Excel solving: Linear Equations Systems (matrix formulation)

From: http://www.excel-easy.com/examples/system-of-linear-equations.html

5	1	8
4	-2	0
6	7	4

A

5x + 1y + 8z = 46
4x -2y =12
6x + 7y + 4z =50

46
12
50

B

x
y
z

X

A.X = B ==> X = InvA.B

For Excel Matrix Function, use CTRL + SHIFT + ENTER !!!!!!

-0.030303	0.19697	0.060606	4
-0.060606	-0.106061	0.121212	2
0.151515	-0.109848	-0.05303	3

InvA

X

==MINVERSE(F6:H8)

==MMULT(C17:E19;F10:F12)

Check	5x + 1y + 8z: 46
	4x -2y: 12
	6x + 7y + 4z: 50

In one step:

=TRANSPOSE(MMULT(MINVERSE(D28:F30);G28:G30))

4	2	3		Check	=MMULT(D28:F28;TRANSPOSE($D$27:$F$27))
5	1	8	46	46	
4	-2	0	12	12	
6	7	4	50	50	

5x + 1y + 8z = 46
4x -2y =12
6x + 7y + 4z =50

Use for Reaction solving using balances (Ex: CH4 combustion)

$1.CH_4 + a.O_2 \rightarrow b.CO_2 + c.H20$

or

$-1.CH_4 - a.O_2 + b.CO_2 + c.H_20 = (0)$

or

$x.O_2 + y.CO_2 + z.H_20 = 1.CH_4$

Similarly to:

5x + 1y + 8z = 46

4x -2y =12

6x + 7y + 4z =50

For C, H, O Balances

=TRANSPOSE(MMULT(MINVERSE(E16:G18);D16:D18))

	CH4		O2	CO2	H20		
	-1	-(Known)	-2	1	2	Check	
C Balance	1	1	0	1	0	1	=MMULT(E16:G16;TRANSPOSE($E$15:$G$15))
H Balance	4	4	0	0	2	4	
O Balance	0	0	2	2	1	0	

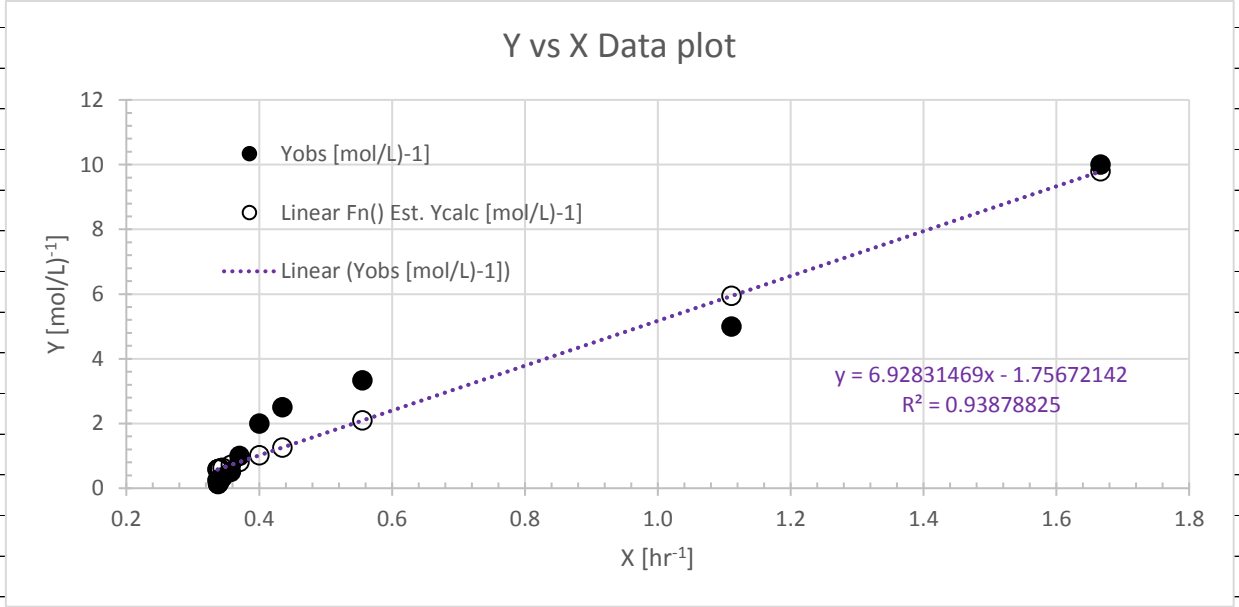
$-1.CH_4 - 2.O_2 + 1.CO_2 + 2H_20 = (0)$

