

## 1 Problems from 2018 Homework

### Problem 1

Consider the *Directed Acyclic Graph* (DAG)  $G(V,E,W)$  in Fig. 1.

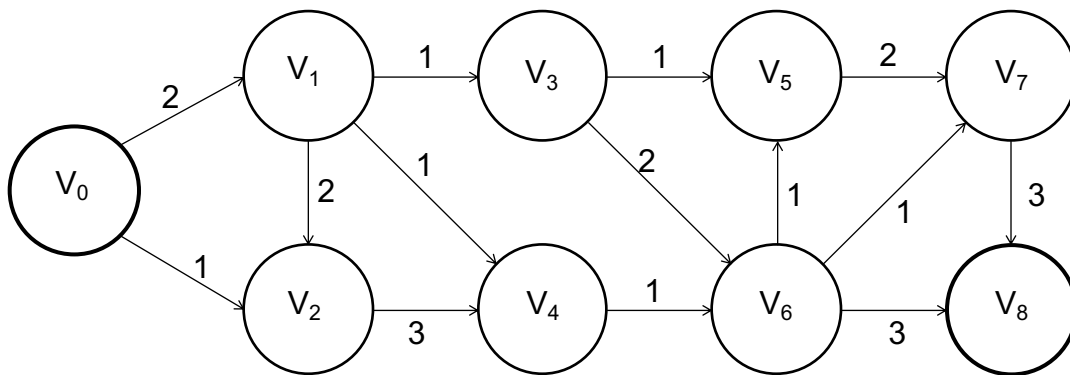


Figure 1: A directed acyclic graph

Consider vertex  $V_0$  as the **source** and vertex  $V_8$  as the **sink**. Find the shortest path from  $V_0$  to  $V_8$  by applying the following algorithms:

- (a) Dijkstra algorithm.
- (b) Bellman-Ford algorithm.

### Problem 2

Given the following equations:

$$\begin{aligned}x &= (a \times b \times c + d) \times e + f \\y &= k + g \times h + g \times j \times h \times i \\z &= x + y\end{aligned}$$

- (a) Draw the data-flow graph using the operations as they appear in the expression, without any optimization. Assume additions and multiplications have 2 inputs.
- (b) Apply **tree height** reduction to the data-flow graph drawn in (a).
- (c) Discuss on the different resources usage between graph in (a) and graph in (b).
- (d) Assume that  $a=2$ ,  $b=3$ ,  $c=2$ ,  $d=2$ ,  $h=3$ ,  $j=4$  and  $i=8$  are constant. Apply **constant propagation** and **operator strength** reduction to the graph obtained in (b). Draw the resulting data-flow graph.

### Problem 3

Given the sequencing graph in Fig.2, use *force directed scheduling* to select the time step at which the colored operation should be scheduled to reduce concurrency. Assume all operations have unit delays and consider an upper bound on the latency found in the Homework.

### Problem 4

$$F = a'bcd' + bc + a'bc'd' + ab' + abd'$$

Given the Boolean function  $F$ :

- (a) Compute the truth table both in binary and hexadecimal notation.
- (b) Draw the min-terms on the cube.
- (c) List all the primes (also on the cube).
- (d) List all the essential primes.
- (e) Find a minimum cover using McCluskey's method (prime implicant table, branch and bound).
- (f) Find a minimum cover using Petrick's method (primes in pos, transform in sop).
- (g) Show the obtained cover on the cube.

