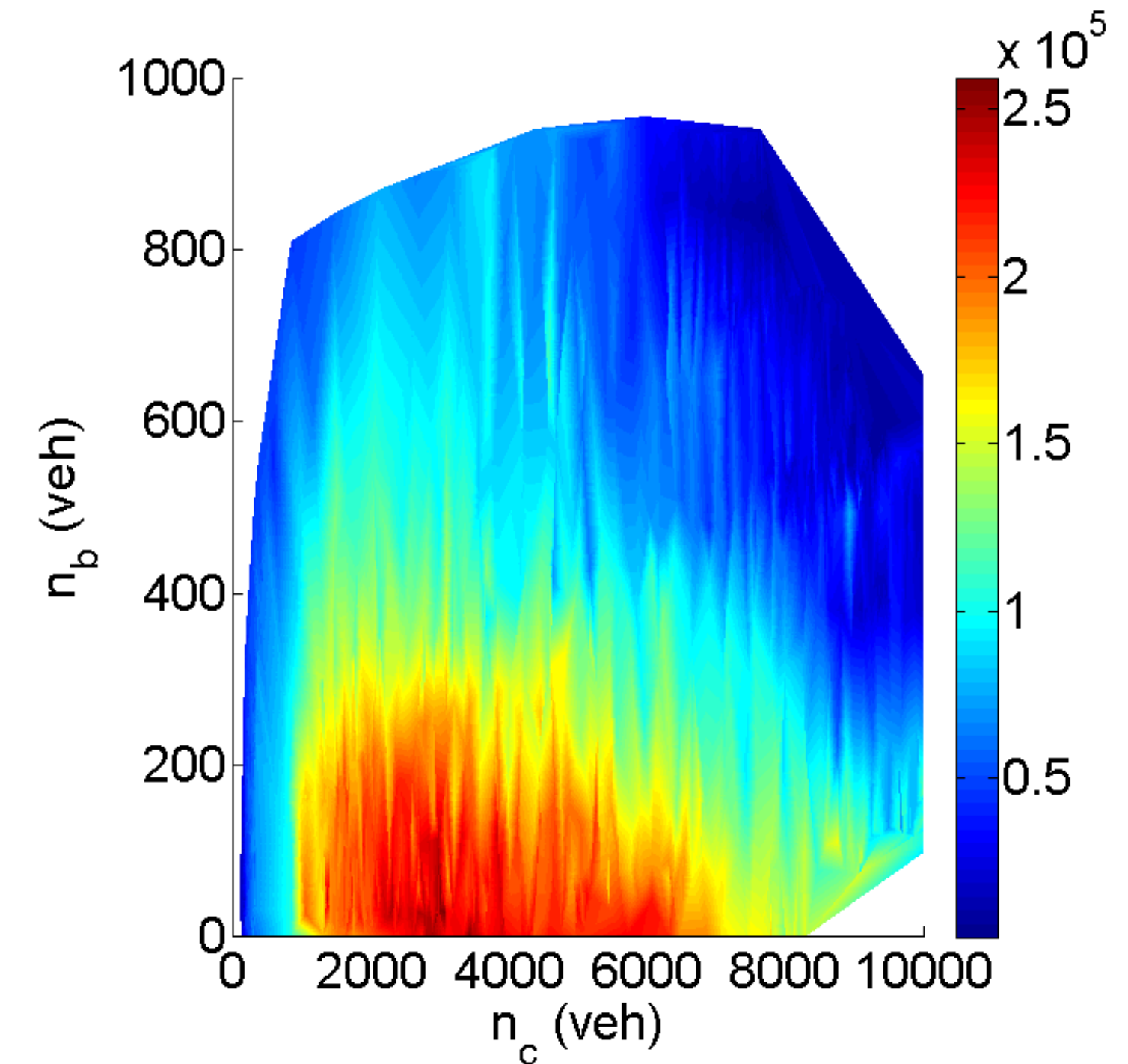
- h_c the occupancy of cars (passengers/veh)
- h_b the occupancy of buses (passengers/veh)
- P_v Production of vehicles (VKT/u.t.)
- P_p Production of passengers (VKT/u.t.)