or

$1.CH_4 + 2.O_2 \rightarrow 1.CO_2 + 2.H20$

Excel linear function estimation Y = A.X + B

X	Yobs	Linear Fn() Est. Ycalc
[hr^{-1}]	[mol/L)$^{-1}$]	[mol/L)$^{-1}$]
1.6667	10.0000	9.7905
1.1111	5.0000	5.9414
0.5556	3.3333	2.0923
0.4348	2.5000	1.2556
0.4000	2.0000	1.0146
0.3704	1.0000	0.8093
0.3571	0.5000	0.7177
0.3448	0.3333	0.6324
0.3413	0.2500	0.6079
0.3373	0.2500	0.5800
0.3401	0.2000	0.5998
0.3390	0.1667	0.5919
0.3378	0.1429	0.5839
0.3378	0.1250	0.5839



(Produce at least same graph, or better one !)

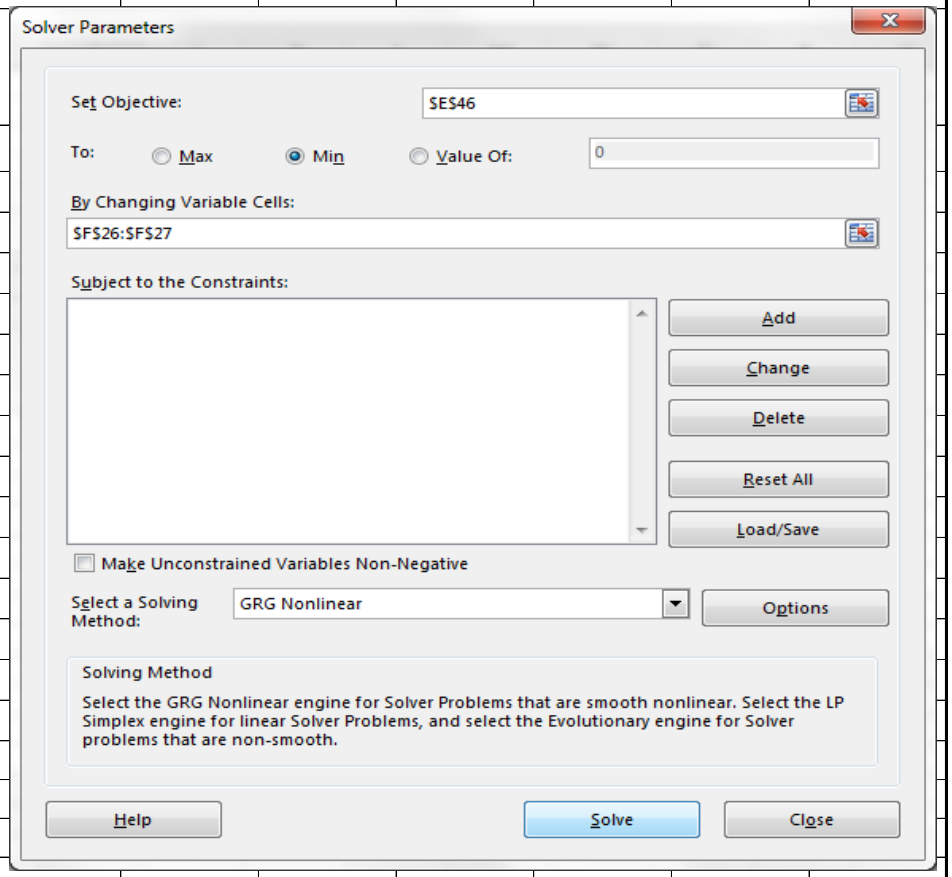
Slope A:	6.92831469	=SLOPE(C6:C19;B6:B19)
Intercept B:	-1.75672142	=INTERCEPT(C6:C19;B6:B19)
R_Squared:	0.93878825	=RSQ(C6:C19;B6:B19)

