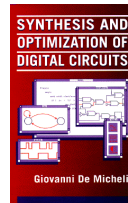


# *Sequential Logic Synthesis*

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# Module 1

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## ◆ Objective

- ▲ Motivation and assumptions for sequential synthesis
- ▲ Finite-state machine design and optimization

# Synchronous logic circuits

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## ◆ Interconnection of

- ▲ Combinational logic gates
- ▲ Synchronous delay elements
  - ▼ Edge-triggered, master/slave

## ◆ Assumptions

- ▲ No direct combinational feedback
- ▲ Single-phase clocking

## ◆ Extensions to

- ▲ Multiple-phase clocking
- ▲ Gated latches

# Modeling synchronous circuits

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- ◆ **Circuit are modeled in hardware languages**
  - ▲ **Circuit model may be directly related to FSM model**
    - ▼ **Description by: switch-case**
  - ▲ **Circuit model may be structural**
    - ▼ **Explicit definition of registers**
- ◆ **Sequential circuit models can be generated from high-level models**
  - ▲ **Control generation in high-level synthesis**

# Modeling synchronous circuits

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## ◆ State-based model:

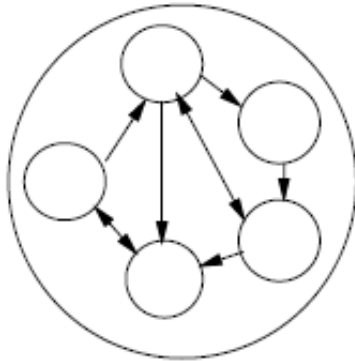
- ▲ Model circuits as **finite-state machines (FSMs)**
- ▲ Represent by state tables/diagrams
- ▲ Apply exact/heuristic algorithms for:
  - ▼ State minimization
  - ▼ State encoding

## ◆ Structural model

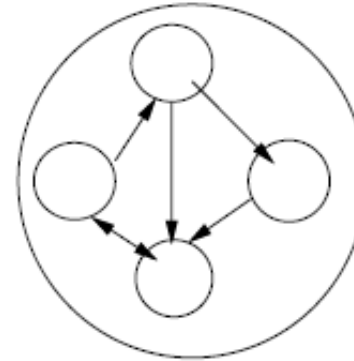
- ▲ Represent circuit by **synchronous logic network**
- ▲ Apply
  - ▼ Retiming
  - ▼ Logic transformations

# State-based optimization

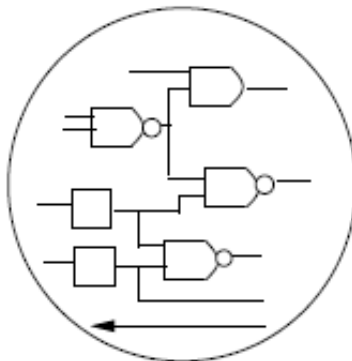
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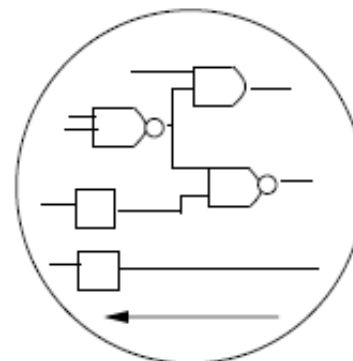
**FSM Specification**



**State Minimization**



**State Encoding**



**Combinational Optimization**

# Modeling synchronous circuits

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- ◆ **Advantages and disadvantages of models**
- ◆ **State-based model**
  - ▲ **Explicit notion of state**
  - ▲ **Implicit notion of area and delay**
- ◆ **Structural model**
  - ▲ **Implicit notion of state**
  - ▲ **Explicit notion of area and delay**
- ◆ **Transition from a model to another is possible**
  - ▲ **State encoding**
  - ▲ **State extraction**

# Sequential logic optimization

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## ◆ Typical flow

### ▲ Optimize FSM state model first

- ▼ Reduce complexity of the model
- ▼ E.g., apply state minimization
- ▼ Correlates to area reduction

### ▲ Encode states and obtain a structural model

- ▼ Apply retiming and transformations
- ▼ Achieve performance enhancement

### ▲ Use state extraction for verification purposes



# Formal finite-state machine model

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- ◆ A set of primary input patterns  $X$
- ◆ A set of primary output patterns  $Y$
- ◆ A set of states  $S$
- ◆ A state transition function:  $\delta: X \times S \rightarrow S$
- ◆ An output function:
  - ▲  $\lambda: X \times S \rightarrow Y$  for **Mealy** models
  - ▲  $\lambda: S \rightarrow Y$  for **Moore** models

# State minimization

---

## ◆ Classic problem

- ▲ Exact and heuristic algorithms are available
- ▲ Objective is to reduce the number of states and hence the area

## ◆ Completely-specified finite-state machines

- ▲ No *don't care* conditions
- ▲ Polynomial-time solutions

## ◆ Incompletely-specified finite-state machines

- ▲ Unspecified transitions and/or outputs
  - ▼ Usual case in synthesis
- ▲ Intractable problem:
  - ▼ Requires binate covering

# State minimization for completely-specified FSMs

---

## ◆ Equivalent states:

- ▲ Given any input sequence, the corresponding output sequence match

## ◆ Theorem:

- ▲ Two states are equivalent if and only if:
  - ▼ They lead to identical outputs and their next-states are equivalent

## ◆ Equivalence is transitive

- ▲ Partition states into equivalence classes
- ▲ Minimum finite-state machine is unique

# State minimization for completely-specified FSMs

---

- ◆ **Stepwise partition refinement:**
  - ▲ **Initially:**
    - ▼ All states in the same partition block
  - ▲ **Iteratively:**
    - ▼ Refine partition blocks
  - ▲ **At convergence:**
    - ▼ Partition blocks identify equivalent states
- ◆ **Refinement can be done in two directions**
  - ▲ Transitions *from* states in block to other states
    - ▼ Classic method. Quadratic complexity
  - ▲ Transitions *into* states of block under consideration
    - ▼ Inverted tables. Hopcroft's algorithm.

# Example of refinement

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## ◆ Initial partition:

▲  $\Pi_1$  : States belong to the same block when outputs are the same for any input

## ◆ Iteration:

▲  $\Pi_{k+1}$  : States belong to the same block if they were previously in the same block and their next states are in the same block of  $\Pi_k$  for any input

