Last Name .... First Name..................

Neural Networks and Biological Modelling Exam
23 June 2010

- Write your name in legible letters on top of this page.
- The exam lasts 160 min.
- Write **all** your answers in a legible way on the exam (no extra sheets).
- No documentation is allowed apart from 1 sheet A5 of **handwritten** notes.
- No calculator is allowed.
- Have your student card displayed before you on your desk.

Evaluation:

1. .... / 11 pts
2. .... / 12 pts
3. .... / 22 pts
4. .... / 11 pts

Total: .... / 56 pts

1 Ion Channel (11 points)

Consider the following model for an ion channel:

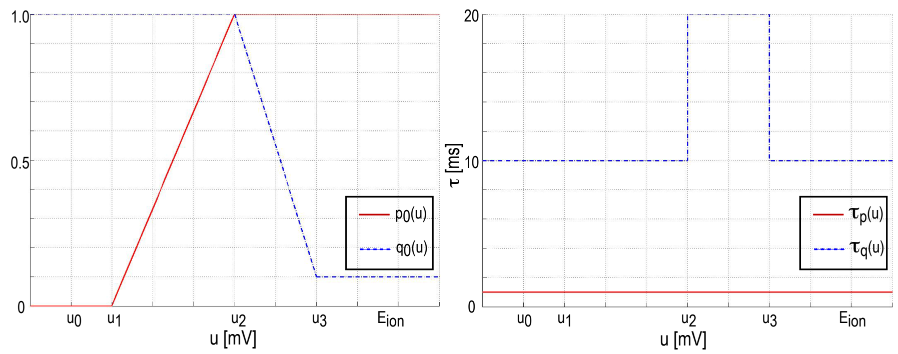
$$I_{\text{ion}} = \bar{g}_{\text{ion}} \left[u(t) - E_{\text{ion}} \right] p^4 \, q \tag{1}$$

where $u(t)$ is the membrane potential of the neuron, $\bar{g}_{\text{ion}}$ and E_{ion} are two constants. The variables p and q obey the following equations:

$$\frac{dp}{dt} = -\frac{p - p_0(u)}{\tau_p(u)} \tag{2}$$

$$\frac{dq}{dt} = -\frac{q - q_0(u)}{\tau_q(u)} \tag{3}$$

with p_0, q_0, τ_p and τ_q as shown in the figure below.



a) [2pts] What is the biological interpretation of the following parameters and variables:

$\bar{g}_{\text{ion}}$: ....

E_{ion}: ....

p: ....

q: ....

b) [3pts] How does the channel react to a voltage step? Suppose that for $t < 0$, the neuron is at u_0. At $t = 0$ the voltage is clamped to a new value u_3mV for 1 second.

- For $t < 0$ ms, the channel is ....

  because ....

  ....

- At $t = 0$ ms, the channel is ....

  because ....

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- At $t = 1$ ms, the channel is ....

  because ....

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- At $t = 3$ ms, the channel is ....

  because ....

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- At $t = 10$ ms, the channel is ....

  because ....

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- At $t = 20$ ms, the channel is ....

  because ....

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c) [2pts] Sketch the time course of the current I_{ion} that you expect for $0 < t < 50$ ms.

d) [2pts] Suppose you want to study the importance of the variable p under the same stimulation protocol as in (b). Calculate the response of $p(t)$ (1pt).

$p(t) =$..

Sketch the evolution of $p(t)$ that you expect for $0 < t < 50$ ms (1pt).

e) [1pt] Sketch the behavior of $p(t)$, $p^2(t)$ and $p^4(t)$ for $0 < t < 1$ ms.

f) [1pt] What is the difference between $p(t)$ and $p^4(t)$? Explain what this means qualitatively in terms of the dynamics of the ion channel.

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2 Network dynamics and population activity in one dimension (12pts)

We consider the population activity $A = g(h)$ of a single population, where the input potential h evolves according to the differential equation

$$\tau \frac{dh}{dt} = -h + J\, g(h) + h_0 \tag{4}$$

with a coupling weight J.

a) [1pt] What is the meaning of the parameter h_0

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b) [2pts] What are the assumptions and/or approximations necessary to arrive at the above equation, starting from a population of integrate-and-fire neurons.

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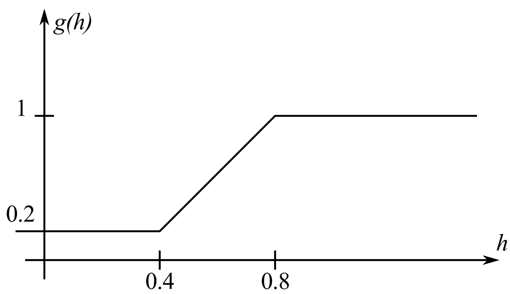
Assume (as shown on the right)
$g(h) = 0.2$ for $h < 0.4$
$g(h) = 2h - 0.6$ for $0.4 \leq h \leq 0.8$
$g(h) = 1$ for $h > 0.8$



c) [2pts] Set $h_0 = 0$ and $J = 1$. Determine analytically the position of the fixed point or fixed points.

