Measuring systems

Problem set 10

Data analysis

Exercise 1 (Estimate of the average)

We can begin by expressing the number of samples *n* that will be recorded for each measurement and the standard deviation of the population (standard measurement for a single measurement) σ , considering the known reference velocity ω_{ref}:

$$n = f_{ech} \cdot \Delta T = 200 \qquad \text{and} \qquad \sigma = \sigma_{\overline{\omega}} \cdot \sqrt{f_{ech} \cdot \Delta T}$$

a) $\sigma = \sigma_{\overline{\omega}} \cdot \sqrt{f_{ech} \cdot \Delta T} = 4.95\ °/s$

b) If we quadruple the number of measurements *N*, we find the standard deviation σ_{4N} for the calculated average:

$$\sigma_{\overline{\omega},4N} = \sigma_{\overline{\omega},N}$$

The standard deviation of the average depends on the number of points used to calculate the average itself. It has nothing to do with how often we repeat this.

c) If the acquisition time is ΔT_2, for a standard deviation $\sigma_{N,2}$ of the averages:

$$\sigma = \sigma_{\overline{\omega}} \cdot \sqrt{f_{ech} \cdot \Delta T} = \sigma_{\overline{\omega}}' \cdot \sqrt{f_{ech} \cdot \Delta T_2} \qquad \Longrightarrow \qquad \Delta T_2 = \left(\frac{\sigma_{\overline{\omega}}}{\sigma_{\overline{\omega}}'}\right)^2 \cdot \Delta T = 5.44\ s$$

d) The standard deviation of the averages $\sigma_{N,3}$, for a sampling frequency which is two times smaller and the acquisition period of ΔT, will be:

$$\sigma = \sigma_{\overline{\omega}}'' \cdot \sqrt{\frac{f_{ech} \cdot \Delta T}{2}} = \sigma_{\overline{\omega}} \cdot \sqrt{f_{ech} \cdot \Delta T} \qquad \Longrightarrow \qquad \sigma_{\overline{\omega}}'' = \sqrt{2} \cdot \sigma_{\overline{\omega}} = 0.5\ °/s$$

Exercise 2 (Types of error, confidence level and number of measurements)

a) The accuracy of P_{atm} is expressed as the systematic error ϵ_μ. Since an error on the central tendency is described as a systematic error, it must be sufficiently far from the central tendency (e.g. $\epsilon_\mu > 3\sigma_p$ for a confidence level of 99%. This is a somewhat arbitrary number and you could have chosen another confidence level but you need to declare the confidence level associated with your choice of *z*× σ_p).

$$\epsilon_\mu = \mu_P - P_{atm} = 0.6\ bar$$

Fidelity (precision) is expressed as the statistical error ϵ_σ according to the chosen confidence level for P_{atm}. ϵ_σ associated with p_0 and p_1 :

$$\epsilon_{\sigma,p_0} = \sigma_P = 0.15\ bar \qquad\qquad \epsilon_{\sigma,p_1} = 2 \cdot \sigma_P = 0.3\ bar$$

The statistical error ϵ_{σ,p_2} for p_2 is calculated according to the table of the normal distribution:

$$\alpha = \frac{1-p_2}{2} = 0.05$$

$$z_\alpha = 1.646$$

$$\epsilon_{\sigma,p_2} = z_\alpha \cdot \sigma_P = 0.25\ bar$$

The total error ϵ_{tot} of P_{atm} corresponds to the sum of ϵ_{μ} and ϵ_{σ} :

$\epsilon_{tot,p_0} = 0.85\ bar$ $\epsilon_{tot,p_1} = 1.0\ bar$ $\epsilon_{tot,p_2} = 0.95\ bar$

b) The statistical error $\hat{\epsilon}_{\sigma,p_2}$ on the average for p_2, knowing that $n = 200$:

$$\hat{\epsilon}_{\sigma,p_2} = \frac{\epsilon_{\sigma,p_2}}{\sqrt{n}} = 0.0175\ bar$$

c) $N = {z_{\alpha}}^2 \cdot \left(\frac{s_P}{\delta}\right)^2 \gtrsim 609$