Measuring Systems

Problem set n° 6

Estimation and noise reduction

Exercise 1 (Differential amplifier and uncertainty of the measurement)

We know the expression for the differential voltage U_d and the common mode voltage U_{cm} at the input of the amplifier:

$$U_d = U_a - U_b = -\frac{1}{4} \cdot \frac{\Delta R}{R} \cdot U_s \qquad \text{and} \qquad U_{cm} = \frac{U_a + U_b}{2} = \frac{U_s}{2}$$

- The residual voltage U_o at the output of the amplifier when the bridge is in equilibrium is:

$$U_d = 0 \qquad \Longrightarrow \qquad U_o = A_d \cdot \frac{U_{cm}}{CMMR} = \frac{A_d}{2} \cdot \frac{U_s}{CMMR} = 50 \text{ mV}$$

- The uncertainty $\varepsilon_{\Delta R/R}$ on the relative variation of the resistance corresponding to the residual voltage is:

$$U_o = -\frac{A_d}{4} \cdot \varepsilon_{\Delta R/R} \cdot U_s \qquad \Longrightarrow \qquad \varepsilon_{\Delta R/R} = -\frac{4 \cdot U_o}{A_d \cdot U_s} = -\frac{2}{CMMR} = -0.2\ ‰$$

Exercise 2 (power and RMS of thermal noise)

a) R_{eq} is the equivalent resistance of the network. We then have : $P_i = \frac{U_{noise}^2}{4R_{eq}} = \frac{4k_B T R_{eq} \Delta f}{4R_{eq}} = k_B T \Delta f$.

We get P_1=12.4*10^{-18} W=12.4 aW and P_2 = 20*10^{-15} W=20 fW.

Note: Only the temperature and bandwidth influence the total thermal noise power.

b) We have an equivalent resistance $R_{eq} = \frac{1}{\frac{1}{R_1+R_2}+\frac{1}{R_3}} = 524\Omega$ and a thermal noise

$$U_{noise} = 4k_B T R_{eq} \Delta f = 0.92\mu V$$

Exercise 3 (Intrinsic noise)

Case 1: positive part of the cycle

During positive arts of the cycle ($U_+(t) \geq 0$), parasitic voltages due to the intrinsic sources of noise are the following:

- Johnson noise : $\sigma_{T,n}\big|_U = \sqrt{4 \cdot k_B \cdot \Delta f \cdot R_{eq} \cdot T} = 0.33\ \mu V$ avec : $R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}$
- Shot noise : $\sigma_{G,n}\big|_I = \sqrt{2 \cdot e \cdot \Delta f \cdot I_{ave}} = \sqrt{\frac{4 \cdot e \cdot \Delta f \cdot \hat{U}}{R_{eq} \cdot \pi}}$

  with : $I_{ave} = \frac{U_{ave}}{R_{eq}} = \frac{\hat{U}}{R_{eq}} \cdot \frac{1}{\pi} \cdot \int_0^{\pi} \sin(x)\,.\,dx = \frac{2 \cdot \hat{U}}{R_{eq} \cdot \pi}$

  $$\Longrightarrow \quad \sigma_{G,n}\big|_U = R_{eq} \cdot \sigma_{G,n}\big|_I = \sqrt{\frac{4 \cdot R_{eq} \cdot e \cdot \Delta f \cdot \hat{U}}{\pi}} = 2.61\ \mu V$$

- $1/f$ noise: $\sigma_{\frac{1}{f},n}\Big|_U = \sqrt{K_{\frac{1}{f}} \cdot ln\left(\frac{f_{max}}{f_{min}}\right)} = 0.14\ \mu V$

The total parasitic voltage is: $e_{nR}|_+ = \sqrt{\sigma_{T,n}\big|_U^{\,2} + \sigma_{G,n}\big|_U^{\,2} + \sigma_{\frac{1}{f},n}\Big|_U^{\,2}} = 2.63\ \mu V$

Case 2: negative part of the cycle

During the negative alternations ($U_-(t) < 0$) there is no current flowing through the diode, so the associated shot noise is 0. Thus, the total parasitic voltage is: $e_{nR}|_- = \sqrt{\sigma_{T,n}\big|_U^{\,2} + \sigma_{\frac{1}{f},n}\Big|_U^{\,2}} = 0.36\ \mu V$.