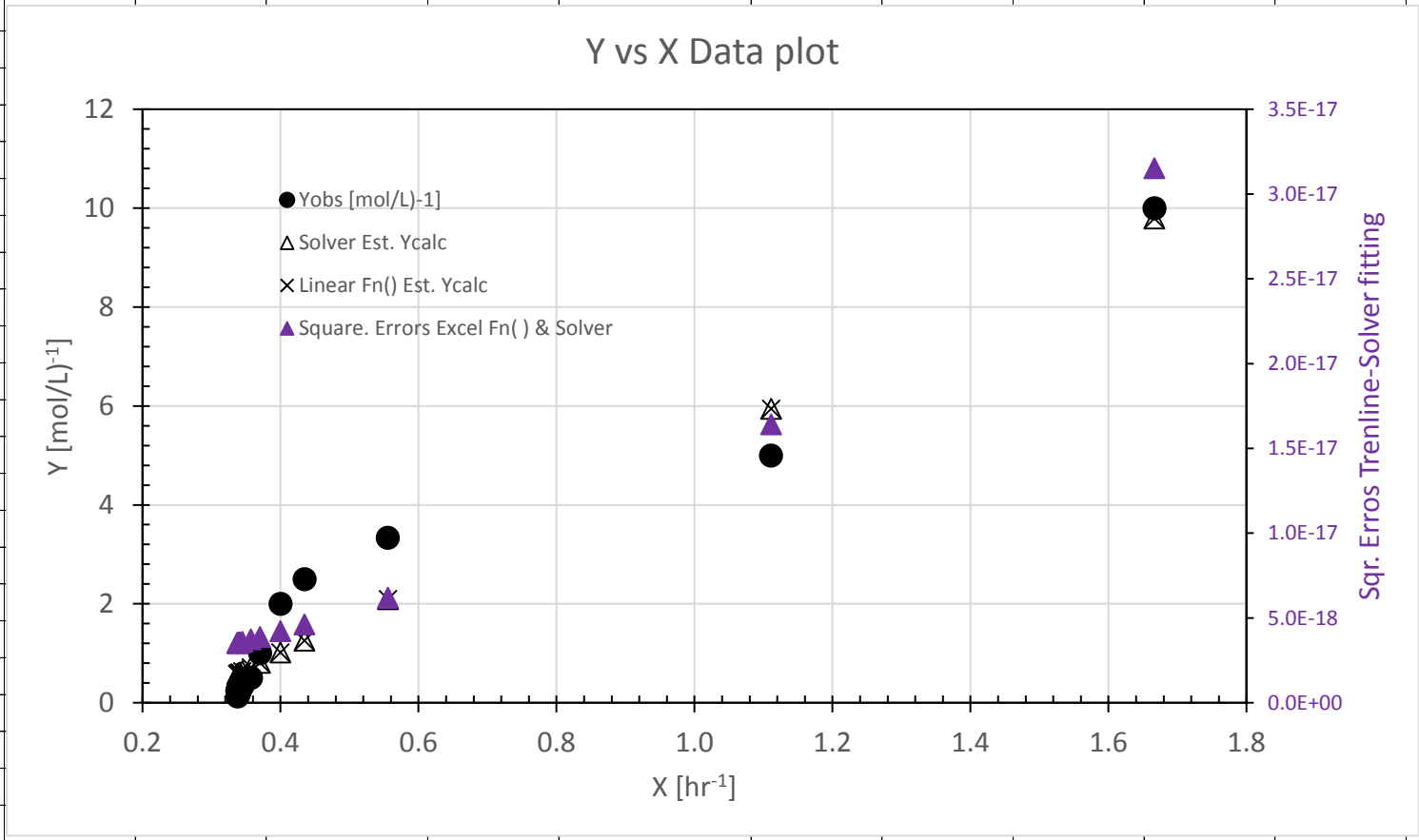
Excel Solver estimation (using Square Errors minisation)