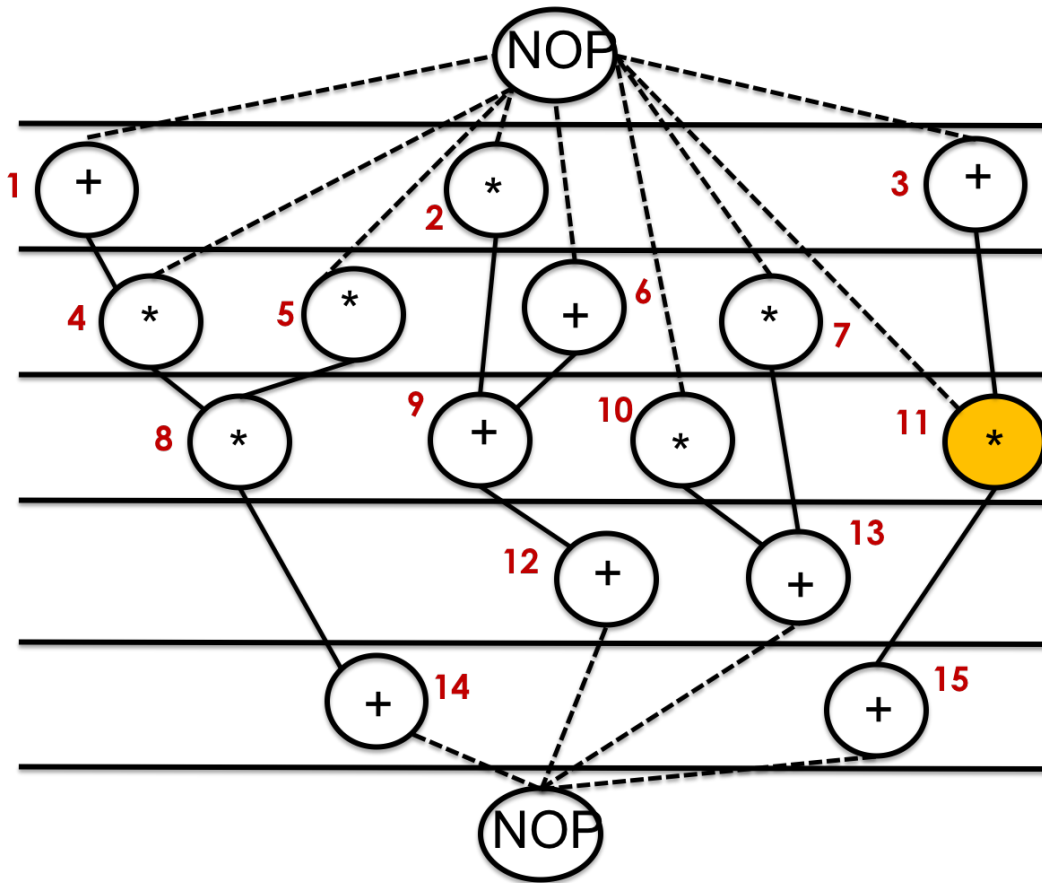


Figure 2: Sequencing graph.

## 2 Problems from previous years

2.1 In graph coloring, two vertices connected by an edge can be assigned to the same color.

YES NO

2.2 Forests are graphs with chromatic number equal to 2.

YES NO

**2.3 We typically color the vertices of compatibility graphs using the *LEFT\_EDGE* algorithm.**

YES NO

**2.4 Both the Dijkstra and the Bellman-ford algorithms are suitable for graphs that contain negative cycles.**

YES NO

**2.5 The left edge algorithm has exponential run-time complexity.**

YES NO

**2.6 The mobility of an operation in a graph is constant whatever the latency bounds are applied for ASAP and ALAP.**

YES NO

### Problem 1

Write the Control-Flow Expression that executes the three codes in parallel.

Code 1	Code 2	Code 3
<b>if <i>b</i> then</b>	<b>begin</b>	<b>always {</b>
<b>while <i>a</i> do</b>	<b>wait !<i>a</i></b>	<b>if !<i>a</i> then</b>
<i>P1</i> ;	<i>P2</i> ;	<i>P5</i> ;
<b>end while</b>	<b>while <i>b</i> do</b>	<b>else</b>
<b>else</b>	<i>P3</i> ;	<i>P4</i> ;
<i>P2</i> ;	<b>end while</b>	<b>end if</b>
<b>end if</b>	<b>end</b>	<b>}</b>

### Problem 2

For the sequencing graph from Fig.3:

- (a) draw the scheduled graph using List scheduling algorithm with at most three multipliers and one adder (at the same time per level). Consider that the mul-