3-dimensional MFD for bus-car systems



$$Q = Q_c + Q_b = n_c * v_c + n_b * v_b$$

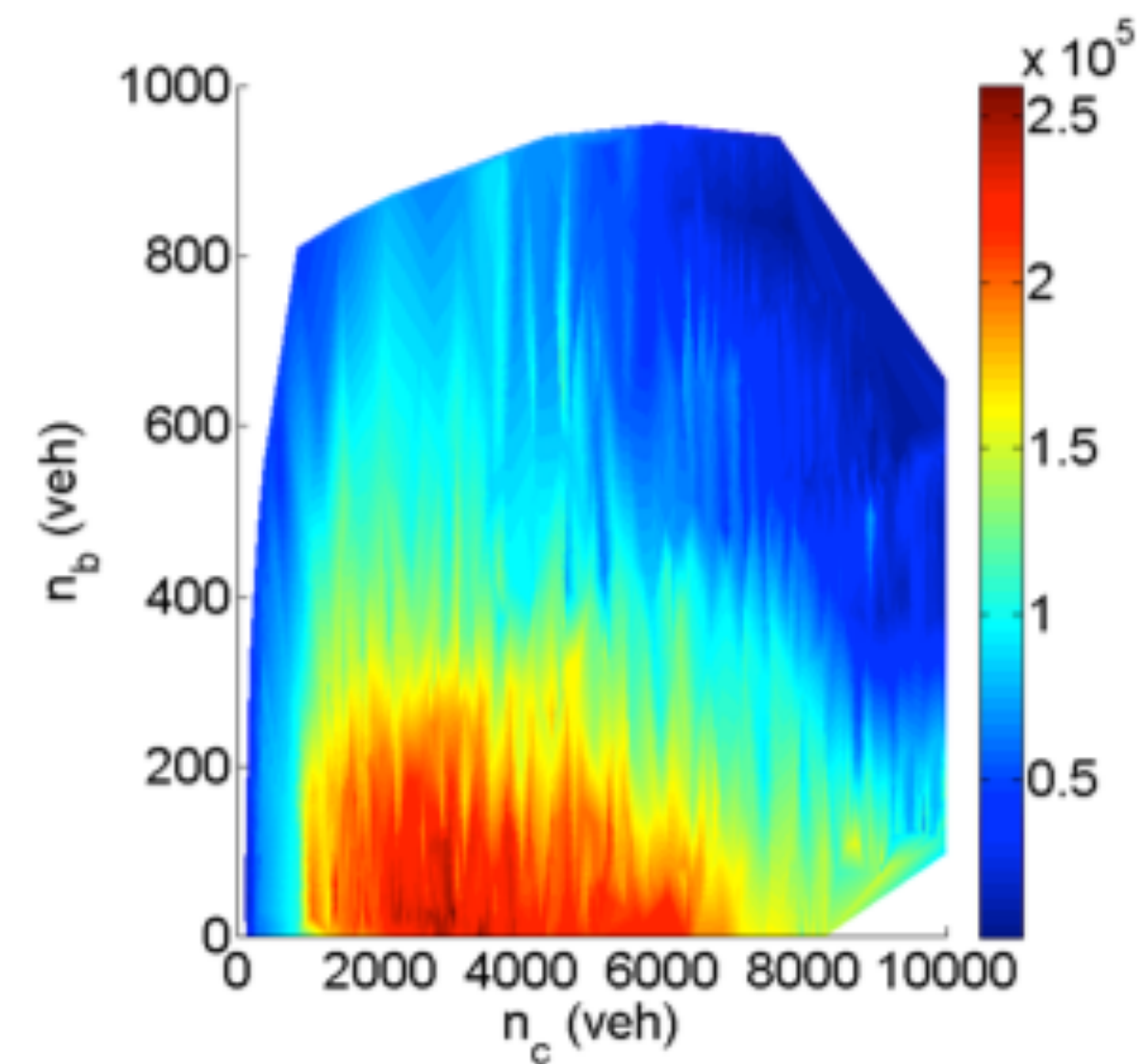
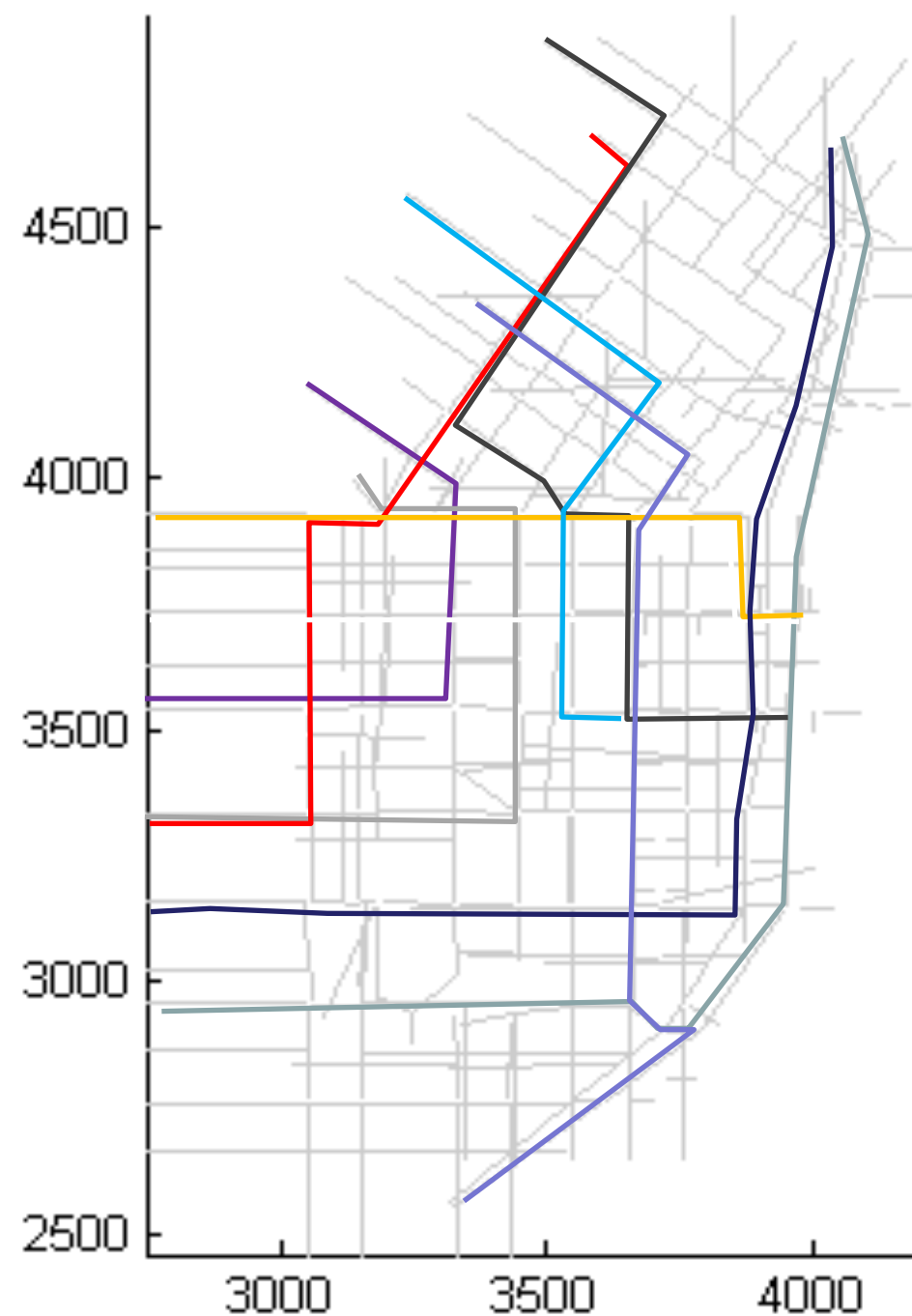
$$P_p = h_c P_c + h_b P_b$$