Slope guess A:	6.92831469	Ycalc=A.X+B
Intercept guess B:	-1.75672142	

X	Yobs	Solver Est. Ycalc	Sqr. Err. (Solver Est.)	Square. Errors Excel Fn( ) & Solver
[hr^{-1}]	[$(mol/L)^{-1}$]	[$(mol/L)^{-1}$]		
1.6667	10.0000	9.79046973	0.043903	3.15296E-17
1.1111	5.0000	5.94140601	0.886245	1.64244E-17
0.5556	3.3333	2.09234229	1.540059	6.20151E-18
0.4348	2.5000	1.25558931	1.548558	4.62519E-18
0.4000	2.0000	1.01460445	0.971004	4.21401E-18
0.3704	1.0000	0.80932106	0.036358	3.87884E-18
0.3571	0.5000	0.71767668	0.047383	3.73369E-18
0.3448	0.3333	0.63235261	0.089413	3.60104E-18
0.3413	0.2500	0.6078911	0.128086	3.56346E-18
0.3373	0.2500	0.5799783	0.108886	3.52081E-18
0.3401	0.2000	0.5998482	0.159879	3.55114E-18
0.3390	0.1667	0.59185983	0.180789	3.53893E-18
0.3378	0.1429	0.58392543	0.194541	3.52682E-18
0.3378	0.1250	0.58392543	0.210613	3.52682E-18

Sum of Square Errors:	6.145717





(Produce at least same graph, or better one !)

Ordinary least squares regression (matrix computation)

https://en.wikipedia.org/wiki/Ordinary_least_squares

Yobs	Xobs	Cst
[mol/L]$^{-1}$]	[hr^{-1}]	
10	1.66666667	1
5	1.11111111	1
3.33333	0.55555556	1
2.5	0.43478261	1
2	0.4	1
1	0.37037037	1
0.5	0.35714286	1
0.33333	0.34482759	1
0.25	0.34129693	1
0.25	0.33726813	1
0.2	0.34013605	1
0.16667	0.33898305	1
0.14286	0.33783784	1
0.125	0.33783784	1

$$y = X\beta + u$$

$$\hat{\beta} = (X^\top X)^{-1} X^\top y$$

For Linear fitting Y = aX + b

(Xt.X)	5.742759	7.273817
	7.273817	14

=MMULT(TRANSPOSE(C8:D21);C8:D21)

Xt.Y	27.00957
	25.80119

=MMULT(TRANSPOSE(C8:D21);B8:B21)

	B	a
		b
X	Xobs	1 Y
	...	1 ...
	...	1 ...
	...	1 ...
(Xt.X)-1	0.509272	-0.2646
	-0.2646	0.208902

=MINVERSE(G17:H18)

[(Xt.X)-1].[Xt.Y]

6.928315
-1.75672

=MMULT(K17:L18;G21:G22)

In one step:

(Xt.X)-1.Xt.Y

6.928314691
-1.756721421

=MMULT(MINVERSE(MMULT(TRANSPOSE(C8:D21);C8:D21));MMULT(TRANSPOSE(C8:D21);B8:B21))