## ◆ Convergence:

▲  $\Pi_{k+1} = \Pi_k$

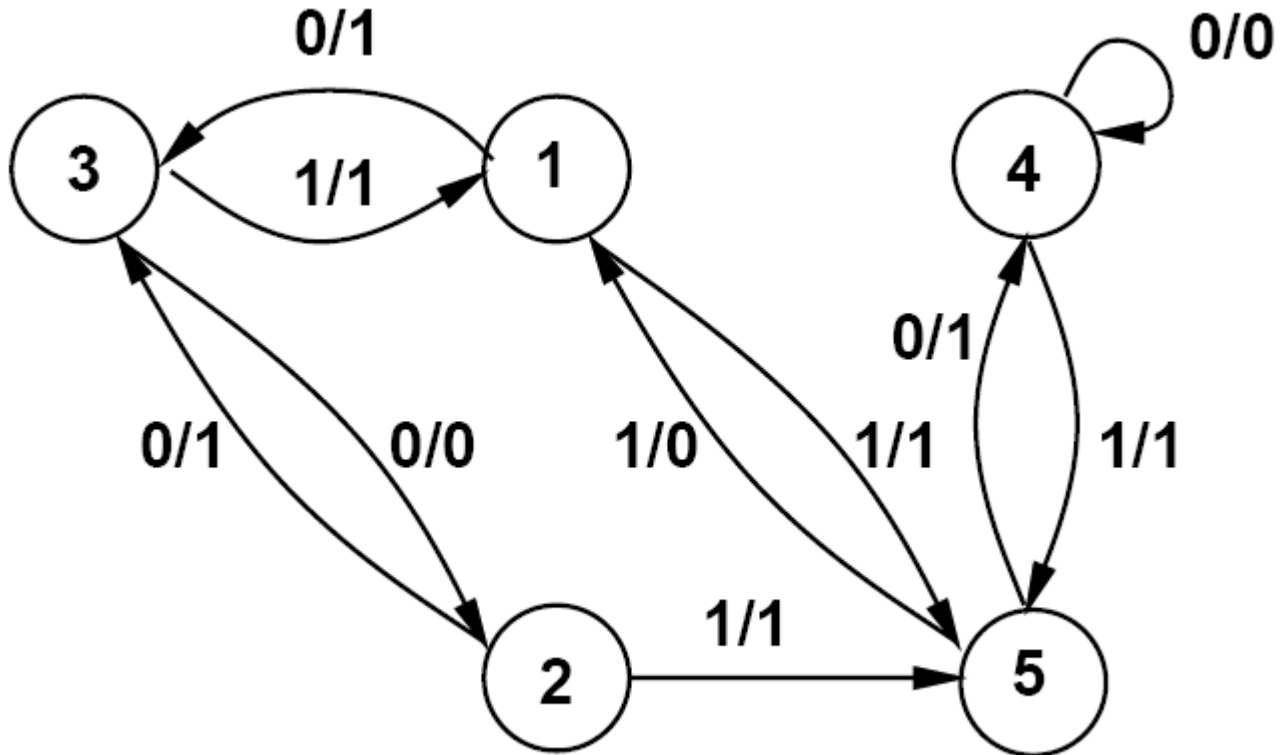
# Example

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INPUT	STATE	N-STATE	OUTPUT
0	$s_1$	$s_3$	1
1	$s_1$	$s_5$	1
0	$s_2$	$s_3$	1
1	$s_2$	$s_5$	1
0	$s_3$	$s_2$	0
1	$s_3$	$s_1$	1
0	$s_4$	$s_4$	0
1	$s_4$	$s_5$	1
0	$s_5$	$s_4$	1
1	$s_5$	$s_1$	0

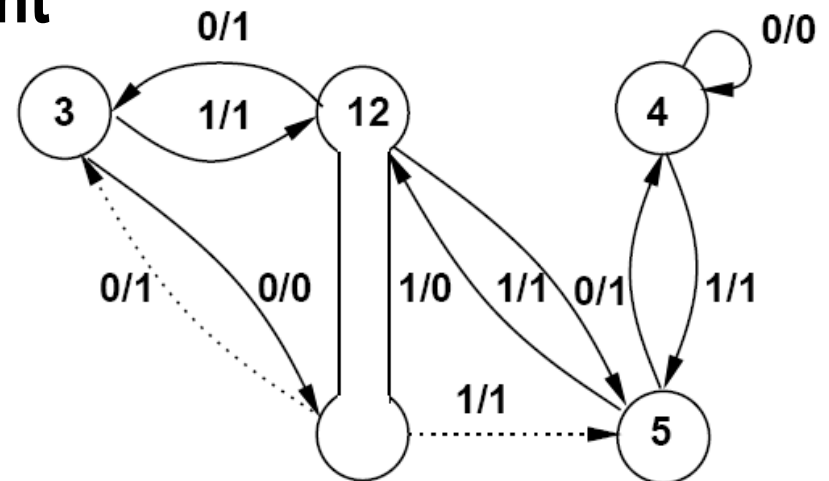
# Example

---



# Example

- ◆  $\Pi_1 = \{ \{ s_1, s_2 \}, \{ s_3, s_4 \}, \{ s_5 \} \}$
- ◆  $\Pi_2 = \{ \{ s_1, s_2 \}, \{ s_3 \}, \{ s_4 \}, \{ s_5 \} \}$
- ◆  $\Pi_2$  is a partition into equivalence classes
  - ▲ No further refinement is possible
  - ▲ States  $\{ s_1, s_2 \}$  are equivalent





# State minimization for incompletely-specified finite-state machines

---

## ◆ Applicable input sequences

- ▲ All transitions are specified

## ◆ Compatible states

- ▲ Given any applicable input sequence, the corresponding output sequence match

## ◆ Theorem:

- ▲ Two states are compatible if and only if:

- ▼ They lead to identical outputs

- ◆ (when both are specified)

- ▼ And their next state is compatible

- ◆ (when both are specified)

# State minimization for incompletely-specified finite-state machines

---

- ◆ **Compatibility is not an equivalence relation**
- ◆ **Minimum finite-state machine is not unique**
- ◆ **Implication relation make the problem intractable**
  - ▲ **Two states may be compatible, subject to other states being compatible.**
  - ▲ **Implications are binate satisfiability clauses**
    - ▼  $a \rightarrow b = a' + b$

# Example

---

INPUT	STATE	N-STATE	OUTPUT
0	$s_1$	$s_3$	1
1	$s_1$	$s_5$	*
0	$s_2$	$s_3$	*
1	$s_2$	$s_5$	1
0	$s_3$	$s_2$	0
1	$s_3$	$s_1$	1
0	$s_4$	$s_4$	0
1	$s_4$	$s_5$	1
0	$s_5$	$s_4$	1
1	$s_5$	$s_1$	0

# Trivial method

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## ◆ Consider all possible *don't care* assignments

### ▲ $n$ *don't care* imply

▼  $2^n$  completely specified FSMs

▼  $2^n$  solutions

## ◆ Example:

### ▲ Replace \* by 1

▼  $\Pi_1 = \{ \{ s_1, s_2 \}, \{ s_3 \}, \{ s_4 \}, \{ s_5 \} \}$

### ▲ Replace \* by 0

▼  $\Pi_1 = \{ \{ s_1, s_5 \}, \{ s_2, s_3, s_4 \} \}$

# Compatibility and implications

## Example

- ◆ Compatible states  $\{s_1, s_2\}$
- ◆ If  $\{s_3, s_4\}$  are compatible
  - ▲ Then  $\{s_1, s_5\}$  are also compatible
- ◆ Incompatible states  $\{s_2, s_5\}$

INPUT	STATE	N-STATE	OUTPUT
0	$s_1$	$s_3$	1
1	$s_1$	$s_5$	*
0	$s_2$	$s_3$	*
1	$s_2$	$s_5$	1
0	$s_3$	$s_2$	0
1	$s_3$	$s_1$	1
0	$s_4$	$s_4$	0
1	$s_4$	$s_5$	1
0	$s_5$	$s_4$	1
1	$s_5$	$s_1$	0

# Compatibility and implications

## ◆ Compatible pairs:

- ▲  $\{s_1, s_2\}$
- ▲  $\{s_1, s_5\} \leftarrow \{s_3, s_4\}$
- ▲  $\{s_2, s_4\} \leftarrow \{s_3, s_4\}$
- ▲  $\{s_2, s_3\} \leftarrow \{s_1, s_5\}$
- ▲  $\{s_3, s_4\} \leftarrow \{s_2, s_4\} \cup \{s_1, s_5\}$

## ◆ Incompatible pairs

- ▲  $\{s_2, s_5\}$
- ▲  $\{s_3, s_5\}$
- ▲  $\{s_1, s_4\}$
- ▲  $\{s_4, s_5\}$
- ▲  $\{s_1, s_3\}$

INPUT	STATE	N-STATE	OUTPUT
0	$s_1$	$s_3$	1
1	$s_1$	$s_5$	*
0	$s_2$	$s_3$	*
1	$s_2$	$s_5$	1
0	$s_3$	$s_2$	0
1	$s_3$	$s_1$	1
0	$s_4$	$s_4$	0
1	$s_4$	$s_5$	1
0	$s_5$	$s_4$	1
1	$s_5$	$s_1$	0

# Compatibility and implications

---

- ◆ **A class of compatible states** is such that all state pairs are compatible
- ◆ **A class is maximal**
  - ▲ If not subset of another class
- ◆ **Closure property**
  - ▲ A set of classes such that all compatibility implications are satisfied
- ◆ **The set of maximal compatibility classes**
  - ▲ Has the closure property
  - ▲ May not provide a minimum solution

# Maximum compatibility classes

---

## ◆ Example:

▲  $\{s_1, s_2\}$

▲  $\{s_1, s_5\} \leftarrow \{s_3, s_4\}$

▲  $\{s_2, s_3, s_4\} \leftarrow \{s_1, s_5\}$

## ◆ Cover with all MCC has cardinality 3



# Exact problem formulation

---

- ◆ **Prime compatibility classes:**

- ▲ Compatibility classes having the property that they are not subset of other classes implying the same (or subset) of classes

- ◆ **Compute all prime compatibility classes**

- ◆ **Select a minimum number of prime classes**

- ▲ Such that all states are covered

- ▲ All implications are satisfied

- ◆ **Exact solution requires binate cover**

- ◆ **Good approximation methods exists**

- ▲ Stamina

# Prime compatibility classes

---

## ◆ Example:

▲  $\{s_1, s_2\}$

▲  $\{s_1, s_5\} \leftarrow \{s_3, s_4\}$

▲  $\{s_2, s_3, s_4\} \leftarrow \{s_1, s_5\}$

## ◆ Minimum cover:

▲  $\{s_1, s_5\}$  ,  $\{s_2, s_3, s_4\}$

▲ Minimum cover has **cardinality 2**

# State encoding

---

- ◆ **Determine a binary encoding of the states**
  - ▲ **Optimizing some property of the representation (mainly area)**
- ◆ **Two-level model for combinational logic**
  - ▲ **Methods based on symbolic optimization**
    - ▼ **Minimize a symbolic cover of the finite state machine**
    - ▼ **Formulate and solve a constrained encoding problem**
- ◆ **Multiple-level model**
  - ▲ **Some heuristic methods that look for encoding which privilege cube and/or kernel extraction**
  - ▲ **Weak correlation with area minimality**

# Example

---

INPUT	P-STATE	N-STATE	OUTPUT
0	s1	s3	0
1	s1	s3	0
0	s2	s3	0
1	s2	s1	1
0	s3	s5	0
1	s3	s4	1
0	s4	s2	1
1	s4	s3	0
0	s5	s2	1
1	s5	s5	0

# Example

---

- **Minimum symbolic cover:**

*	s1s2s4	s3	0
1	s2	s1	1
0	s4s5	s2	1
1	s3	s4	1

- **Encoded cover :**

*	1**	001	0
1	101	111	1
0	*00	101	1
1	001	100	1

# Summary

## finite-state machine optimization

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- ◆ **FSM optimization has been widely researched**
  - ▲ **Classic and newer approaches**
- ◆ **State minimization and encoding correlate to area reduction**
  - ▲ **Useful, but with limited impact**
- ◆ **Performance-oriented FSM optimization has mixed results**
  - ▲ **Performance optimization is usually done by structural methods**

# Module 2

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## ◆ Objective

- ▲ Structural representation of sequential circuits
- ▲ Retiming
- ▲ Extensions

# Structural model for sequential circuits

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## ◆ Synchronous logic network

- ▲ Variables

- ▲ Boolean equations

- ▲ Synchronous delay annotation

## ◆ Synchronous network graph

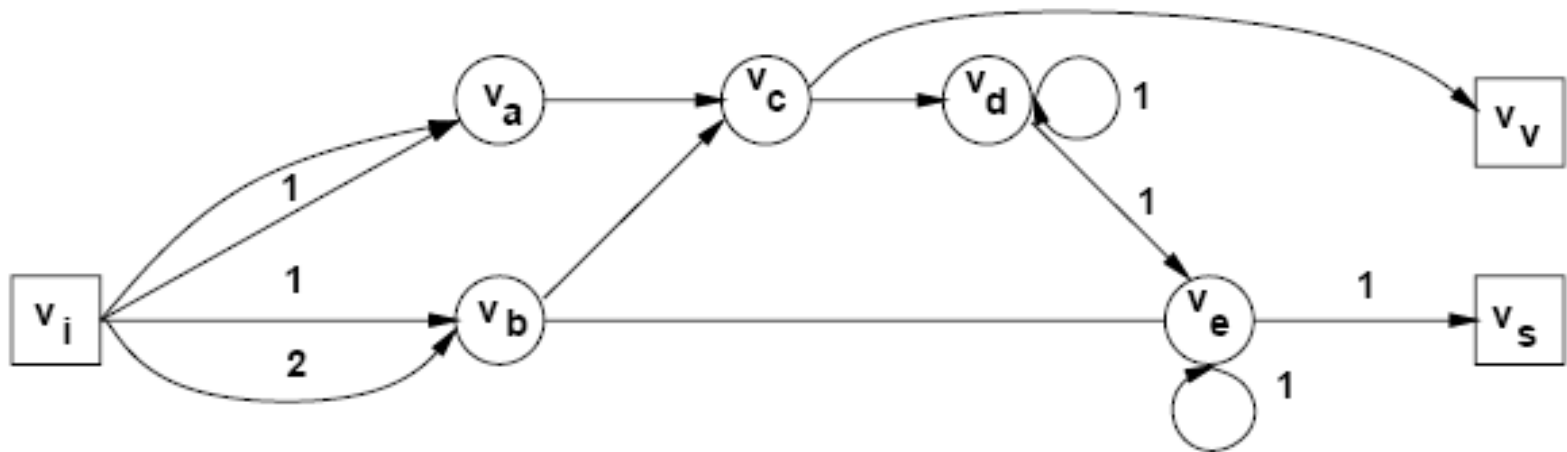
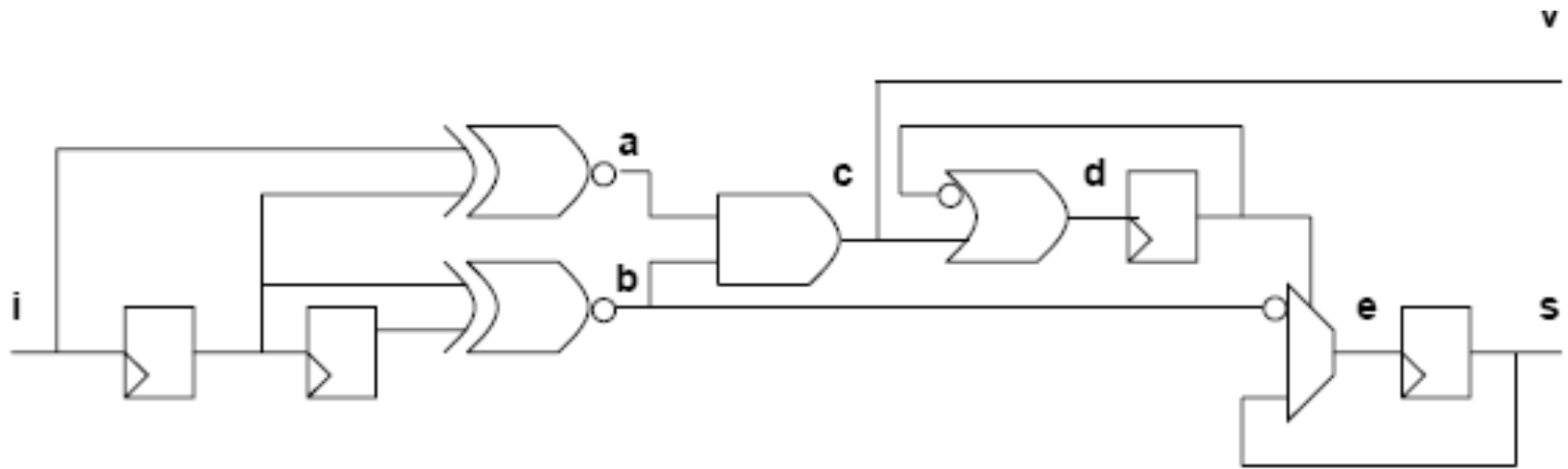
- ▲ Vertices  $\leftrightarrow$  equations  $\leftrightarrow$  I/O, gates

- ▲ Edges  $\leftrightarrow$  dependencies  $\leftrightarrow$  nets

- ▲ Weights  $\leftrightarrow$  synchronous delays  $\leftrightarrow$  registers



# Example



# Example

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$$a^{(n)} = i^{(n)} \bar{\oplus} i^{(n-1)}$$

$$b^{(n)} = i^{(n-1)} \bar{\oplus} i^{(n-2)}$$

$$c^{(n)} = a^{(n)} b^{(n)}$$

$$d^{(n)} = c^{(n)} + d'^{(n-1)}$$

$$e^{(n)} = d^{(n)} e^{(n-1)} + d'^{(n)} b'^{(n)}$$

$$v^{(n)} = c^{(n)}$$

$$s^{(n)} = e^{(n-1)}$$

$$a = i \bar{\oplus} i \textcircled{1}$$

$$b = i \textcircled{1} \bar{\oplus} i \textcircled{2}$$

$$c = a b$$

$$d = c + d \textcircled{1}'$$

$$e = d e \textcircled{1} + d' b'$$

$$v = c$$

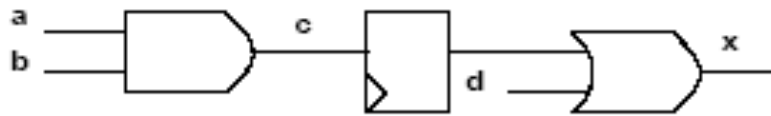
$$s = e \textcircled{1}$$

# Approaches to sequential synthesis

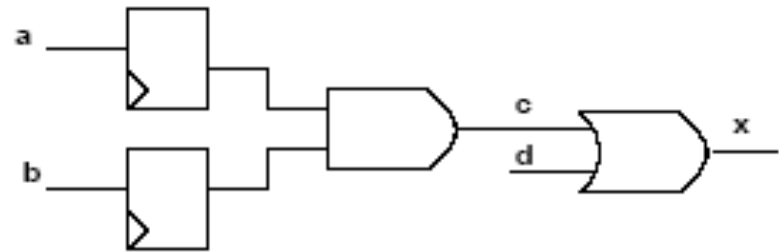
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- ◆ **Optimize combinational logic only**
  - ▲ Freeze circuit at register boundary
  - ▲ Modify equation and network graph topology
- ◆ **Retiming**
  - ▲ Move register positions. Change weights on graph
  - ▲ Preserve network topology
- ◆ **Synchronous transformations**
  - ▲ Blend combinational transformations and retiming
  - ▲ Powerful, but complex to use

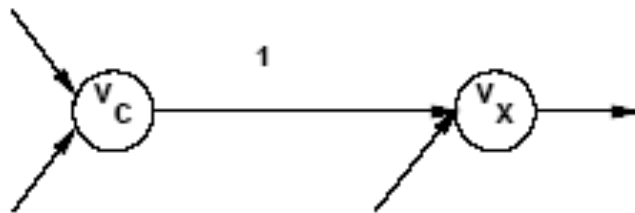
# Example of local retiming



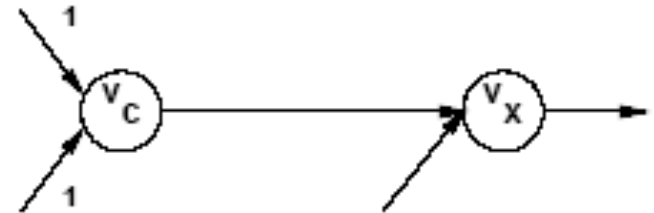
(a)



(c)



(b)



(d)

# Retiming

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- ◆ **Global optimization technique**
- ◆ **Change register positions**
  - ▲ **Affects area:**
    - ▼ Retiming changes register count
  - ▲ **Affects cycle-time**
    - ▼ Changes path delays between register pairs
- ◆ **Retiming algorithms have polynomial-time complexity**

# Retiming assumptions

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- ◆ **Delay is constant at each vertex**
  - ▲ No fanout delay dependency
- ◆ **Graph topology is invariant**
  - ▲ No logic transformations
- ◆ **Synchronous implementation**
  - ▲ Cycles have positive weights
    - ▼ Each feedback loop has to be broken by at least one register
  - ▲ Edges have non-negative weights
    - ▼ Physical registers cannot anticipate time
- ◆ **Consider topological paths**
  - ▲ No false path analysis

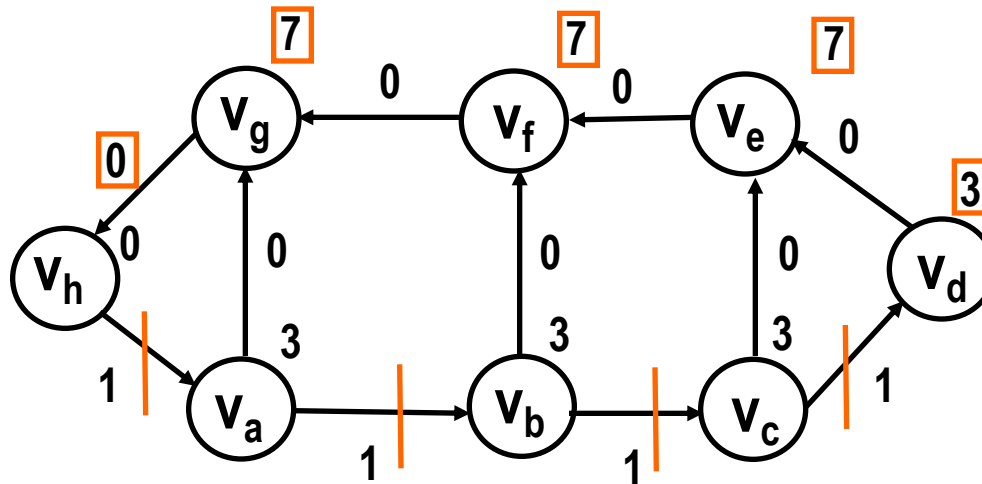
# Retiming

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- ◆ Retiming of a vertex  $v$ 
  - ▲ Integer  $r_v$
  - ▲ Registers moved from output to input:  $r_v$  positive
  - ▲ Registers moved from input to output:  $r_v$  negative
- ◆ Retiming of a network
  - ▲ Vector whose entries are the retiming at various vertices
- ◆ A family of I/O equivalent networks are specified by:
  - ▲ The original network
  - ▲ A set of vectors satisfying specific constraints
    - ▼ Legal retiming

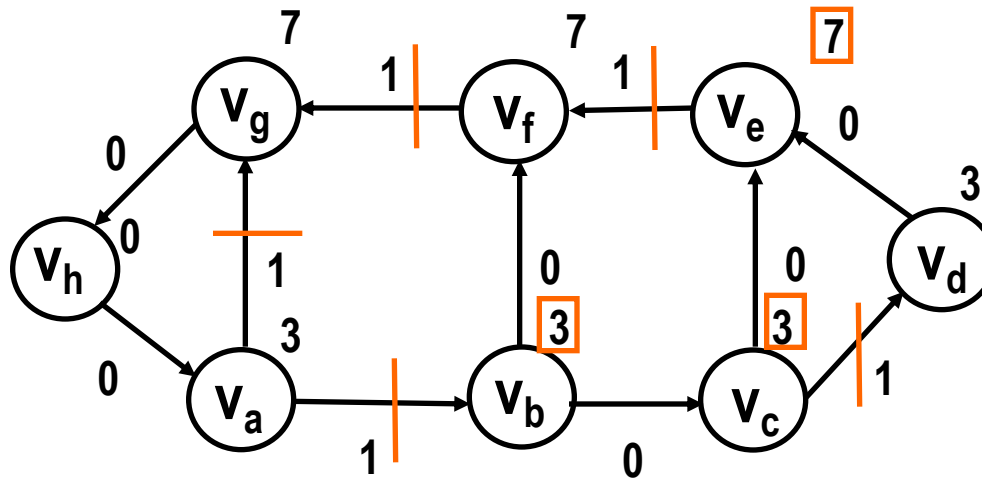
# Example

Original graph



Delay: 24

Retimed graph



Delay: 13



# Definitions and properties

## ◆ Definitions:

▲  $w(v_i, v_j)$  weight on edge  $(v_i, v_j)$

▲  $(v_i, \dots, v_j)$  path from  $v_i$  to  $v_j$

▲  $w(v_i, \dots, v_j)$  weight on path from  $v_i$  to  $v_j$

▲  $d(v_i, \dots, v_j)$  combinational delay on path from  $v_i$  to  $v_j$

## ◆ Properties:

▲ Retiming of an edge  $(v_i, v_j)$

▼  $\hat{w}_{ij} = w_{ij} + r_j - r_i$

▲ Retiming of a path  $(v_i, \dots, v_j)$

▼  $\hat{w}(v_i, \dots, v_j) = w(v_i, \dots, v_j) + r_j - r_i$

▲ Cycle weights are invariant



# Legal retiming

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## ◆ A retiming vector is legal iff:

### ▲ No edge weight is negative

▼  $\hat{w}_{ij}(v_i, v_j) = w_{ij}(v_i, v_j) + r_j - r_i \geq 0$  for all  $i, j$

### ▲ Given a clock period $\varphi$ :

### ▲ Each path $(v_i, \dots, v_j)$ with $d(v_i, \dots, v_j) > \varphi$ has at least one register:

▼  $\hat{w}(v_i, \dots, v_j) = w(v_i, \dots, v_j) + r_j - r_i \geq 1$  for all  $i, j$

### ▲ Equivalently, each combinational path delay is less than $\varphi$

# Refined analysis

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## ◆ Least-register path

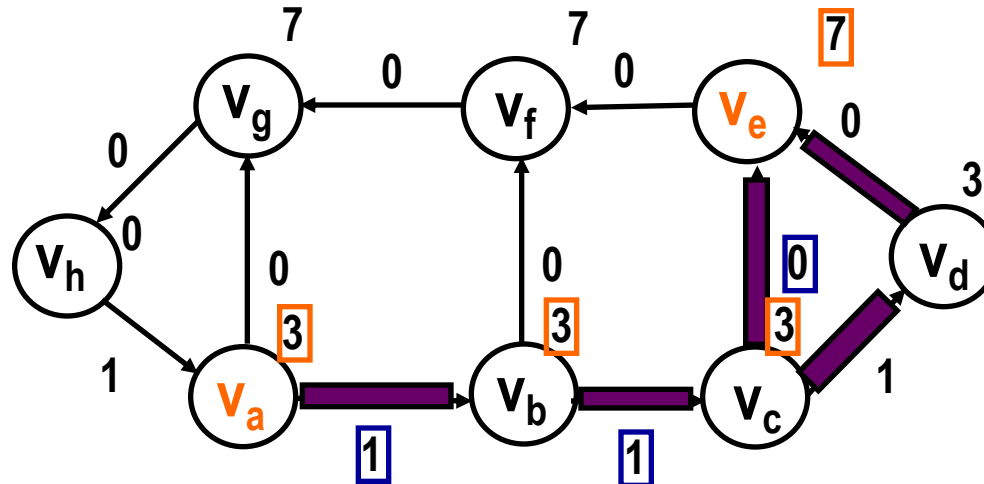
▲  $W(v_i, v_j) = \min w(v_i, \dots, v_j)$  over all paths between  $v_i$  and  $v_j$

## ◆ Critical delay:

▲  $D(v_i, v_j) = \max d(v_i, \dots, v_j)$  over all paths between  $v_i$  and  $v_j$   
with weight  $W(v_i, v_j)$

◆ There exist a vertex pair  $(v_i, v_j)$  whose delay  $D(v_i, v_j)$  bounds the cycle time

# Example



•Vertices:  $v_a, v_e$

•Paths:  $(v_a, v_b, v_c, v_e)$  and  $(v_a, v_b, v_c, v_d, v_e)$

• $W(v_a, v_e) = 2$

• $D(v_a, v_e) = 16$

# Minimum cycle-time retiming problem

---

- ◆ Find the minimum value of the clock period  $\varphi$  such that there exist a retiming vector where:

- ▲  $r_i - r_j \leq w_{ij}$  for all  $(v_i, v_j)$

- ▼ All registers are implementable

- ▲  $r_i - r_j \leq W(v_i, v_j) - 1$  for all  $(v_i, v_j)$  such that  $D(v_i, v_j) > \varphi$

- ▼ All timing path constraints are satisfied

## ◆ Solution

- ▲ Given a value of  $\varphi$

- ▲ Solve linear constraints  $A r \leq b$

- ▼ Mixed integer-linear program

- ▲ A set of inequalities has a solution if the constraint graph has no positive cycles

- ▼ Bellman-Ford algorithm – compute longest path

- ▲ Iterative algorithm

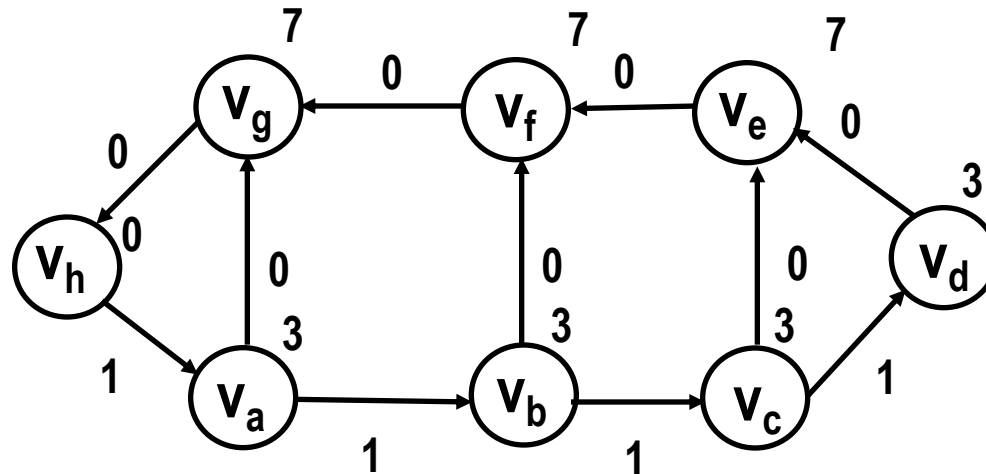
- ▼ Relaxation

# Minimum cycle-time retiming algorithm

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- ◆ Compute *all pair* path weights  $W(v_i, v_j)$  and delays  $D(v_i, v_j)$ 
  - ▲ Warshall-Floyd algorithms with complexity  $O(|V|^3)$
- ◆ Sort the elements of  $D(v_i, v_j)$  in decreasing order
  - ▲ Because an element of  $D$  is the minimum  $\varphi$
- ◆ Binary search for a  $\varphi$  in  $D(v_i, v_j)$  such that
  - ▲ There exists a legal retiming
  - ▲ Bellman-Ford algorithm with complexity  $O(|V|^3)$
- ◆ Remarks
  - ▲ Result is a global optimum
  - ▲ Overall complexity is  $O(|V|^3 \log |V|)$

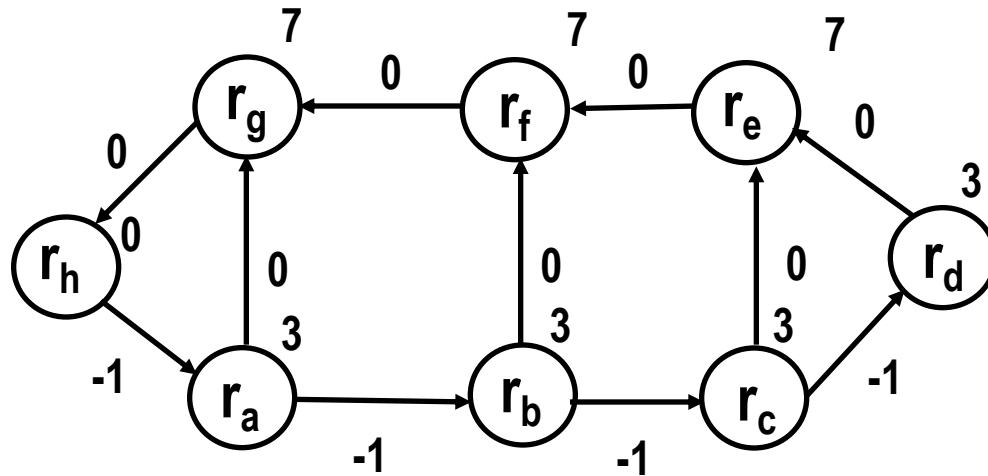
# Example: original graph



## •Constraints (first type):

- $r_a - r_b \leq 1$  or equivalently  $r_b \geq r_a - 1$
- $r_c - r_b \leq 1$  or equivalently  $r_c \geq r_b - 1$
- ...

# Example: constraint graph



## •Constraints (first type):

- $r_a - r_b \leq 1$  or equivalently  $r_b \geq r_a - 1$
- $r_c - r_b \leq 1$  or equivalently  $r_c \geq r_b - 1$
- ...





# Example

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◆ Sort elements of **D**:

▲ 33,30,27,26,24,23,21,20,19,17,16,14,13,12,10,9,7,6,3

◆ Select  $\varphi = 19$

▲ 33,30,27,26,24,23,21,20,19,17,16,14,13,12,10,9,7,6,3

▲ Pass: legal retiming found

◆ Select  $\varphi = 13$

▲ 33,30,27,26,24,23,21,20,19,17,16,14,13,12,10,9,7,6,3

▲ Pass: legal retiming found

◆ Select  $\varphi < 13$

▲ 33,30,27,26,24,23,21,20,19,17,16,14,13,12,10,9,7,6,3

▲ Fail: no legal retiming found

◆ Fastest cycle time is  $\varphi = 13$ . Corresponding retiming vector is used

# Example $\varphi = 13$

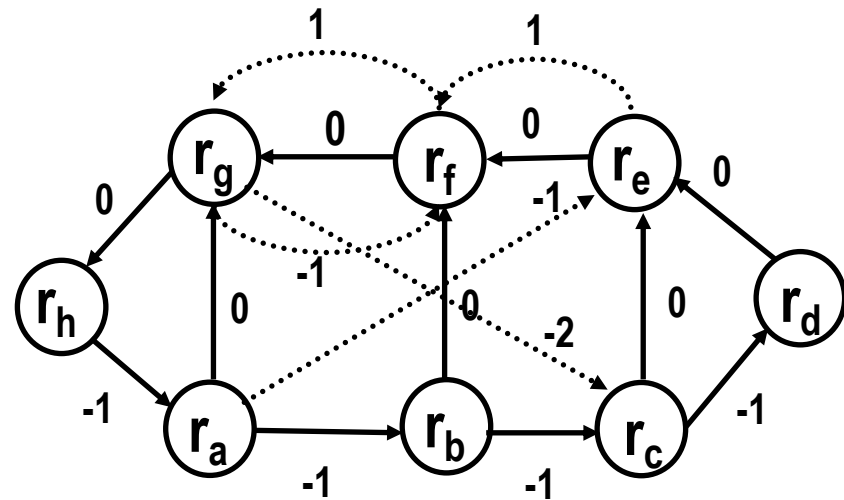
$$r_a - r_e \leq 2 - 1 \text{ or equivalently } r_e \geq r_a - 1$$

$$r_e - r_f \leq 0 - 1 \text{ or equivalently } r_f \geq r_e + 1$$

$$r_f - r_g \leq 0 - 1 \text{ or equivalently } r_g \geq r_f + 1$$

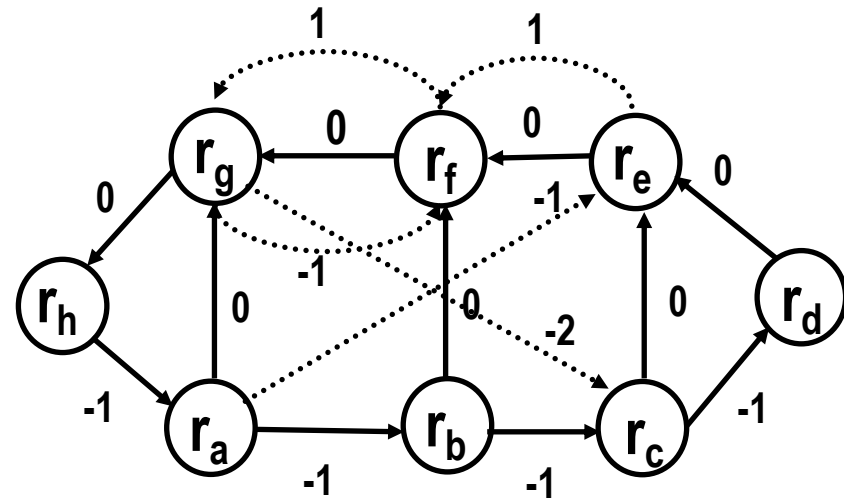
$$r_g - r_f \leq 2 - 1 \text{ or equivalently } r_f \geq r_g - 1$$

$$r_g - r_c \leq 3 - 1 \text{ or equivalently } r_c \geq r_g - 2$$



# Example $\varphi = 13$

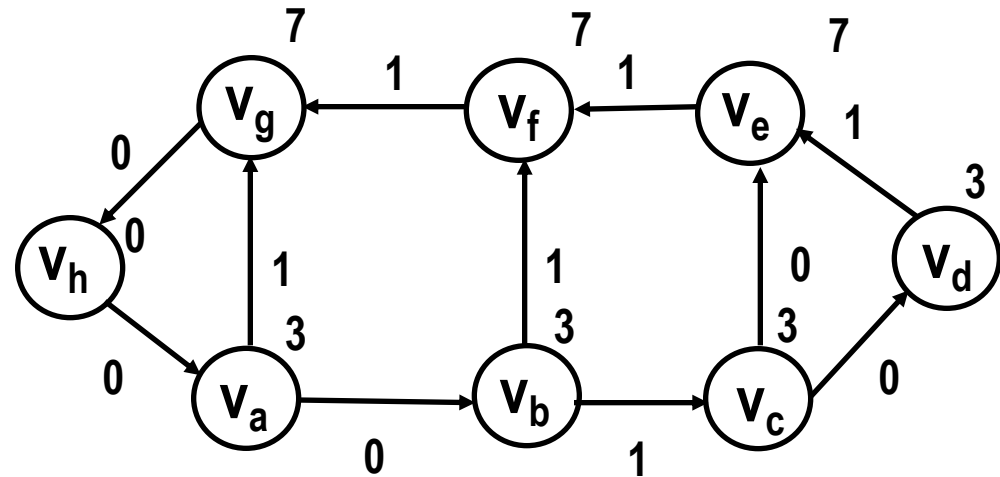
◆ Constraint graph:



