2: Ultrasound imaging and x-rays

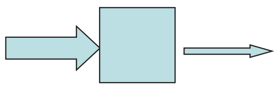
1. How does ultrasound imaging work ?
2. What is ionizing electromagnetic radiation ?
   - Definition of ionizing radiation
3. How are x-rays produced ?
   - Bremsstrahlung
   - Auger electron

After this course you
1. understand the basic principle of ultrasound imaging
2. Are able to estimate the influence of frequency on resolution and penetration.
3. are capable of calculating echo amplitudes based on acoustic impedance;
4. know which parts of the electromagnetic spectrum are used in bio-imaging
5. know the definition of ionizing radiation;
6. understand the principle of generation of ionizing radiation and control of energy and intensity of x-ray production;

2-1. What are the main fates of US waves in matter ?

1. Attenuation

Sound wave travels through the substance but loses energy I(x)



Material	α [dB/cm MHz]
Water	0.002
Blood	0.2
Tissue	0.7
Bone	15
Lung	40

$$I(x) = I_0 e^{-\alpha k x f}$$

Attenuation coefficient α [dB/(cm Mhz)]

α is usually given in dB: dB=10logI(x)/I_0

[3dB=2fold increase in I(x): $10^{0.3}=2$

Unit conversion: k=ln10/10]

Typically α~0.5dB/(cm MHz)

→ 6MHz signal will lose 3dB per cm of travel (2 fold loss in wave energy)

2. Refraction

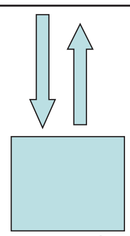
Sound wave bends as it hits an interface at an oblique angle

3. Scatter

Sound wave dispersed in all directions

4. Reflection

Sound wave bounces back to probe



Reflection (echo formation) is key to imaging

What is the basic principle of US imaging ?

The <u>basic principle</u> of imaging using sound waves :

1. Emit sound pulse
   (length [1-5 μs] is a multiple of cycle time 1/f)
2. Measure time and intensity of echo
3. Reconstruct using known wave propagation velocity c

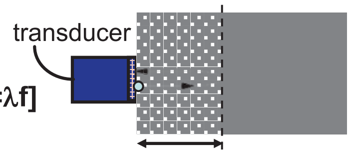
Distance of tissue boundary from probe (transducer)

Ultrasound: frequency f=1-20MHz
(not 20kHz)

Sound wave propagation velocity c [c=λf]

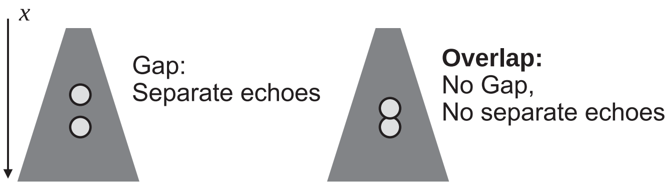
~330m/s (air) = 0.33 mm/μs

~1.45-1.6 mm/μs (tissue) ⇒ (1cm~7μs)
(increases with density ρ, bone ~ 4 mm/μs)



Distance=speed x time/2

What determines the resolution in US imaging ?

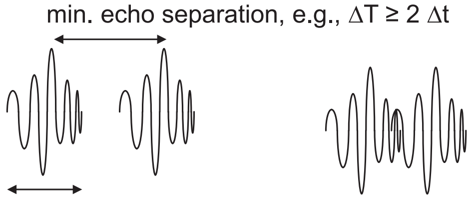


$T_1=2x_1/c$

$T_2=2x_2/c$

$\Delta T=T_1-T_2=2\Delta x/c$

min. echo separation, e.g., $\Delta T \geq 2\,\Delta t$



Pulse duration Δt = N/f

Wavelength λ determines minimal resolution

1. To have defined frequency:
   Pulse length = N/f ∝ λ
2. Separation of return echoes, e.g.
   ΔT > 2 pulse length

1. Resolution

   increases with f

2. Penetration (cf. attenuation)

   decreases with f

When does an acoustic echo occur ?

Acoustic impedance and reflection ratio

Acoustic impedance Z

Definition:

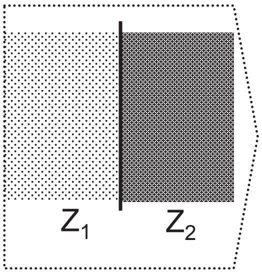
Z= ρ**c** [kg/m^{2}s=rayls]

Tissue	Z (rayls)
Air	0.0004×10^6
Lung	0.18×10^6
Fat	1.34×10^6
Water	1.48×10^6
Kidney	1.63×10^6
Blood	1.65×10^6
Liver	1.65×10^6
Muscle	1.71×10^6
Skull bone	7.8×10^6

Amount of reflected wave energy

$I_{ref}=I_0 R_I$

At interface between objects with different acoustical properties



Reflection coefficient

$$R_I = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$

Probability of reflection + transmission is = 1:

Transmission $T_I = 1 - R_I$

What are the reflection coefficients R_I between tissues ?

