

Fundamentals of Traffic Operations and Control

Nikolas Geroliminis

Exercise solutions

Ramp metering and coordinated ramp metering

Author: Işık İlber Sirmatel

Question 1: Ramp metering

a) Outside the peak hour there is no need for ramp metering since mainline capacity (6000 veh/h) is greater than the total demand (5700 veh/h), which is the sum of the mainline demand (5400 veh/h) and the ramp demand (300 veh/h).

During the peak hour, the total demand exceeds the mainline capacity, so we need ramp metering to prevent congestion in the mainline. To calculate the optimal metering rate α , we equate the mainline capacity to the demand restricted by ramp metering:

$$6000 \text{ veh/h} = 5400 \text{ veh/h} + 1000 \text{ veh/h} \cdot (0.2 + \alpha \cdot 0.8).$$

Solving the above equation for α , we obtain $\alpha = 0.5$.

b) We can calculate the delay graphically by drawing the time-space diagram of the SPV lane. At the beginning of the peak hour when we start to employ ramp metering with $\alpha = 0.5$ as found in a), a part of the demand for the SPV lane will be accumulating in a queue. At the end of the peak hour, ramp metering will not be active (as there is no need) and thus the queue will start dissolving and will disappear completely around 2.3 hours after the start of the peak hour. This whole procedure can be expressed graphically by fig. 1.

The figure can be constructed as follows: During the peak hour (lasting 1 hour), the SPV demand ($0.8 \cdot 1000 = 800$ veh/h) will be entering the SPV lane (blue line, slope = 800 veh/h), whereas only half of it ($\alpha \cdot 800 = 400$ veh/h) is allowed to exit into mainline (red line, slope = 400 veh/h). After peak hour ends, the SPV demand is $0.8 \cdot 300 = 240$ veh/h (yellow line, slope = 240 veh/h), whereas the capacity available to the SPV demand is $6000 - 5400 - 0.2 \cdot 300 = 540$ veh/h (purple line, slope = 540 veh/h). Calculating the area of the parallelogram formed by the four lines, we find 466.67 veh.h, which is the total delay experienced by the SPV users.

c) We can also draw the time-space diagram of the ramp for the case where ramp metering is applied to both the HOV and SPV lanes (i.e., the vehicles are treated the same), as shown in fig. 2. In this case, the optimal metering rate can be calculated from the following equation

$$6000 \text{ veh/h} = 5400 \text{ veh/h} + \alpha_c \cdot 1000 \text{ veh/h}.$$

Solving the above equation for α_c , we obtain $\alpha_c = 0.6$.

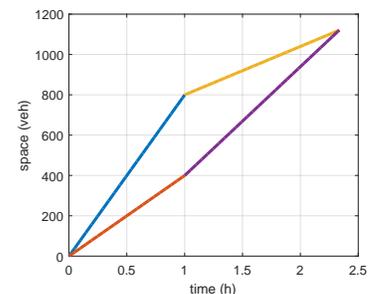


Figure 1: Time-space diagram of SPV lane.

The figure can be constructed (similarly to fig. 1) as follows: During the peak hour (lasting 1 hour), the ramp demand (1000 veh/h) will be entering the ramp (blue line, slope = 1000 veh/h), whereas only 60% of it ($\alpha_c \cdot 1000 = 600$ veh/h) is allowed to exit into mainline (red line, slope = 600 veh/h). After peak hour ends, the ramp demand is 300 veh/h (yellow line, slope = 300 veh/h), whereas the capacity available to the ramp demand is $6000 - 5400 = 600$ veh/h (purple line, slope = 600 veh/h).

Calculating the area of the triangle formed by the four lines, we find 466.67 veh.h, which is the total delay experienced by all vehicles using the ramp. As SPVs carry 1 passenger each, whereas HOVs carry 2, the total delay for **b**) and **c**) (in passenger hours travelled) can be calculated as follows:

$$\text{delay}_b = 1 \text{ pax/veh} \cdot 466.67 \text{ veh.h} = 466.67 \text{ pax.h}$$

$$\begin{aligned} \text{delay}_c &= 0.8 \cdot 1 \text{ pax/veh} \cdot 466.67 \text{ veh.h} + 0.2 \cdot 2 \text{ pax/veh} \cdot 466.67 \text{ veh.h} \\ &= 560 \text{ pax.h,} \end{aligned}$$

from where we obtain the time saving as follows:

$$\text{delay}_c - \text{delay}_b = 93.33 \text{ pax.h.}$$

d) The maximum queue length will occur at the end of the peak hour, and can be calculated as follows (for **b**):

$$1 \text{ h} \cdot (800 \text{ veh/h} - 400 \text{ veh/h}) = 400 \text{ veh,}$$

which can also be read from fig. 1 as the vertical distance between the blue and red lines for time = 1 h.

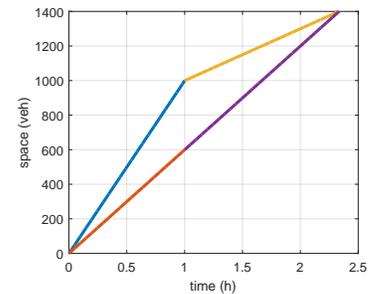


Figure 2: Time-space diagram of ramp.

Question 2: Coordinated ramp metering

a) The bottleneck will occur at the beginning of the peak hour (i.e., when demand exceeds mainline capacity), and will be located downstream of where ramp 2 joins the mainline.

b) As the bottleneck would occur near ramp 2, this ramp should start metering first. The optimal metering rate α_2 can be calculated from the following equation:

$$6000 \text{ veh/h} = 4600 \text{ veh/h} + 800 \text{ veh/h} + \alpha_2 \cdot 1000 \text{ veh/h}.$$

Solving the above equation for α_2 , we obtain $\alpha_2 = 0.6$.

c) Due to metering at ramp 2, vehicles will start queueing with a rate of $(1 - \alpha_2) \cdot 1000 \text{ veh/h} = 400 \text{ veh/h}$, which will result in the queue of ramp 2 reaching capacity in $t_2 = 100 \text{ veh}/400 \text{ veh/h} = 15 \text{ minutes}$. At this point it is impossible for ramp 2 to continue metering as it has run out of storage capacity. Thus ramp 2 stops and ramp 1 starts metering to prevent congestion in the mainline. The optimal metering rate α_1 can be calculated from the following equation:

$$6000 \text{ veh/h} = 4600 \text{ veh/h} + \alpha_1 \cdot 800 \text{ veh/h} + 1000 \text{ veh/h}.$$

Solving the above equation for α_1 , we obtain $\alpha_1 = 0.5$.

d) This time, due to metering at ramp 1, vehicles will start queueing with a rate of $(1 - \alpha_1) \cdot 800 \text{ veh/h} = 400 \text{ veh/h}$, which will result in the queue of ramp 1 reaching capacity in $t_1 = 100 \text{ veh}/400 \text{ veh/h} = 15 \text{ minutes}$. During this time, since both the demand and discharge of ramp 2 are 1000 veh/h , its queue length will not change, staying at its maximum value 100 veh . Thus, both ramps will be at their storage capacity $t_2 + t_1 = 30 \text{ minutes}$ after the beginning of the peak hour.

At this point it is impossible for these two ramps to continue metering as they are out of storage capacity. If available, ramps upstream of ramp 1 should start metering to prevent congestion in the mainline, otherwise congestion is unavoidable as demand exceeds capacity.