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## Measuring systems

Problem set 9

***Data analysis***

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### Exercise1 (Confidence Interval)

a):  $\bar{x} = \frac{A}{N} = 0.059\text{mm}$ ,  $\sigma = \sqrt{\frac{B}{N}} = 3.027\text{mm}$ .

b)  $I_{confidence,95\%} = IC_{95\%} = \pm 2\sigma = 6.05\text{mm}$

### Exercise 2 (Systematic error)

It is known that the absolute value  $|\underline{Z}_L|$  of the coil impedance  $L$  is two times smaller than the absolute value  $|\underline{Z}_C|$  of the capacitor with capacitance  $C$  at a given frequency  $f$ :

$$|\underline{Z}_L| = L \cdot \omega = \frac{|\underline{Z}_C|}{2} = \frac{1}{2 \cdot \omega \cdot C} \quad (\text{with : } \omega = 2 \cdot \pi \cdot f) \quad \Rightarrow \quad \underline{Z}_L = \frac{j}{2 \cdot \omega \cdot C}$$

For the real voltage  $|\underline{U}_{C,real}|$  without the voltmeter we find:

$$\underline{U}_{C,real} = \frac{\underline{Z}_C}{\underline{Z}_L + \underline{Z}_C} \cdot \underline{U} = \frac{\frac{1}{j \cdot \omega \cdot C}}{\frac{j}{2 \cdot \omega \cdot C} + \frac{1}{j \cdot \omega \cdot C}} \cdot \underline{U} = \frac{1}{\frac{j^2}{2} + 1} \cdot \underline{U} = 2 \cdot \underline{U} \quad \Rightarrow \quad |\underline{U}_{C,real}| = 2 \cdot |\underline{U}|$$

For measuring the voltage with the voltmeter  $|\underline{U}_{C,volt}|$ , we obtain:

$$\begin{aligned} \underline{U}_{C,volt} &= \frac{(\underline{Z}_C || R_i)}{\underline{Z}_L + (\underline{Z}_C || R_i)} \cdot \underline{U} = \frac{\frac{R_i}{1 + j \cdot \omega \cdot R_i \cdot C}}{\frac{j}{2 \cdot \omega \cdot C} + \frac{R_i}{1 + j \cdot \omega \cdot R_i \cdot C}} \cdot \underline{U} = \frac{R_i}{\frac{R_i}{2} + \frac{j}{2 \cdot \omega \cdot C}} \cdot \underline{U} = \frac{2}{1 + \frac{j}{\omega \cdot R_i \cdot C}} \cdot \underline{U} \\ |\underline{U}_{C,volt}| &= \sqrt{1 + \left( \frac{1}{\omega \cdot R_i \cdot C} \right)^2} \cdot |\underline{U}| \end{aligned}$$

The systematic error should less than  $\varepsilon = 1\%$ :

$$\frac{|\underline{U}_{C,volt}| - |\underline{U}_{C,real}|}{|\underline{U}_{C,real}|} = \left( 1 + \left( \frac{1}{\omega \cdot R_i \cdot C} \right)^2 \right)^{-\frac{1}{2}} - 1 \leq 1\% \quad \Rightarrow \quad R_i \geq \frac{1}{2 \cdot \pi \cdot f \cdot C \sqrt{\frac{1}{(1-\varepsilon)^2} - 1}} = 22.34 \text{ k}\Omega$$