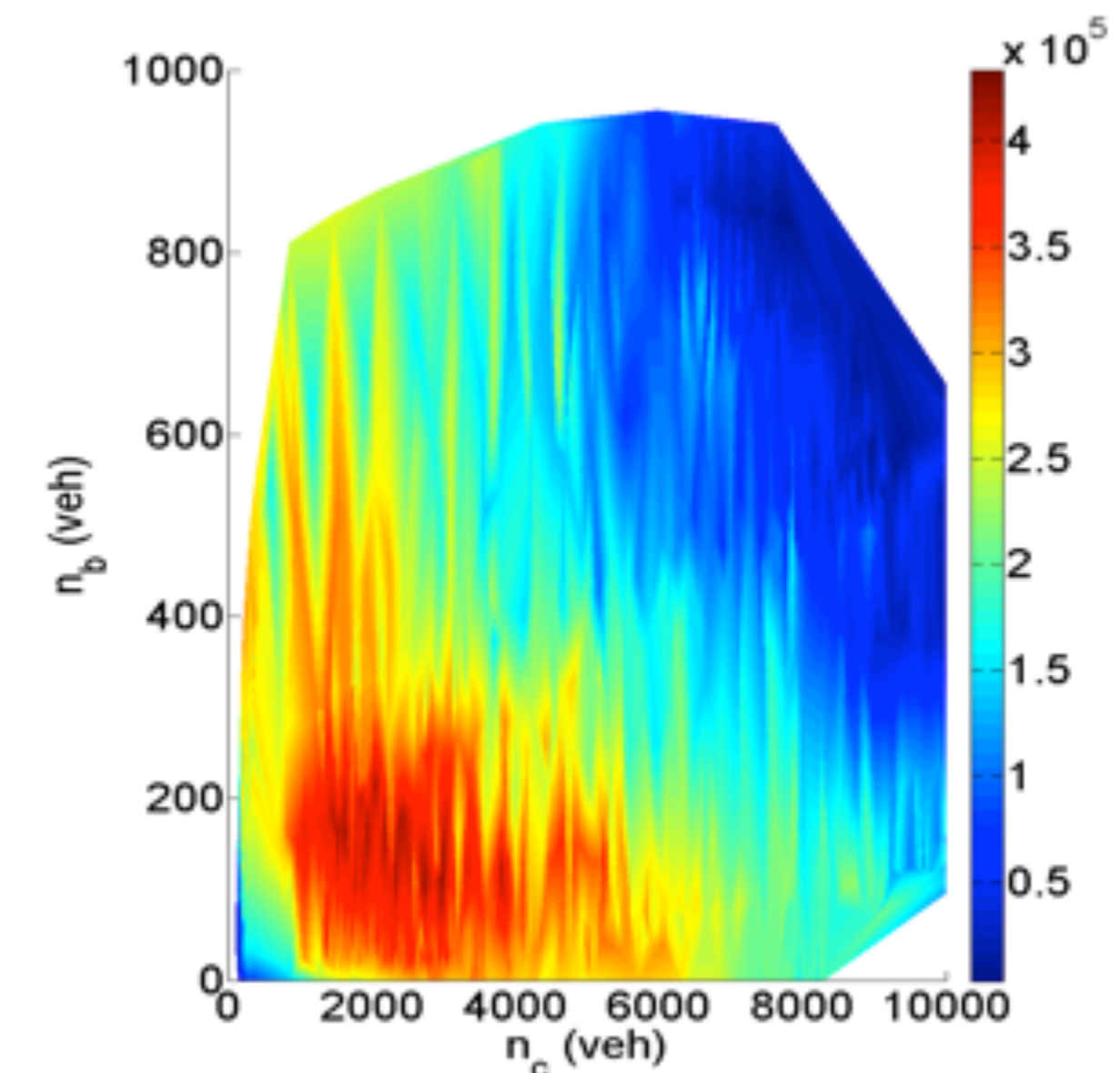
- h_c the occupancy of cars (passengers/veh)
- h_b the occupancy of buses (passengers/veh)
- P_v Production of vehicles (VKT/u.t.)
- P_p Production of passengers (VKT/u.t.)

