Measuring systems

Problem set n° 11

Comparison of measured data

Exercice 1 (Proximity detector)

a) We use a ***bilateral z-test*** (N>30, known standard deviation) in order to determine if there is a systematic error between the theoretical average *D* and the experimentally measured $\bar{d}_A$. We repeat the same for $\bar{d}_B$. The null hypothesis corresponds to:

H_0: "Sensor A does not show a systematic error" ($\bar{d}_A = D$)

We proceed identically for sensor B. We find for z_α, $z_{obs,A}$ and $z_{obs,B}$:

$$z_\alpha = z_{2.5\%} = 1.96 \qquad z_{obs,A} = \frac{\bar{d}_A - D}{\frac{\sigma_A}{\sqrt{N}}} = 1.59 \qquad z_{obs,B} = \frac{\bar{d}_B - D}{\frac{\sigma_B}{\sqrt{N}}}$$

$$= -2.12$$

$$z_{obs,A} \in [-z_\alpha;\ z_\alpha] \qquad \Leftrightarrow \qquad z_{obs,B} \notin [-z_\alpha;\ z_\alpha]$$

According to these results it can be said that sensor B shows a systematic error with a risk of error 2α (H_0 is rejected in this case) while the systematic error of sensor A (H_0 not rejected) is not significant.

b) We want to compare the two experimental averages $\bar{d}_A$ and $\bar{d}_B$. We choose again a ***bilateral z-test*** (N>30, known standard deviation) in order to verify the null hypothesis H_0 :

H_0: "The sensors do not differ from each other with respect to their average" ($\bar{d}_A = \bar{d}_B$)

For z_α we find the same value as in a) and for $z_{obs,B-A}$:

$$z_{obs,B-A} = \frac{\bar{d}_A - \bar{d}_B}{\sqrt{\frac{{\sigma_B}^2 + {\sigma_A}^2}{N}}} = -2.55 \qquad \Rightarrow \quad z_{obs,B-A} \notin [-z_\alpha;\ z_\alpha]$$

Thus the hypothesis H_0 is rejected and it can be said that the sensors are significantly different from each other with respect to their average with a risk of error 2α.

c) We choose a ***unilateral z-test*** (N>30, known standard deviation) to find out if the experimental average $\bar{d}_A$ is greater than the experimental average $\bar{d}_B$. The null hypothesis is H_0:

H_0: "The average of B is greater or equal to that of A" ($\bar{d}_A \geq \bar{d}_B$)

We obtain the same value for $z_{obs,B-A}$ as in b) and for $-z_\alpha$ we find:

$$-z_\alpha = -z_{5\%} = -1.65 \qquad z_{obs,B-A} \notin [-z_\alpha;\, \infty[\;\Rightarrow\; \bar{d}_A > \bar{d}_B$$

We thus reject the hypothesis H_0 and we can affirm that the average $\bar{d}_A$ is significantly greater than the average $\bar{d}_B$ with a risk of error α.

Exercise 2 (Accelerometers)

We compare the two experimental averages μ_1 and μ_2, in order to find out the error risk α for which there is a systematic error between the two accelerometers. For this purpose we may use a **bilateral z-test** (N>> 30 and the standard deviation is not known) with a null hypothesis H_0:

H_0 : << The sensors do not exhibit systematic error with respect to each other>> ($\mu_1 = \mu_2$)
In order to test this hypothesis we calculate the $z_{obs,2-1}$ and the corresponding risk of error $2\alpha_{obs,2-1}$:

$$z_{obs,2-1} = \frac{\mu_1 - \mu_2}{\sqrt{\frac{(S_2^2 + S_1^2)}{N}}} = -2.6 \;\rightarrow\; \alpha_{obs,2-1} = 0.47\ \% \text{ (from the table)}$$

The obtained risk of error is $2\alpha_{obs,2-1}$ = 0.94%.