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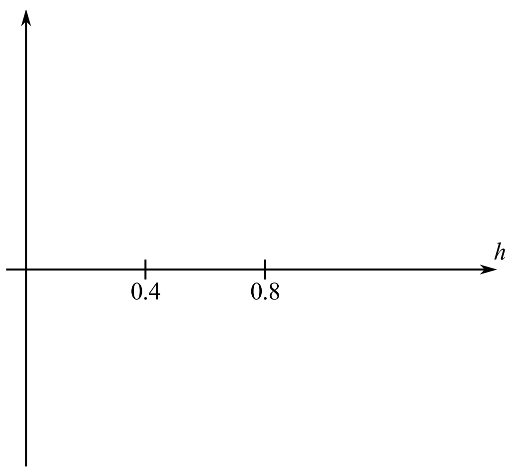
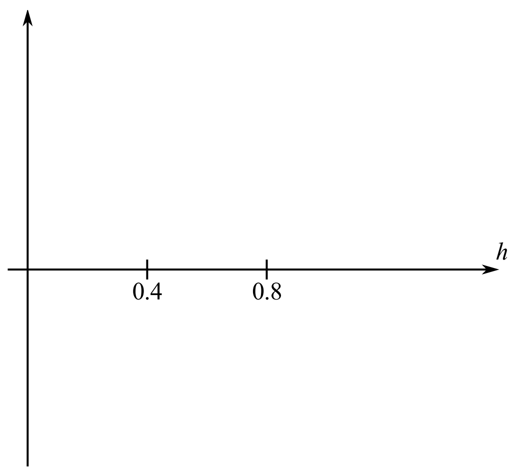
d) [3pts] In the space below:

- Plot dh/dt as a function of h. [1pt]
- Draw arrows for the flow. [1pt]
- Determine graphically the stability of the fixed point or fixed points, and mark the result on the plot (write "S" for stable and "U" for unstable). [1pt]

e) [2pts] Still with same conditions, draw qualitatively the function $A(t) = g(h(t))$, on the axes below. Use four different initial conditions, that enable you to characterize the behavior of the system.

f) [2pts] Keep $J = 1$ but take h_0 as a free parameter. Construct the fixed point or fixed points graphically using the two empty axes sets below. Pick two different values of h_0 so that the solution or set of solutions is qualitatively different, and mark the fixed points in the two graphs.





3 Network dynamics and population activity in two dimensions (22pts)

We study a network analogous to the one in the previous exercise. However, we assume that the interaction weights J are subject to fatigue (also called synaptic depression) when they are used intensively. We write $J = J_0\, x$ with some variable x and a fixed parameter J_0 The system of equations is:

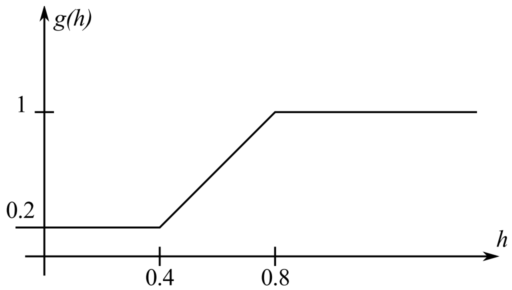
$$\tau \frac{dh}{dt} = -h + J_0\, x\, g(h) \tag{5}$$

$$\tau_x \frac{dx}{dt} = 2.8 - x - 9.0\, x\, g(h) \tag{6}$$

$\tau = 1$ and $\tau_x > \tau$ are time constants (arbitrary units). You may assume that $15 < \tau_x < 30$.

J_0 is a parameter (later we consider $J_0 = 3$ and $J_0 = 2.5$).

As in the previous exercise:
$g(h) = 0.2$ for $h < 0.4$
$g(h) = 2h - 0.6$ for $0.4 \le h \le 0.8$
$g(h) = 1$ for $h > 0.8$
(as shown on the right)



a) [2pts] Consider a fixed activity $g(h) = A_0$ and describe in words the evolution of the variable x, starting from an inititial condition $x = 1$. What is the time scale of evolution? Is there a fixed point? etc. Continue the following two sentences.