◆ Longest path from source

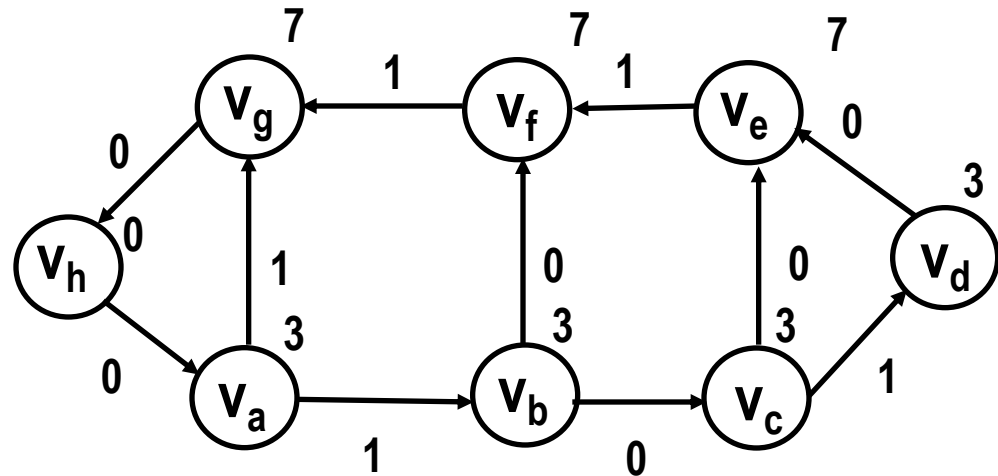
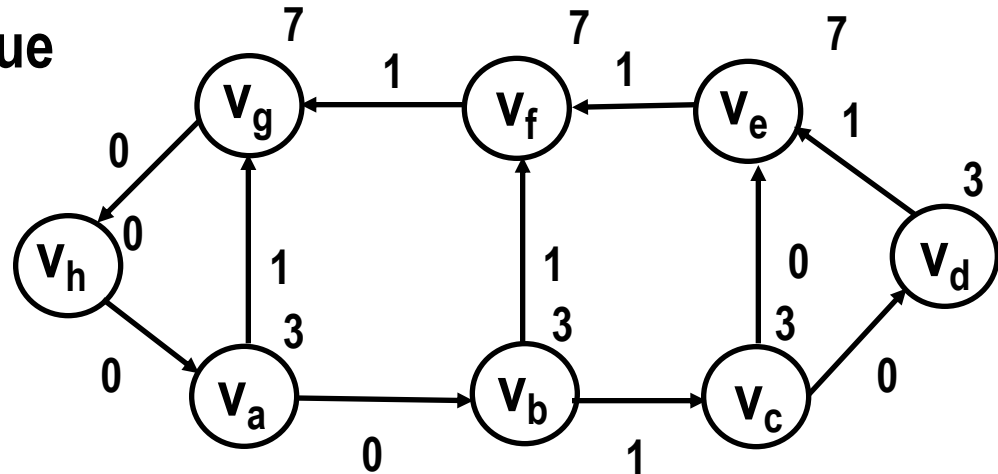
▲  $-[12342100]$

◆ Retimed graph



# Example $\varphi = 13$

◆ The solution is not unique



# Relaxation-based retiming

---

## ◆ Most common algorithm for retiming

- ▲ Avoids storage of matrices **W** and **D**
- ▲ Applicable to large circuits

## ◆ Rationale

- ▲ Search for decreasing  $\varphi$  in fixed step
  - ▼ Look for values of  $\varphi$  compatible with peripheral circuits
- ▲ Use efficient method to determine legality
  - ▼ Network graph is often very sparse
- ▲ Can be coupled with topological timing analysis

# Relaxation-based retiming

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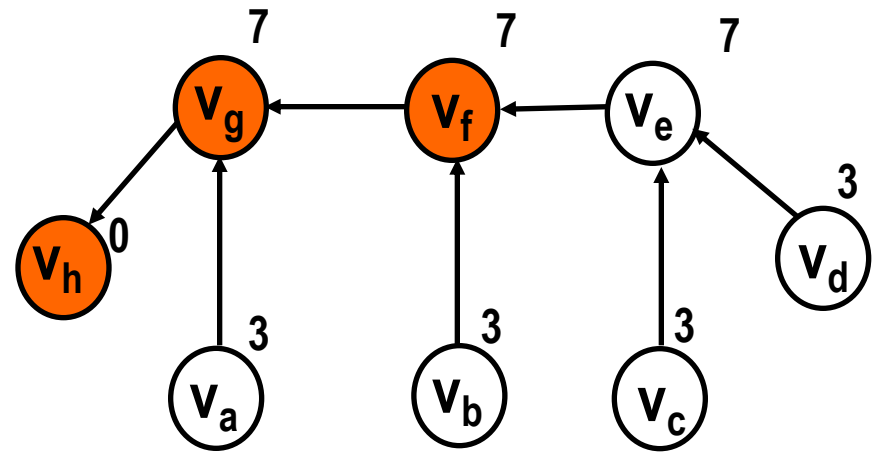
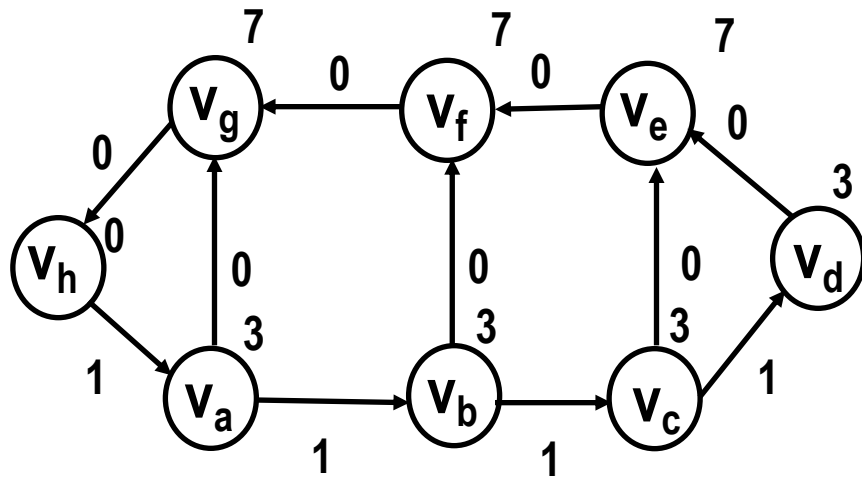
- ◆ Start with a given cycle-time  $\varphi$
- ◆ Look for paths with excessive delays
- ◆ Make such paths shorter
  - ▲ By bringing the terminal register closer
  - ▲ Some other paths may become longer
  - ▲ Namely, those path whose tail has been moved
- ◆ Use an iterative approach

# Relaxation-based retiming

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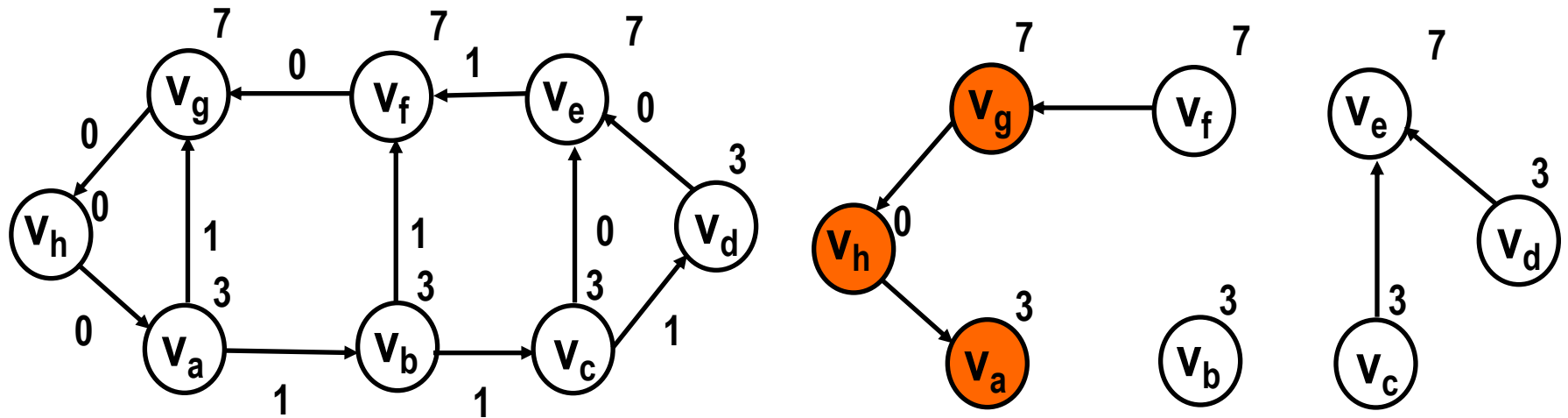
- ◆ Define data ready time at each node
  - ▲ Total delay from register boundary
- ◆ Iterative approach
  - ▲ Find vertices with *data ready*  $> \varphi$
  - ▲ Retime these vertices by 1
- ◆ Algorithm properties
  - ▲ If at some iteration there is no vertex with *data ready*  $> \varphi$ , a legal retiming has been found
  - ▲ If a legal retiming is not found in  $|V|$  iterations, then no legal retiming exists for that  $\varphi$