3-dimensional MFD for bus-car systems

VEHICULAR FLOW



PASSENGER FLOW



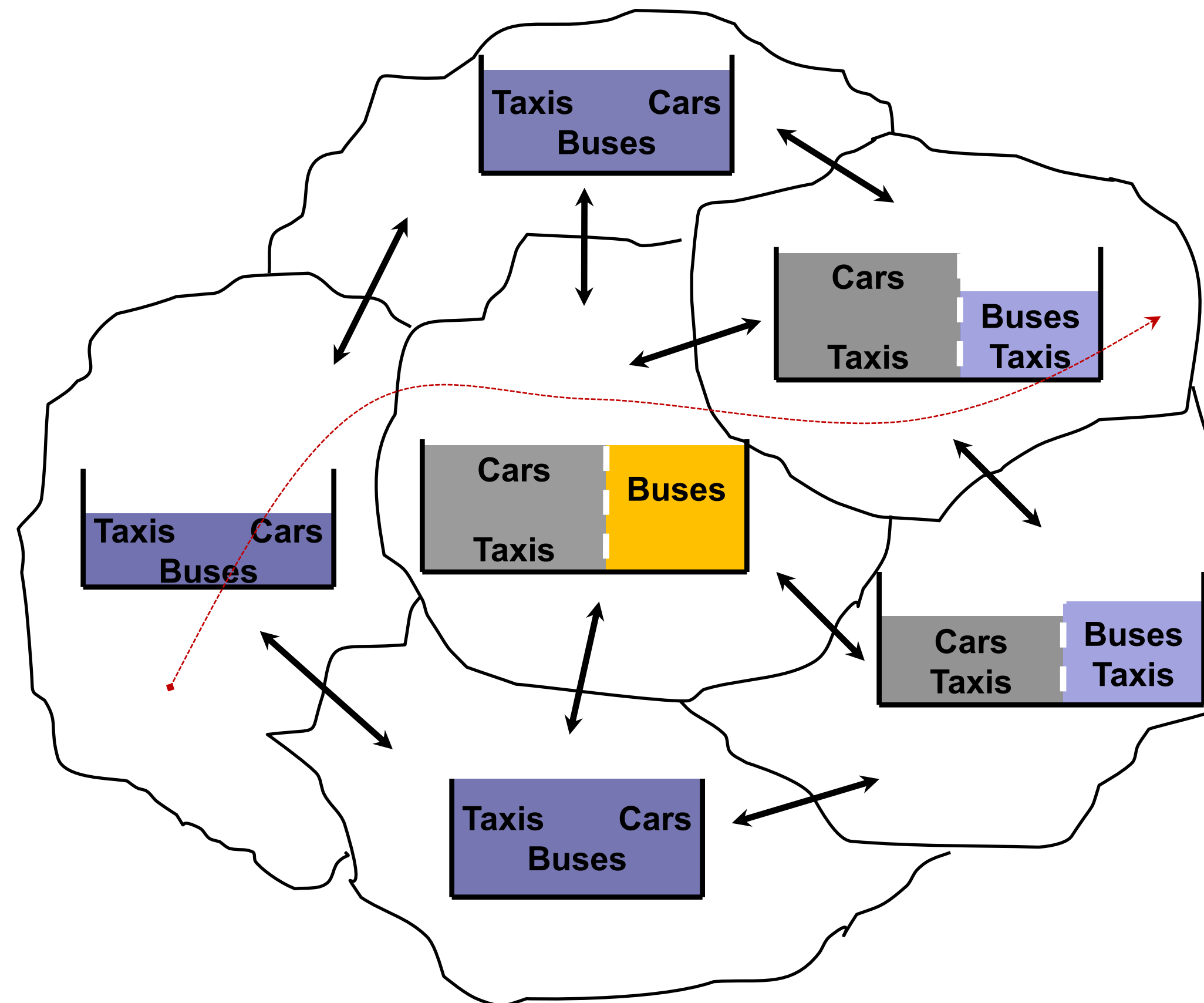
h_c the occupancy of cars (passengers/veh)
 h_b the occupancy of buses (passengers/veh)
 P_v Production of vehicles (VKT/u.t.)
 P_p Production of passengers (VKT/u.t.)