"For $A_0 = 0$ the variable x evolves

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"For $A_0 = 1$ the variable x evolves

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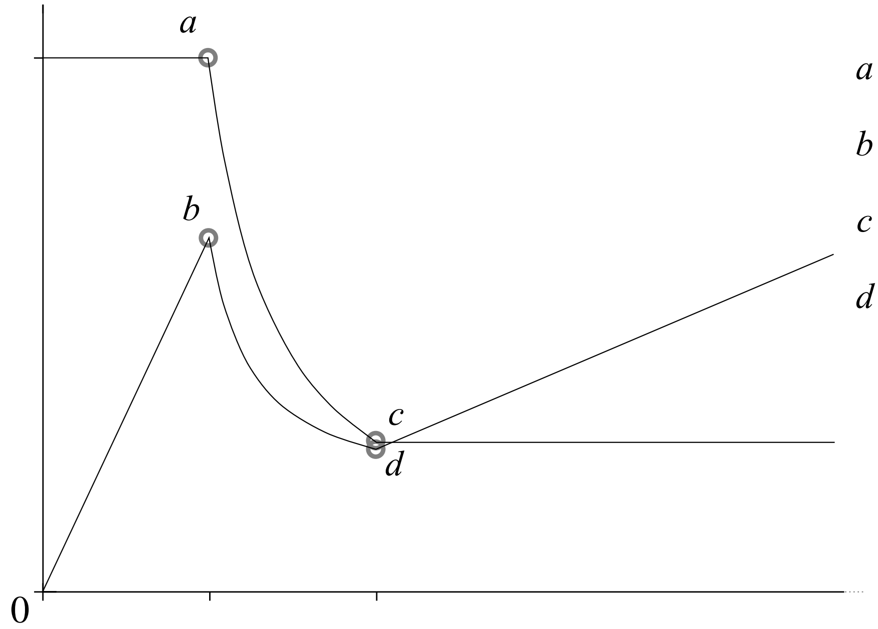
b) [3pts] Assume $J_0 = 3$. The nullclines are shown below, without axis labels or scale.

- Add the axis labels.
- Label the 2 nullclines.
- Give the numerical values for the 4 circled points a, b, c and d.



$a = (\quad ; \quad)$

$b = (\quad ; \quad)$

$c = (\quad ; \quad)$

$d = (\quad ; \quad)$

c) [1pt] Evaluate the differential equations at $(h, x) = (0, 1)$ and draw an arrow in the above graph. (The direction of the arrow should qualitatively point in the right direction, i.e. towards the right quadrant).

d) [2pts] Add in your graph flow arrows along the nullclines (at least four on each nullcline).

e) [2pts] Draw two trajectories. One starting at (1,0) and another one starting at (0,1).

f) [4pts] Determine analytically the stability of the fixed point, using Equations 5 and 6 repeated here for convenience:

$$\frac{dh}{dt} = \frac{1}{\tau}\left[-h + J_0\, x\, g(h)\right] \tag{7}$$

$$\frac{dx}{dt} = \frac{1}{\tau_x}\left[2.8 - x - 9.0\, x\, g(h)\right] \tag{8}$$

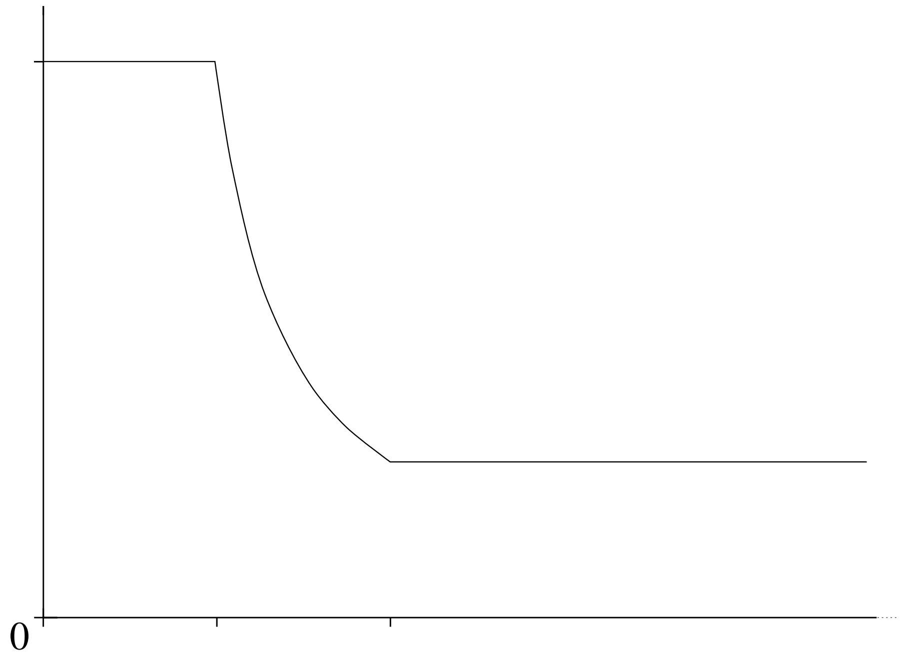
Give the two eigenvalues.

