$$Accumulation_i = \sum R_i^{IN} - \sum R_i^{OUT} + \sum R_i^{Conversions}$$

At Steady State, Accumulation = 0

R_i: $kgC_i.hr^{-1}$

r_i Volumetric rate = R_i / $V_{reactor}$

r_i: $kgC_i \,.\, m^{-3}.\, hr^{-1}$

$R_i = r_i \times V_r$

q_i Specific rate = R_i / Biomass amont

q_i Specific rate = r_i / Biomass concentration

q_i: $kgC_i.kgX^{-1}.hr^{-1}$

$r_i = q_i \times C_X$

$$Y_{SX} = \left|\frac{r_X}{-r_S}\right| = \left|\frac{\mu.C_X}{-q_S.C_X}\right| = \left|\frac{\mu}{-q_S}\right|$$

$$Y_{SP} = \left|\frac{q_P}{-q_S}\right| = \left|\frac{q_P / \mu}{-q_S / \mu}\right| = \left|\frac{1/Y_{PX}}{1/Y_{SX}}\right| = \left|\frac{Y_{SX}}{Y_{PX}}\right|$$

$$Y_{ij} = \left|\frac{rate_j}{rate_i}\right| = \left|\frac{R_j}{R_i}\right| = \left|\frac{R_j / V_R}{R_i / V_R}\right| = \left|\frac{r_j}{-r_i}\right| = \left|\frac{q_j.C_X}{q_i.C_X}\right| = \left|\frac{q_j}{q_i}\right|$$