The Manley Rowe relations

Now we turn to the conservation of energy (intensity).

The intensity is defined as:

$$I_i = 2n_i \epsilon_0 c\, A_i A_i^*$$

the change in I_i $\frac{dI_i}{dz}$: $\frac{dI_i}{dz} = 2n_i \epsilon_0 c \left\{ A_i^* \frac{dA_i}{dz} + A_i \frac{dA_i^*}{dz} \right\}$) insert $\frac{dA_i}{dz}$

$$\frac{dI_1}{dz} = \frac{2n_i \epsilon_0 c\, \chi^{(2)} \omega_1^2}{k_1 c^2} \left(i A_1^* A_3 A_2^* e^{-i\Delta k z} + c.c. \right)$$

$k = \frac{n\omega}{c}$

Note: $\underbrace{ie^{-ix}}_{i(\cos x - i\sin x)} - ie^{ix} = i(\cos x - i\sin x) - i(\cos x + i\sin x) = 2\sin x$

$= -2\,\mathrm{Im}\left(e^{-ix}\right)$

$$\frac{dI_1}{dz} = -4\epsilon_0 \chi^{(2)} \omega_1\, \mathrm{Im}\left(A_3 A_1^* A_2^* e^{-i\Delta k z}\right)$$

$\frac{c\omega_1^2}{k_1 c^2} = \frac{c\omega_1^2}{c^2} \cdot \frac{c}{n\omega_1}$

$$\frac{dI_2}{dz} = -4\epsilon_0 \chi^{(2)} \omega_2\, \mathrm{Im}\left(A_3 A_1^* A_2^* e^{-i\Delta k z}\right)$$

$$\frac{dI_3}{dz} = -4\epsilon_0 \chi^{(2)} \omega_3\, \mathrm{Im}\left(A_3^* A_1 A_2 e^{i\Delta k z}\right) = 4\epsilon_0 \chi^{(2)} \omega_3\, \mathrm{Im}\left(A_3 A_1^* A_2^* \cdot e^{-i\Delta k z}\right)$$

$I = I_1 + I_2 + I_3$ is the total intensity (energy per area per time)

$$\frac{dI}{dz} = \frac{dI_1}{dz} + \frac{dI_2}{dz} + \frac{dI_3}{dz} = -4\epsilon_0 \chi^{(2)}\, \mathrm{Im}\left(A_3^* A_1 A_2 e^{-i\Delta k z}\right) \cdot \underbrace{(\omega_1 + \omega_2 - \omega_3)}_{=0},$$

Thus there is conservation of energy, but also:

$$\frac{d}{dz}\left(\frac{I_1}{\omega_1}\right) = \frac{d}{dz}\left(\frac{I_2}{\omega_2}\right) = -\frac{d}{dz}\left(\frac{I_3}{\omega_3}\right)$$

The ~~an~~ change in photons is coupled

$\frac{I_1}{\omega_1}$: $\frac{\text{photons}}{\text{area} \cdot \text{time}}$

Exercise 11

a. For THG $\chi^{(3)}_{ijkl}(3\omega;\omega,\omega,\omega)$

there are all indistinguishable freq components

therefore χ has only one element

The general form

$$\chi_{ijkl} = \chi_{1122}\delta_{ij}\delta_{kl} + \chi_{1212}\delta_{ik}\delta_{jl} + \chi_{1221}\delta_{il}\cdot\delta_{jk}$$

becomes $\chi_{1122} = \chi_{1212} = \chi_{1221}$

$$\chi_{ijkl} = \chi_{1122}\{\delta_{ij}\delta_{kl} + \delta_{ik}\cdot\delta_{jl} + \delta_{il}\cdot\delta_{jk}\}$$ with value 1 or 3

b. For Kleinman symmetry we would have the same perturbation

c. Here we have

$$\chi_{ijkl}(\omega;\omega,\omega,-\omega)$$

can be freely permuted $\Big\}$ 1122 = 1212

So we have $\chi_{ijkl} = \chi_{1122}\cdot\{\delta_{ij}\delta_{kl} + \delta_{ik}\cdot\delta_{jl}\} + \chi_{1221}\cdot\delta_{il}\cdot\delta_{jk}$

For $P^{(3)}(\omega)$: $P^{(3)}_i(\omega) = 3\epsilon_0\chi_{ijkl}(\omega)E_j(\omega)E_k(\omega)E_l(-\omega)$

combine: first term i and j and i and k are the same (ω)

one of the incoming beams is $-\omega$

Write it out: $P^{(3)}_i(\omega) = 3\epsilon_0\Big[\chi_{1122}\Big[\underbrace{E_j(\omega)\delta_{ij}}_{E_i(\omega)}\cdot\underbrace{E_k(\omega)\cdot E_l(-\omega)\delta_{kl}}_{\vec{E}(\omega)\cdot\vec{E}(-\omega)} + \underbrace{E_k(\omega)\delta_{ik}}_{E_i(\omega)}\cdot\underbrace{E_j(\omega)\cdot E_l(-\omega)\delta_{jl}}_{\vec{E}(\omega)\cdot\vec{E}(-\omega)}\Big]$

= inner product

$\vec{E}(\omega)\cdot\vec{E}(-\omega) = E^*(\omega)$ ← counter propagating → $= E^*(\omega)$

Both terms have opposite rotational directions.

$$+\ \chi_{1221}\underbrace{E_l(-\omega)\delta_{il}}_{E_i(-\omega) = E_i^*}\cdot\underbrace{E_j(\omega)E_k(\omega)\delta_{jk}}_{\vec{E}(\omega)\vec{E}(\omega)}\Big]$$