Result: (1pt) The fixed point is (stable/unstable/saddle)...........................

(3pts) The two eigenvalues are ....

(Space for calculations)

g) [2pts] We now set $J_0 = 2.5$. One of the nullclines is shown below. Construct the other nullcline. To get the nullcline it is convenient to evaluate the condition of the nullcline at $h = 0$, $h = 0.4$ $h = 0.8$, $h = 2.5$.



h) [2pts] Add arrows on the nullclines in your graph above, at least four arrows on each nullcline.

i) [2pts] There is a fixed point in the range $0.4 < h < 0.8$. (There may or may not be other ones as well). Assume that this fixed point is unstable. In your graph above, draw a trajectory starting at (1,0).

j) [2pts] Give in words a physical/biophysical/intuitive explanation of your result. Make use of the insights you gained in the previous exercise, and from part a) of this exercise.

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4 Noise in spiking neurons (11 points)

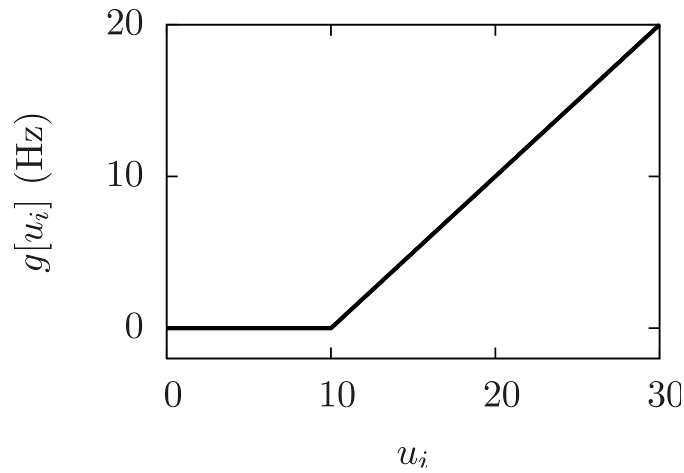
We consider a network composed of $N = 100$ fully interconnected neurons. The spiking behaviour of neuron i is described by a Poisson process with instantaneous firing rate $\rho_i(t)$ related to the membrane potential:

$$\rho_i(t) = g[u_i(t)]$$

where g is the transfer function drawn in the figure below. The dynamics of u_i obeys

$$\tau_m \frac{du_i}{dt} = -u_i(t) + w \sum_j X_j(t) \qquad (9)$$

$X_j(t) = \sum_{t_j^f} \delta(t - t_j^f)$ denotes the spike train of neuron j. δ is the Dirac function. In the following, we assume 10 ms $< \tau_m <$ 20 ms.



a) [2pts] Write the solution of Eq. 9 in the form $u_i(t) = \int_?^? \ldots\, dt$. (Of course, u_i should not appear on the right hand side!)

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b) [2pts] Assume that all neurons are independent Poisson processes with the same constant firing rate ρ.: Using the result of the previous question, show that $\langle u_i(t) \rangle = Nw\rho$. (Here $\langle \cdot \rangle$ denotes the average over all possible spike trains in the network up to time t).

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c) [2pts] Assume that we can approximate $\langle g[u_i(t)] \rangle$ by $g[\langle u_i(t) \rangle]$. Based on the result from Question b), derive a condition on the product wN for the existence of a (non-zero) fixed point in the network activity.

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d) [1pt] Assume for w a value of $\frac{2}{N}$. Is there a (non-zero) fixed point? If yes, construct the fixed point graphically on the figure of the previous page.

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e) [1pt] Assume the standard deviation of the membrane potential is Δ and the fixed point is u_0. What are the conditions under which approximating $\langle g[u_i(t)]\rangle$ by $g[\langle u_i(t)\rangle]$ is correct?

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f) [3pts] Assume $w = \frac{J}{N}$, with $J > 2$ and, as in Question b), that the neurons are independent Poisson neurons with constant rate ρ. Use your intuition **or** calculate the variance $\Delta^2 = \langle u_i(t)^2\rangle - \langle u_i(t)\rangle^2$ (space below) to answer the following questions.

How does Δ behave in the following cases:

- "If N increases, Δ . . . ."
- "If τ_m increases, Δ . . . ."
- "If J increases, Δ . . . ."

Hint: recall that the autocorrelation of a Poisson process X with constant rate ν is given by $\langle X(t)X(t+\tau)\rangle = \nu^2 + \nu\delta(\tau)$.

(Space for optional calculations)

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