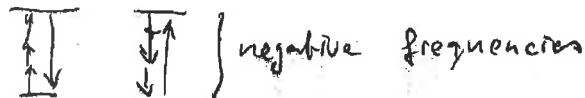


# Exercise 4

- a. There are 81 elements for the Cartesian components  
 $\omega_4 = \omega_1 + \omega_2 + \omega_3 \rightarrow 3!$  combinations per frequency = 6  
 4 frequencies  
 2 up and down



$$81 \times 4 = 324$$

- b.  $\times \frac{1}{2}$  We drop the negative frequencies  
 c. If the material is lossless all frequency components appear with the same efficiency  $\Rightarrow 81$

{ 81 x 4 could be too but  $\chi(\omega_4) = \chi(\omega_1)$  if the response is a product of the other components }

d. 15	XXXX	}	3	4,0	This means that all freq components respond
	YYYY				
	ZZZZ				
	EXXX	}	6	3,1	
	YXXX				
	XXYY				
	XYYY				
	YZZZ				
	XZZZ				
	XXYY	}	3	2,2	
	XXZZ				
	YYZZ				
	XXYZ	}	3	2,1,1	
	YXYZ				
	ZXYZ				

# Exercise 5

$$d_{ijk} = \frac{1}{2} \pi_{ijk}^{(2)}$$

$$d_{ijk} : i_{11} \quad i_{22} \quad i_{33} \quad i_{23} = i_{32} \quad i_{31} = i_{13} \quad i_{12} = i_{21}$$

$$d_{ijk} : i_2 \quad i_2 \quad i_3 \quad i_4 \quad i_5 \quad i_6$$

Elements

a

$11 = 111$	$21 = 211$	$31 = 311$
$12 = 122$	$22 = 222$	$32 = 322$
$13 = 133$	$23 = 233$	$33 = 333$
$14 = 123 = 132$	$24 = 223 = 232$	$34 = 323 = 332$
$15 = 131 = 113$	$25 = 213 = 231$	$35 = 331 = 313$
$16 = 112 = 121$	$26 = 212 = 221$	$36 = 312 = 321$

$$\Rightarrow 3 \times 6$$

b. For Kleinman's symmetry in all indices we have:

$$111, 222, 333; \quad 11, 22, 33 \quad \text{Three identical indices: } 3$$

$$\text{All three indices different: } 123 = 132 = 312 = 321 = 213 = 231$$

$$14 = 36 = 25$$

$$14 = 14 = 36 = 36 = 25 = 25$$

Two are the same:

6

$112 = 121 = 211$	$16 = 21$
$113 = 131 = 311$	$15 = 31$
$221 = 212 = 122$	$26 = 12$
$331 = 313 = 133$	$35 = 13$
$332 = 323 = 233$	$34 = 23$
$223 = 232 = 322$	$24 = 32$

Two in total: