

Exercise 1:

Second-order:

$$\begin{array}{ccc} U_1 & \xrightarrow{\quad} & \boxed{\chi^{(2)}} \\ \downarrow & \xrightarrow{\quad} & \end{array} \begin{array}{c} \xrightarrow{\quad} 2U_1 \\ \xrightarrow{\quad} 2U_2 \\ \xrightarrow{\quad} U_1 + U_2 \\ \xrightarrow{\quad} U_2 - U_1, U_1 - U_2 \end{array} \left. \begin{array}{l} \text{possible} \\ \text{frequencies} \end{array} \right\}$$

Boyd 1.1.2.

→
See next page
for work out.

All of these processes can be used in the generation of light
but also to study materials.

SHG: Surface Spectroscopy

Surface imaging or bulk imaging

SH scattering → particle interfaces

nanoscale structure in bulk liquids.

Mostly probe of electronic structure; resonant or non-resonant

STG: Often used as surface vibrational spectroscopy

to measure a vibrational spectra of interfaces - planar

or particle interfaces. Another STG process is the linear electro-optic effect.

SHG / STG are useful because of their surface specificity. This property follows from the spatial symmetry properties of the material under study.

Third-order processes:

There are many third-order processes; some are used to generate new frequencies, others as switches, or to manipulate light, to probe matter, or as a distortion to experiments.

$$E(t) = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t} + \text{c.c.}$$

$$P^{(2)} = \epsilon_0 \chi^{(2)} E(t)^2$$

$$P^{(2)} = \epsilon_0 \chi^{(2)} \left[E_1^2 e^{-2i\omega_1 t} + E_2^2 e^{-2i\omega_2 t} + 2E_1 E_2 e^{-i(\omega_1 + \omega_2)t} + 2E_1 E_2^* e^{-i(\omega_1 - \omega_2)t} + \text{c.c.} \right] +$$

SHG

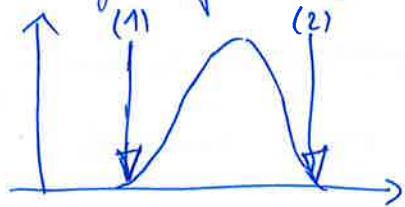
SHG

SFG

DFG

$$+ 2\epsilon_0 \chi^{(2)} [E_1 E_1^* + E_2 E_2^*]$$

~~OR~~ OR (optical rectification)
 $= \text{DFG of the pulse with itself } (2) - (1) \Rightarrow \text{get very low frequencies (THz)}$



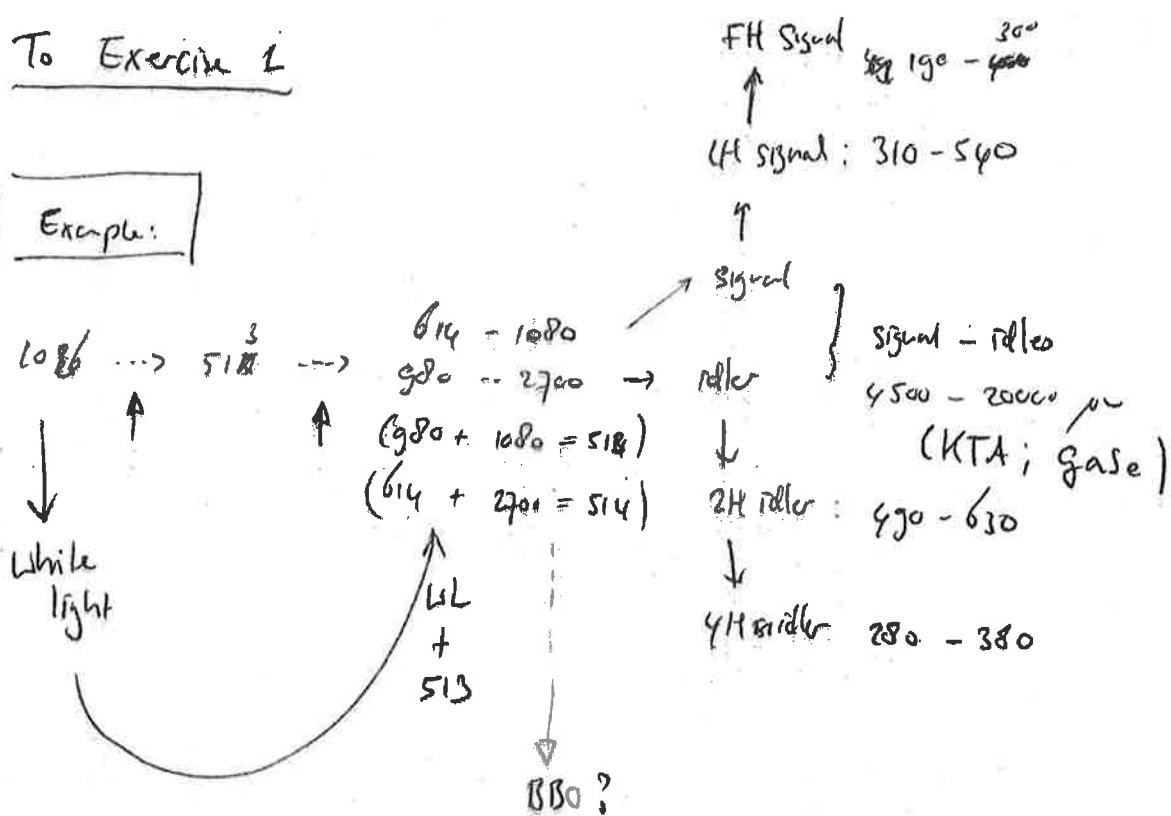
Exercise 1, continued:

Examples:

- Third harmonic generation
- Intensity dependent refractive index
- Kerr effect
- Electric field induced second harmonic generation.

