Neural Networks and Biological Modeling

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Question set "Cable Equation"

Exercise 1: Cable equation

The electrical properties of a neuronal process (axon or dendrite) can be described mathematically by a cable equation with passive membrane. The canonical form is given by

$$\frac{\partial}{\partial t} u(t,x) - \frac{\partial^2}{\partial x^2} u(t,x) = -u(t,x) + r_T i_{\text{ext}}(t,x)\,, \tag{1}$$

where r_T is the transverse resistance.

1.1 Show that when $r_T i_{\text{ext}}(t,x) = \delta(x)\delta(t)$, the solution is given by

$$G_\infty(t,x) = \frac{\Theta(t)}{\sqrt{4\pi\, t}} \exp\left[-t - \frac{x^2}{4\,t}\right]\,. \tag{2}$$

where $\Theta(t)$ is the Heaviside function, $\Theta(t) = 0$ if $t \le 0$ and 1 if $t > 0$, with $d\Theta(t)/dt = \delta(t)$.

(i) What is the physical interpretation of this solution?

(ii) What is the difference to a purely diffusive process?

1.2 Optional homework

Construct a solution of the cable equation for a semi-infinite cable ($0 < x < \infty$) for a charge q injected at time $t = 0$ at position x_0.

To this end, impose a reflecting boundary condition at $x = 0$, that is, $\frac{du}{dx}\mid_{x=0} = 0$. The interpretation of this condition is that no current can flow through $x = 0$.

The condition can be implemented by using the method of images, which consists in adding an imaginary charge q at position $-x_0$.

(i) Discuss a possible biological interpretation if $x = 0$ represents the cell body of the neuron.

(ii) What would be, in this case, the temporal evolution of the somatic membrane potential following a current pulse in the dendrite?