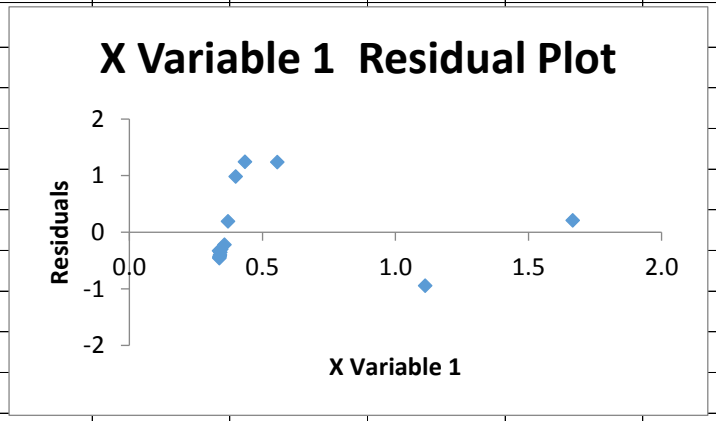
Excel Multi-Regression

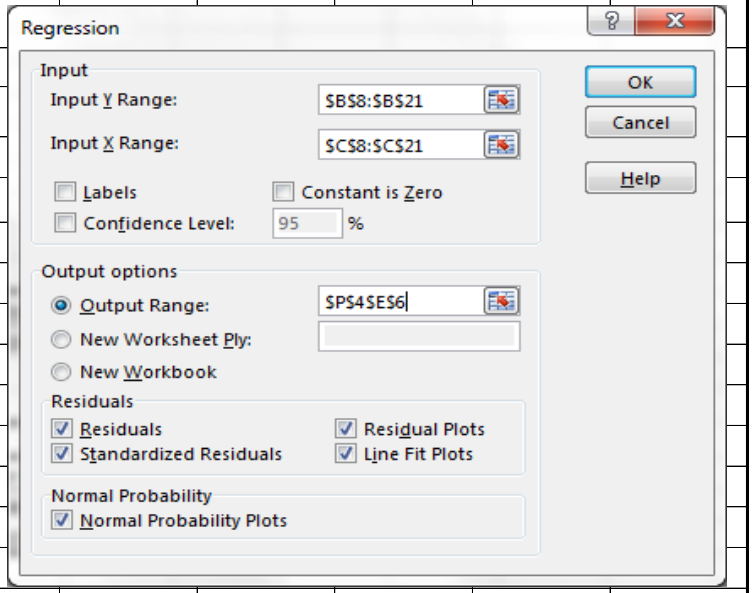
$Y = a \cdot X + b$

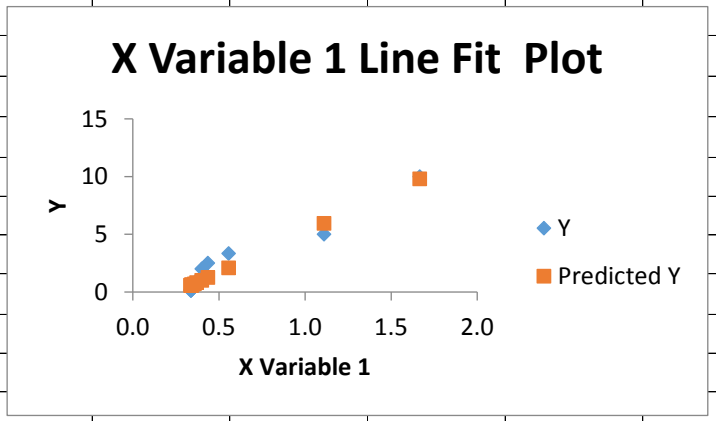
Yobs	Xobs
$[mol/L)^{-1}]$	$[hr^{-1}]$
10.0000	1.6667
5.0000	1.1111
3.3333	0.5556
2.5000	0.4348
2.0000	0.4000
1.0000	0.3704
0.5000	0.3571
0.3333	0.3448
0.2500	0.3413
0.2500	0.3373
0.2000	0.3401
0.1667	0.3390
0.1429	0.3378
0.1250	0.3378

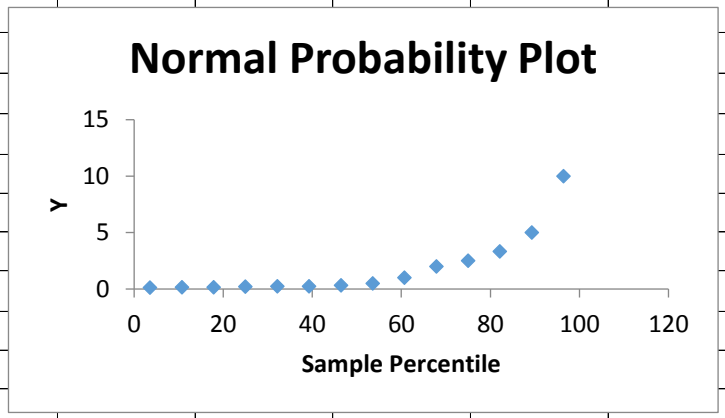
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.9689109
R Square	0.9387882
Adjusted R	0.9336873
Standard E	0.7156417
Observatio	14