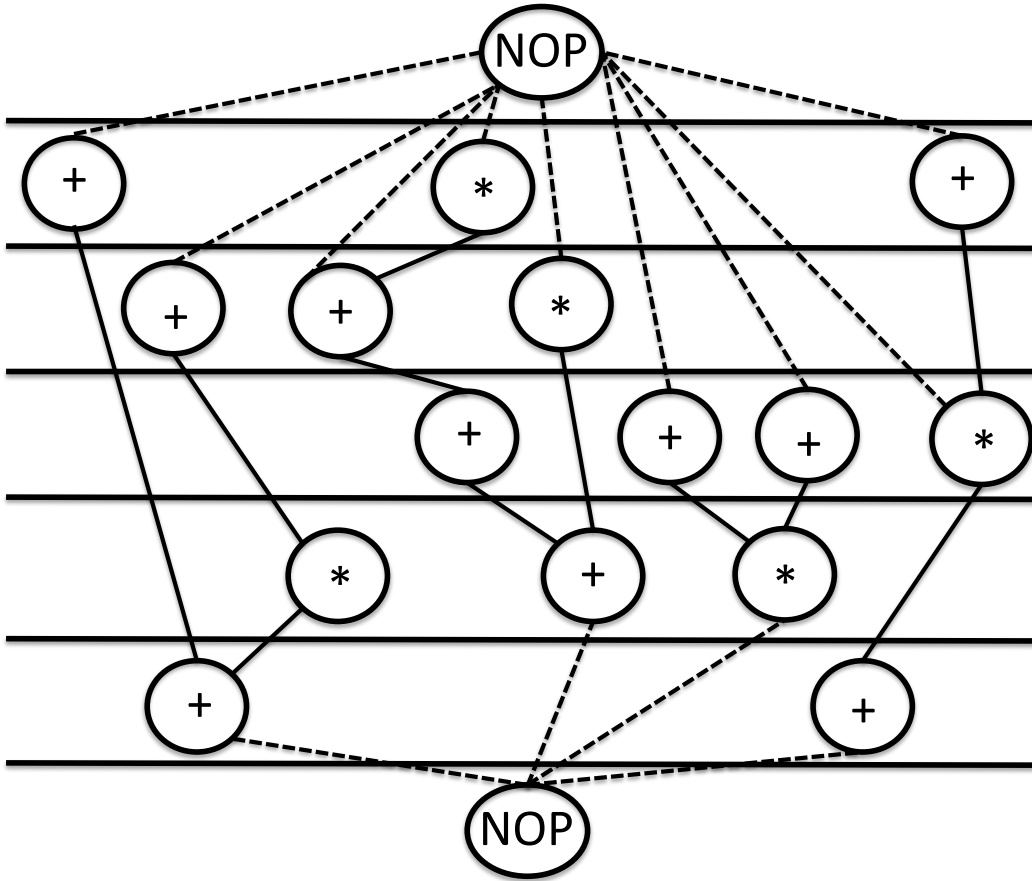


Figure 3: Sequencing graph.

multiplier takes two units of time and the adder one. (Minimum latency subject to resource bound).

- (b) The resource bound is now to three multipliers and two adder. Again, the multiplier takes two units of time and the adder one. Draw the scheduled graph using List scheduling algorithm (Minimum latency subject to resource bound).
- (c) Is any additional resource useful to reduce the latency? If yes which one (adder or multiplier)?

### Problem 3

Given the data flow graph in Figure 4 (from sequencing graph in Figure 3):

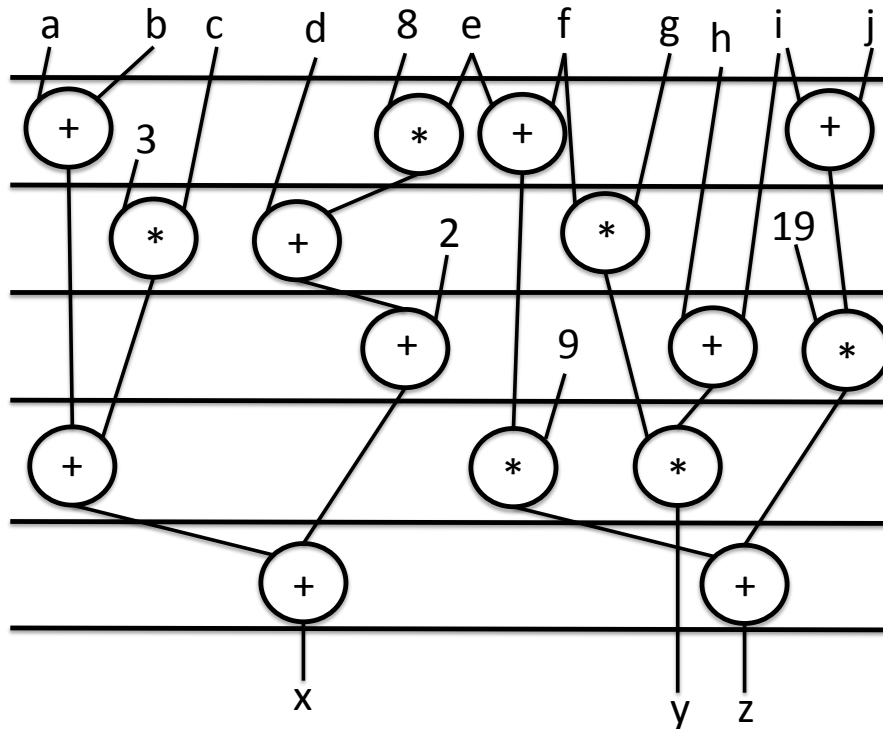


Figure 4: Dataflow graph.

Apply behavioral optimization (minimize latency, all operations complete in one unit of time, no resource bound) by means of:

- (a) Tree-height reduction (use associativity, commutativity, distributivity).
- (b) Operator-strength reduction (multiplication by a constant).

If you need shift operators, you can annotate the required shift (e.g.,  $\{\ll 1\}$  for 1 position left shift) directly to edge affected (shift operations take no explicit resources).

- (c) After behavioral level optimization, apply the ASAP algorithm.

### Problem 4

$$F = x'y' + x'yw + x'yz'w' + xyzw' + xy' + xyw$$

Given the Boolean function  $F$ :

- (a) Draw the min-terms on the cube.
- (b) List all the primes (also on the cube).
- (c) List all the essential primes.
- (d) Find a minimum cover using McCluskey's method (prime implicant table, branch and bound).
- (e) Find a minimum cover using Petrick's method (primes in pos, transform in sop).
- (f) Show the obtained cover on the cube.