To Exercise 2

Example:



Exercise 2

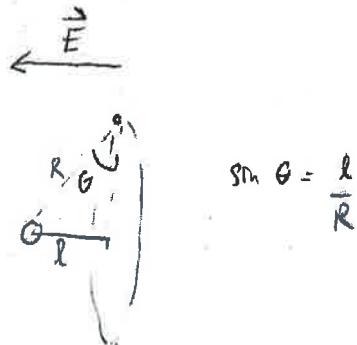
2a.

Attraction of nucleus with electron (Coulombic): centripetal (F_p)

Reaction force due to rotating motion in fixed reference frame: centrifugal (F_f)



- b) $m_e \approx 10^{-31} \text{ kg}$ $m_{Hr} \approx 10^{-27} \text{ kg}$ - the electron moves, the nucleus reacts
fixed



$$F_e = \frac{-e^2}{4\pi\epsilon_0\epsilon R^2}$$

$$\vec{F}_{ind} = -e \cdot \vec{E} = -\frac{e^2 \cdot \sin \theta}{4\pi\epsilon_0\epsilon R^2} = -\frac{e^2 \cdot l}{4\pi\epsilon_0\epsilon R^3} \Rightarrow \frac{e \cdot P_{ind}}{4\pi\epsilon_0\epsilon R^3} = eE$$

$$P_{ind} = -e \cdot l$$

$$= \bar{\lambda} K_0 E$$

$$\uparrow \\ e: -; E: -$$

$$P_{ind} = \underbrace{4\pi\epsilon_0\epsilon R^3}_{K_0} \cdot E$$

- c. We have $P^{(1)} = N \cdot \langle p^{(1)} \rangle$ for a gas this is simply $N \cdot p^{(1)}$

$$P^{(1)} = \epsilon_0 \chi^{(1)} E = N \cdot p^{(1)} = N \cdot 4\pi\epsilon_0\epsilon R^3 \cdot E$$

$$\Rightarrow \chi^{(1)} = 4\pi\epsilon R^3 \cdot N \quad \text{and} \quad \epsilon = 1 + \chi^{(1)}$$

the gas

$$\frac{m^3}{m^3} \frac{\text{Molecules}}{m^3}$$

$$R^3 = (5.29 \cdot 10^{-11})^3 = 1.48 \cdot 10^{-31} \text{ m}^3$$

$$\chi^{(1)} = 0.04$$

$$\epsilon = 1.04$$

$$d = 70.85 \frac{\text{kg}}{\text{m}^3}$$

$$M = 2 \frac{\text{g/mol}}{\text{kg/mol}}$$

$$= 2 \cdot 10^{-3} \frac{\text{kg}}{\text{mol}}$$

$$\left\{ \begin{array}{l} N = 70.85 \cdot 500 \cdot 6.02 \cdot 10^{23} = 2.13 \cdot 10^{29} \frac{\text{molec.}}{\text{m}^3} \\ \frac{\text{kg}}{\text{m}^3} \cdot \frac{\text{mol}}{\text{kg}} \cdot \frac{\text{molec.}}{\text{mol}} \end{array} \right.$$

$$p = e \cdot l \quad \text{dipole moment} \quad [\text{C} \cdot \text{m}]$$

$$P = \underbrace{N \lambda E}_p \quad \text{polarizability (density of dipoles)} \quad \left[\frac{\text{C} \cdot \text{m}}{\text{m}^3} \right]$$

Exercise 3

$$3a. E = \frac{e}{4\pi\epsilon_0 R^2} \rightarrow E = 5 \cdot 10^{11} \frac{V}{m}$$

$$a_0 = 5.29 \cdot 10^{-11} m$$

$$b. I = 2n\epsilon_0 c |E|^2 = 1.8 \cdot 10^{17} W/cm^2$$

$$c. P_{peak} = \frac{P_{avg}}{f \cdot \tau} \quad (\text{rep rate; pulse dur})$$

$\frac{P_{avg}}{f}$ = pulse energy

$$I_{peak} = \frac{P_{peak}}{A} = \frac{P_{avg}}{f \cdot \tau \cdot A}$$

$$0.013 \cdot f = 1$$

$$f = 77 \text{ Hz; so not with 1 MHz.} \\ (\text{due to the small focus}) \quad -1 \text{ MHz.}$$

Note that with this type of ^{pulse energy} intensity (13 mJ) there is typically already white light generation at this narrow focus.