$$P = P(n_c, n_b) = h_c Q_c + h_b Q_b$$

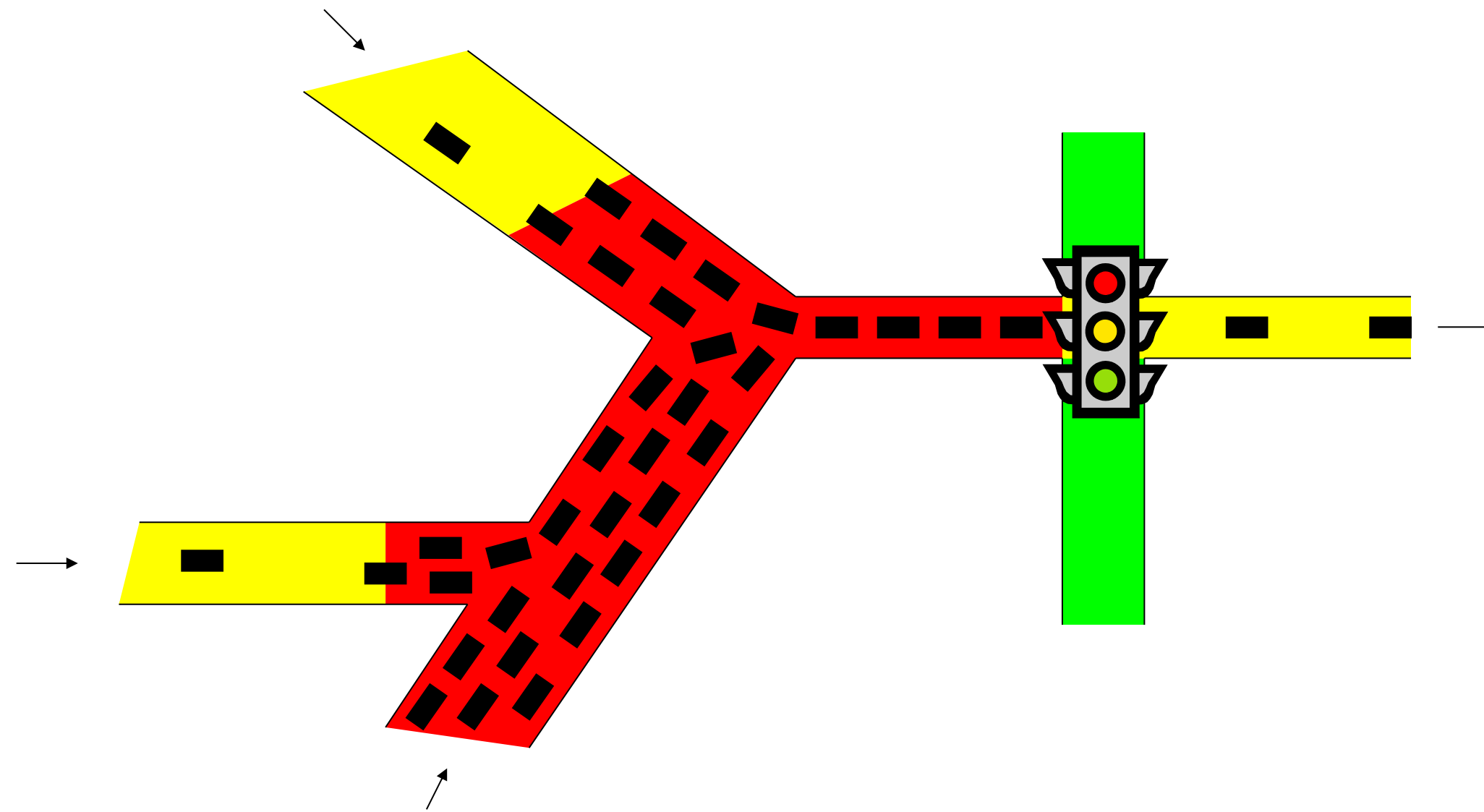
Multimodal multi-reservoir networks



- Spatial allocation of road space
- Special treatment of buses

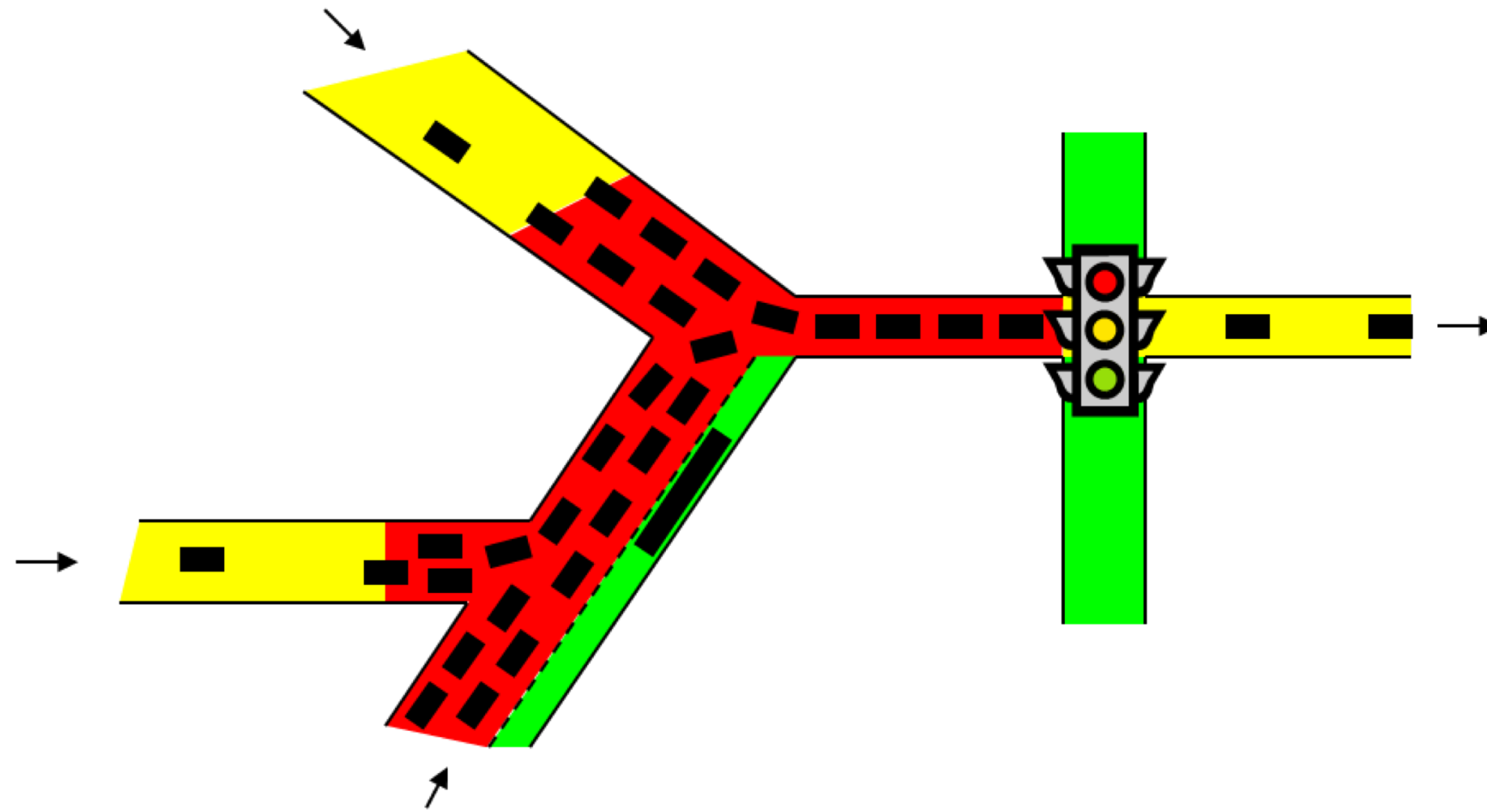


An example of bus-car interactions



Queues form at locations with limited capacity,
but spill-over to other locations

An example of bus-car interactions



Queues form at locations with limited capacity,
but spill-over to other locations