# *Measurement Systems*

Solutions to Problem set n° 5

***Estimating and reducing noise***

## Exercise 1 (Perturbation by conductive coupling – Four-wire method)

a) $\Delta R$ is the difference between the measured value ($u_m/i$) and the real value ($R$).

$$\frac{\Delta R}{R} = \frac{\frac{u_m}{i} - R}{R} = \frac{\frac{i\left(\frac{R_w}{2} + \frac{R_w}{2} + R\right)}{i} - R}{R} = \frac{R_w}{R}$$

b) The voltmeter measures the voltage drop across itself. Since it has a resistance of $R_v$ while a current of $i_2$ flows through it, we have $u_m = R_v i_2$. We then have, from the definition of $\Delta R/R$ and the measurement procedure ($R_m = u_m/i$) :

$$\frac{\Delta R}{R} = \frac{\frac{u_m}{i} - R}{R} = \frac{\frac{R_v i_2}{i} - R}{R}$$

Using Kirchoff's law: $i = i_1 + i_2$ and $(R_w + R_v)i_2 = R i_1$ , we find:

$$\frac{i_2}{i} = \frac{R}{R_w + R_v + R}$$

Finally:

$$\frac{\Delta R}{R} = \frac{R_v \frac{R}{R_w + R_v + R} - R}{R} = -\frac{R + R_w}{R_w + R_v + R}$$

The four-wire method allows us to measure $R$ with a negligible error, even when $R_w \gg R$ as long as $R_v$ (internal impedance of the voltmeter) is very large compared to $R$ and $R_w$.

## Exercise 2 (Grounding)

The original circuit schematic contained multiple ground loops. To find them, you should first assume that all the ground symbols are connected to each other using a connector (usually not shown in drawings but always there in the form of either the ground or a cable in the wall connecting the sockets in which these instruments are plugged in). Pick any one ground and try to trace a loop that connects them to another ground. If you can do this, then you have a ground loop. The correct configuration is the one where the loops are eliminated. The reference signal needs to be connected to the ground in at one point only to make sure the reference corresponds to 0 V. By connecting it to ground in multiple points, you introduce ground loops. The reference signal on Instrument 1 is internally connected to the ground. This you cannot change because it would imply opening the instrument and it would also be against the safety rules which dictate that anything you can easily touch with your hands on an instrument needs to be grounded to keep you from getting electrocuted if there is a fault with the instrument. You can safely disconnect other connections between reference and ground on instruments 2 and 3, Figure 1.

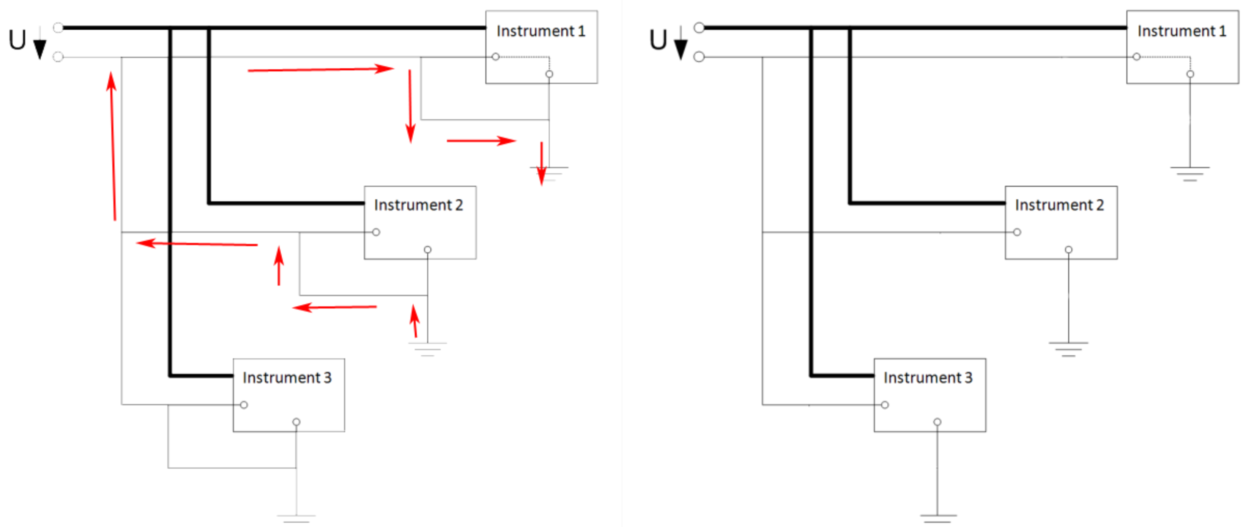


Figure 1: Grounded circuit: Left – with multiple ground loops. One shown using red arrows. Right: correct configuration, with instrument 1 internally referenced to ground (this connection is internal and cannot be changed due to safety issues) and all other connections between the reference signal and ground disconnected.

**Exercise 3 (Magnetic shielding)**

a) In this case, wire 2 is not shielded and magnetic induction takes place due to the current flowing in wire 1. We have thus: $u_{2,0} = j\omega M_{12} i_p = j\omega M i_p$. From this, $U_{2,0,eff} = \omega M_{12} I_{p,eff} = = 2\pi f M I_{p,eff}$=$2\pi \cdot 40 \cdot 10^3 \cdot 0.1 \cdot 10^{-6} \cdot 10 \cdot 10^{-3} = 0.25 mV$.

b) Here, wire 2 is protected by a grounded shielding. We obtain (slides, magnetic shielding): $u_2 = \frac{j \cdot 2\pi \cdot f \cdot M}{1 + j\frac{f}{fc}} i_p$ with $f_c = \frac{1}{2\pi}\frac{R}{L}$ . Therefore $U_{2,eff} = \frac{2\pi \cdot f \cdot M}{\sqrt{1+\left(\frac{f}{fc}\right)^2}} I_{\text{p,eff}}$ . Finally $U_{2,\text{eff}} = 4\mu\text{V}$.

c) $\frac{U_{2,0,eff}}{U_{2,eff}} = 62.84 \Rightarrow k = -35.96\, dB$ . The protection factor $k_{\text{max}}$ defines the effectiveness of the shielding.
Notice that $k_{\text{max}}$ can also be defined by the cutoff frequency of the shielding : $k_{max} = 10 \log \left(\max \left|\frac{u_2}{u_{2,0}}\right|^2\right) = 10 \log \left(\frac{1}{1+\left(\frac{f_{parasite}}{fc}\right)^2}\right)$.