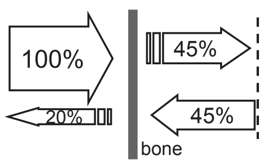
R_I	Fat	Muscle	Skin	Brain	Liver	Blood	Cranial bone	Plexi-glass
Water	0.047	0.02	0.029	0.007	0.035	0.007	0.57	0.35
Fat		0.067	0.076	0.054	0.049	0.047	0.61	0.39
Muscle			0.009	0.013	0.015	0.02	0.56	0.33
Skin					0.006	0.029	0.56	0.32
Brain					0.028	0.00	0.57	0.34
Liver						0.028	0.55	0.32
Blood							0.57	0.35
Cranial bone								0.29

Dolphin fetus

Reflection by solid material

e.g. bone-tissue interface

$\Rightarrow$ Shadow formation: ~45% of energy transmitted



(T_I=1-R_I)

2-2. What is the optimal choice of US frequency ?

SNR Resolution

Resolution:

Δx decreases with increasing frequency f : $\propto$1/f

$\Rightarrow$ Resolution $\propto$ f

SNR:

Signal returned from an echo-generating tissue interface at distance x from transducer

$$S(f,\alpha,x) = S_0 e^{-\alpha k f 2x} R_I$$

is constant

f: US frequency (experimental parameter)

α: attenuation coefficient (tissue parameter)

Find the optimal f …

$\Rightarrow$ Maximize f·S

Maximum is where derivative with respect to f is zero

is constant

$$\frac{d\left(fS(f,\alpha,x)\right)}{df} = R_I S_0 \frac{d}{df} fe^{-\alpha k f 2x}$$

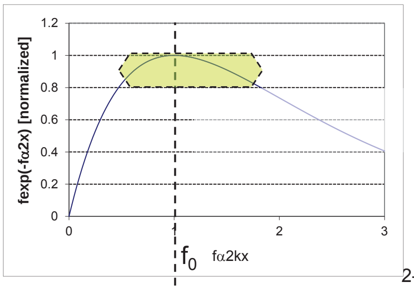
$$\frac{d}{df}\left(fe^{-\alpha k f 2x}\right) = 0$$

$$= e^{-\alpha k f 2x}\underbrace{\left(1 - f\alpha k 2x\right)}_{f_0 = 1/(2\alpha k x)} \mathbf{=0}$$

f_0=1/(2αkx)

The optimal frequency decreases with tissue depth and with increasing absorption

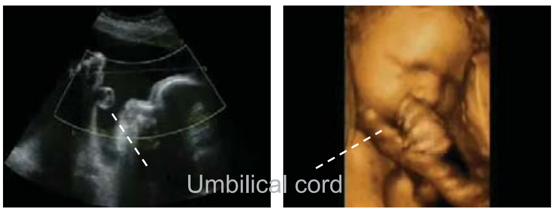
How critical is the choice of f_0 ?



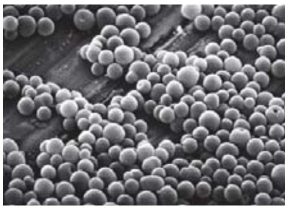
Ex. 3-D US Imaging & Contrast agents

3D US Physical Principle:

1. the transducer is moved during exposure (linear shift, swinging, rotation)
2. received echoes are stored in the memory
3. the image in the chosen plane is reconstructed mathematically



Contrast agents: gas-filled Bubbles



Gas : most contrast (plus resonance and higher harmonic imaging)

(see tiny Z $\rightarrow$ total reflection, R_I~1)

How can Ultrasound detect moving blood ?

Doppler effect

Motion (Doppler): Frequency shift f_D of moving tissue, results in shifted US frequency (demodulation for detection)

(where is this also used?)

Doppler frequency shift f_D

$$f_D = \frac{2 f_0 v_0 \cos\alpha}{c}$$

c: speed of US, e.g. 1500 m/s

v_0: speed of source, e.g. 50 cm/s

f_0: frequency of moving source, e.g. 5MHz

α: Rel. angle at which blood is moving

Example:

$f_D = 2 \cdot 5 \cdot 10^6$ [Hz] 0.5 [m/s]/1500 [m/s]

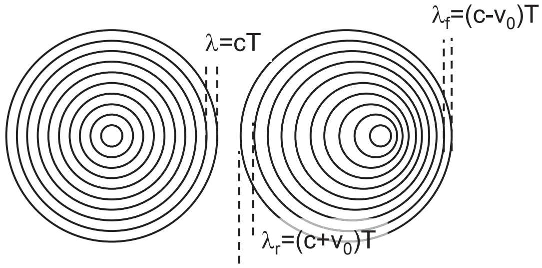
~ 3kHz

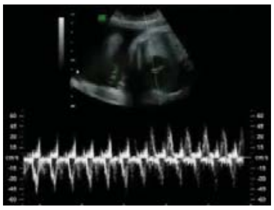
~ 0.05% of f_0

stationary

Source moving with v_0

In a period T, source moves closer by v_0T





2-3. Basis of x-ray imaging

useful relationships Electromagnetic radiation

$c = \lambda\nu$ (c = speed of light = $3 \cdot 10^8$m/s)