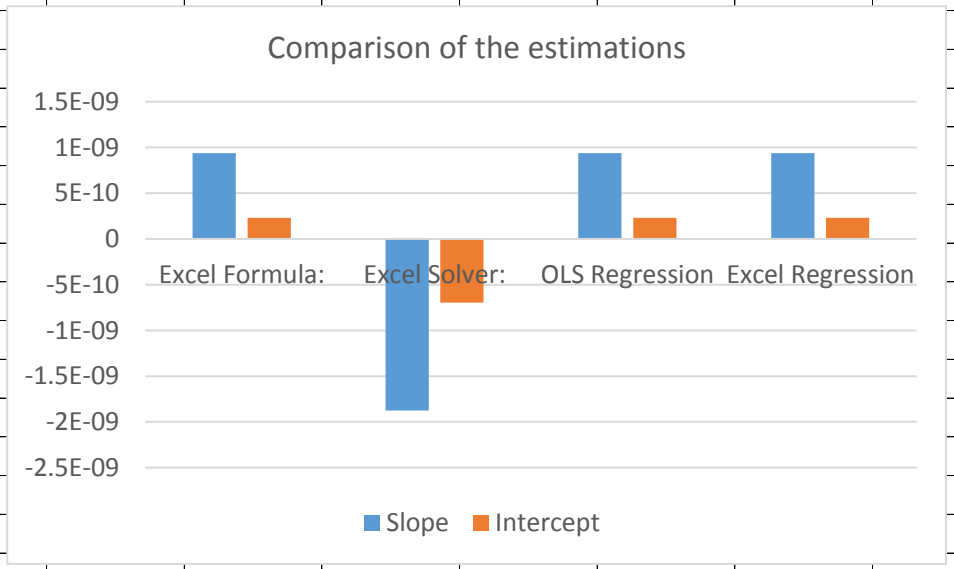




	A	B	C	D	E	F	G	H	I	J	K	L	M
31													
32													
33	ANOVA												
34		*df*	*SS*	*MS*	*F*	*ignificance F*							
35	Regression	1	94.25521	94.25521	184.0408	1.22E-08							
36	Residual	12	6.145717	0.512143									
37	Total	13	100.4009										
38													
39		*Coefficients*	*tandard Erro*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*Upper 95.0%*				
40	Intercept	-1.7567214	0.32709	-5.37076	0.000168	-2.46939	-1.04405	-2.46939	-1.04405				
41	X Variable :	6.9283147	0.510706	13.56616	1.22E-08	5.815583	8.041046	5.815583	8.041046				
42													
43													
44													
45	RESIDUAL OUTPUT					PROBABILITY OUTPUT							
46													
47	*Observation*	*Predicted Y*	*Residuals*	*ndard Residuals*		*Percentile*	*Y*						
48	1	9.7904697	0.20953	0.304742		3.571429	0.125						
49	2	5.941406	-0.941406	-1.36919		10.71429	0.142857						
50	3	2.0923423	1.240991	1.804904		17.85714	0.166667						
51	4	1.2555893	1.244411	1.809877		25	0.2						
52	5	1.0146045	0.985396	1.433164		32.14286	0.25						
53	6	0.8093211	0.190679	0.277324		39.28571	0.25						
54	7	0.7176767	-0.217677	-0.31659		46.42857	0.333333						
55	8	0.6323526	-0.299019	-0.4349		53.57143	0.5						
56	9	0.6078911	-0.357891	-0.52052		60.71429	1						
57	10	0.5799783	-0.329978	-0.47992		67.85714	2						
58	11	0.5998482	-0.399848	-0.58154		75	2.5						
59	12	0.5918598	-0.425193	-0.6184		82.14286	3.333333						
60	13	0.5839254	-0.441068	-0.64149		89.28571	5						
61	14	0.5839254	-0.458925	-0.66746		96.42857	10						

Comparison of Excel linear Formula, Excel Solver, OLS and Excel Regression estimations

	Slope	Intercept
Excel Formula:	6.928314691	-1.7567214
Excel Solver:	6.928314689	-1.7567214
OLS Regression	6.928314691	-1.7567214
Excel Regression	6.928314691	-1.7567214
Average	6.92831469	-1.75672142
Std.Dev.	1.40617E-09	4.63933E-10
RSD	2.02961E-08	-2.6409E-08



Différence betweeen each estimation and the average of all estimations.

	Slope	Intercept
Excel Formula:	9.37455E-10	2.31965E-10
Excel Solver:	-1.8749E-09	-6.959E-10
OLS Regression	9.37445E-10	2.31969E-10
Excel Regression	9.37454E-10	2.31966E-10

(Produce at least same graph, or better one !)

Excel SLOPE()/ INTERCEPT()

I've been looking for this formula in this website so I've made it Q&A style :) Hope this helps.

Slope;

$$\alpha = (n\sum(xy) - \sum x \sum y)/(n\sum x^2 - (\sum x)^2)$$

Offset:

$$\beta = (\sum y - \alpha \sum x)/n$$

Trendline formula:

$$y = \alpha x + \beta$$

source : http://www.ehow.com/how_6454143_calculate-trendline.html

Excel LINEST()

for m and b are based on the following formulas:

$$m = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

$$b = \bar{y} - m\bar{x}$$