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a. $D(\sigma_v) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Non-zero 11, 22, 33, 21, 12

$D_{22} = 1$: this means we need an even number of "2" or y

$\Rightarrow$ xx; yy; zz are non-zero

b $D(C_4) = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $D_{21} = -1$
$D_{12} = D_{33} = 1$

$\Rightarrow D_{21}$ is going to bring down the number of non-zero / different elements

$\chi^{(1)'}_{22} = D_{21} D_{21} \cdot \chi^{(1)}_{11}$ must be true $\left.\begin{matrix} 11 = 22; \\ 22 = 11; \end{matrix}\right\} xx = yy$
$\chi^{(1)'}_{11} = D_{12} D_{12} \chi^{(1)}_{22}$ "

c. The same thing now applies also to the pairs xx, zz and yy, zz.
So that $xx = yy = zz$ $\Rightarrow$ just one value for $\chi^{(1)}$