$E = h\nu = hc/\lambda$ (h = Planck's Constant)

$h = 2\pi \cdot 10^{-34}$Js

$= 4 \cdot 10^{-18}$keVs

1eV = energy of e^- in acquired in 1V electric field

$E = hc/\lambda$

= 1.2keV/nm

With which elements of matter does EM radiation interact mainly ?
(in imaging mainly with electrons)

Electron binding energy

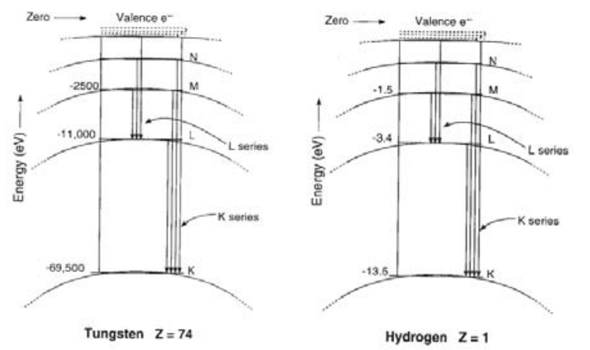


FIGURE 2-5. Energy-level diagrams for hydrogen and tungsten. Energies associated with various electron orbits (not drawn to scale) increase with Z and decrease with distance from the nucleus.

Binding energy

1. decreases with shell distance
2. increases with Z

(Why?)

Lowest K-shell binding e^- energy:

E_K^{min} = 13.6eV (^{1}H)

$h\nu > E_K^{min}$: ionizing

$h\nu < E_K^{min}$: non-ionizing

Electron (some useful constants)

m_e = mass = $9 \cdot 10^{-31}$kg

q_e = charge = $1.6 \cdot 10^{-19}$ C (As)

Rest energy $m_e c^2$ = 511 keV

Ionizing radiation is above 13.6 eV

2-4. How are x-rays generated (scheme) ?

Negatively charged cathode = electron source

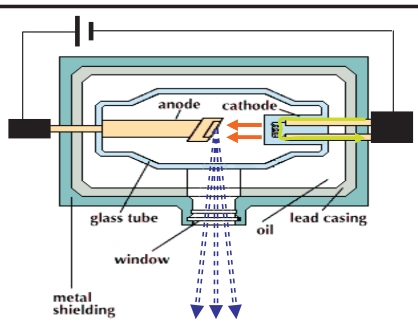
Electrical current (filament current) heats up the cathode (why is that necessary ?)

Electrons are liberated and accelerated by electric field (Energy of e^-= $q\Delta V$)

Anode = metal target (tungsten)

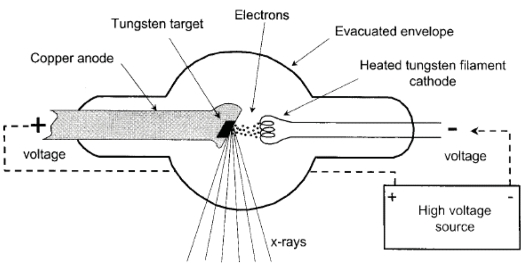
accelerated electrons hit anode ⇒ generate X-rays

(tube current with voltage difference up to 150 kV)



Intensity of beam = Power/Area

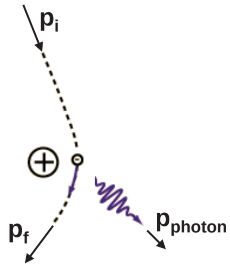
1. Number of X-rays (proportional to tube current)
2. Energy of X-rays $h\nu$ (proportional to voltage)



Emission of x-rays I:
What is Bremsstrahlung ?

Consider the interaction of e^- with stationary atom as collision :

$p_i = p_f + p_{photon}$



Coulomb:

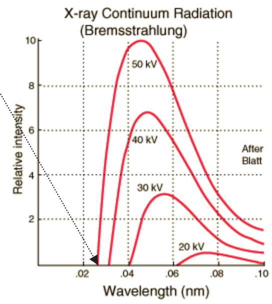
$a \sim q_e Z / m_e r^2$

$P_{Brems} = q_e^2 a^2 / 6\pi\varepsilon_0 c^3$

No info on directionality of radiation

(but maximum energy is defined, how?)

Max. Energy: E_e^i



Decreasing energy

Elastic scattering:

Probability ~ Z^2/E_{e-}^2

Inelastic scattering: ν release

Probability ~ Z^2

High Z: Tungsten is a good target

Emission of x-rays II:
What are Characteristic (fluorescent) X-rays ?

Impacting e^- liberates inner shell e^-

1. Atom is excited (higher energy state)
2. Vacancy
3. Filled by outer shell electron (cascading)
4. Emission of characteristic x-ray

Auger emission

The excited atom can also reduce energy by liberating an additional e- (Auger e-):