# Example $\varphi = 13$ iteration = 1

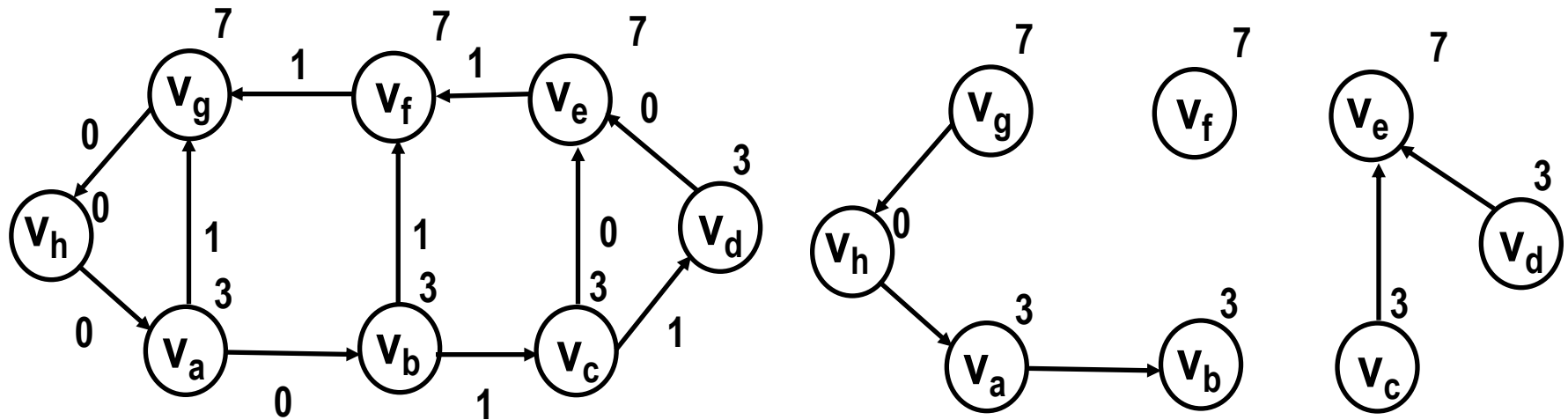




# Example $\varphi = 13$ iteration = 2



# Example $\varphi = 13$ iteration = 3

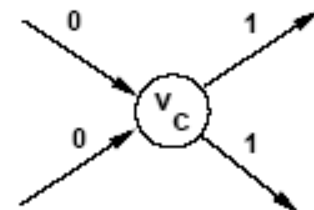
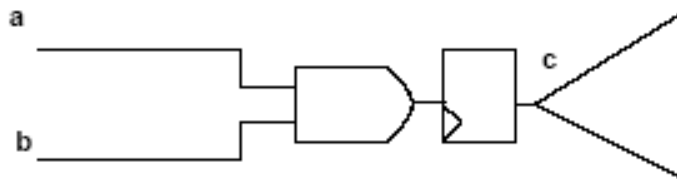
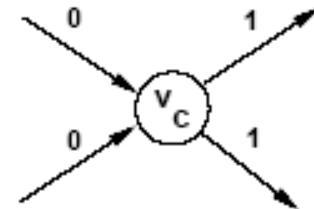
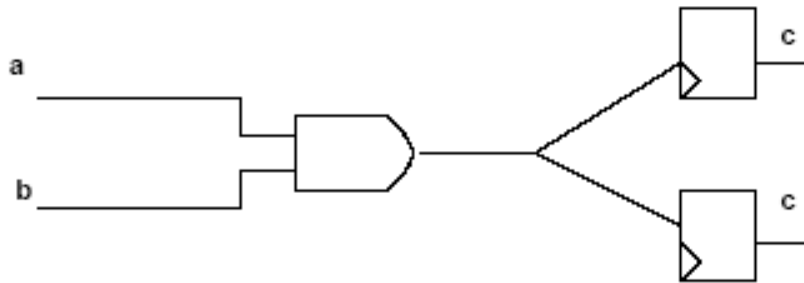
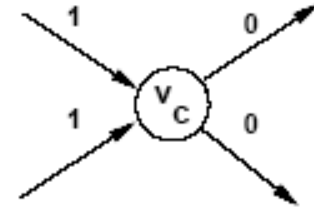
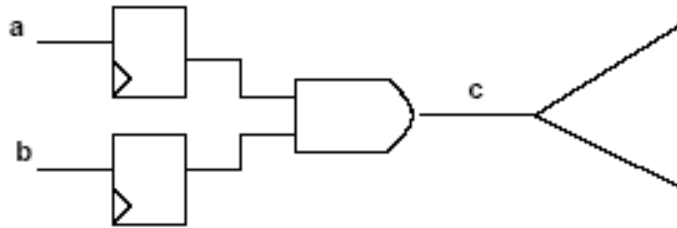


# Retiming for minimum area

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- ◆ Find a retiming vector that minimizes the number of registers
- ◆ Simple area modeling
  - ▲ Every edge with a positive weight denotes registers
  - ▲ Total register area is proportional to the sum of all weights
- ◆ Register sharing model
  - ▲ Every set of positively-weighted edges with common tail is realized by a shift registers with taps
  - ▲ Total register area is proportional to the sum, over all vertices, of the maxima of weights on outgoing edges

# Example



# Minimum area retiming simple model

---

## ◆ Register variation at node $v$

▲  $r_v ( \text{indegree}(v) - \text{outdegree}(v) )$

## ◆ Total area variation:

▲  $\sum r_v ( \text{indegree}(v) - \text{outdegree}(v) )$

## ◆ Area minimization problem:

▲  $\text{Min } \sum r_v ( \text{indegree}(v) - \text{outdegree}(v) )$

▲ Such that  $r_i - r_j \leq w_{ij}$  for all  $( v_i, v_j )$

# Minimum area retiming under timing constraint

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## ◆ Area recovery under timing constraint

▲  $\text{Min } \sum r_v ( \text{indegree}(v) - \text{outdegree}(v) )$  such that:

▲  $r_i - r_j \leq w_{ij}$  for all  $(v_i, v_j)$  and

▲  $r_i - r_j \leq W(v_i, v_j) - 1$  for all  $(v_i, v_j)$  such that  $D(v_i, v_j) > \varphi$

## ◆ Common implementation is by integer linear program

▲ Problem can alternatively be transformed into a matching problem and solved by Edmonds-Karp algorithm

## ◆ Register sharing

▲ Construct auxiliary network and apply this formulation.

▲ Auxiliary network construction takes into account register sharing

# Other problems related to retiming

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## ◆ Retiming pipelined circuits

- ▲ Balance pipe stages by using retiming
- ▲ Trade-off latency versus cycle time

## ◆ Peripheral retiming

- ▲ Use retiming to move registers to periphery of a circuit
- ▲ Restore registers after optimizing combinational logic

## ◆ Wire pipelining

- ▲ Use retiming to pipeline interconnection wires
- ▲ Model sequential and combinational macros
- ▲ Consider wire delay and buffering

# Summary of retiming

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- ◆ **Sequential optimization technique for:**
  - ▲ **Cycle time or register area**
- ◆ **Applicable to**
  - ▲ **Synchronous logic networks**
  - ▲ **Architectural models of data paths**
    - ▼ **Vertices represent complex (arithmetic) operators**
  - ▲ **Exact algorithm in polynomial time**
- ◆ **Extension and issues**
  - ▲ **Delay modeling**
  - ▲ **Network granularity**



# Module 3

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## ◆ Objective

- ▲ Relating state-based and structural models
- ▲ State extraction

# Relating the sequential models

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## ◆ State encoding

- ▲ Maps a state-based representation into a structural one

## ◆ State extraction

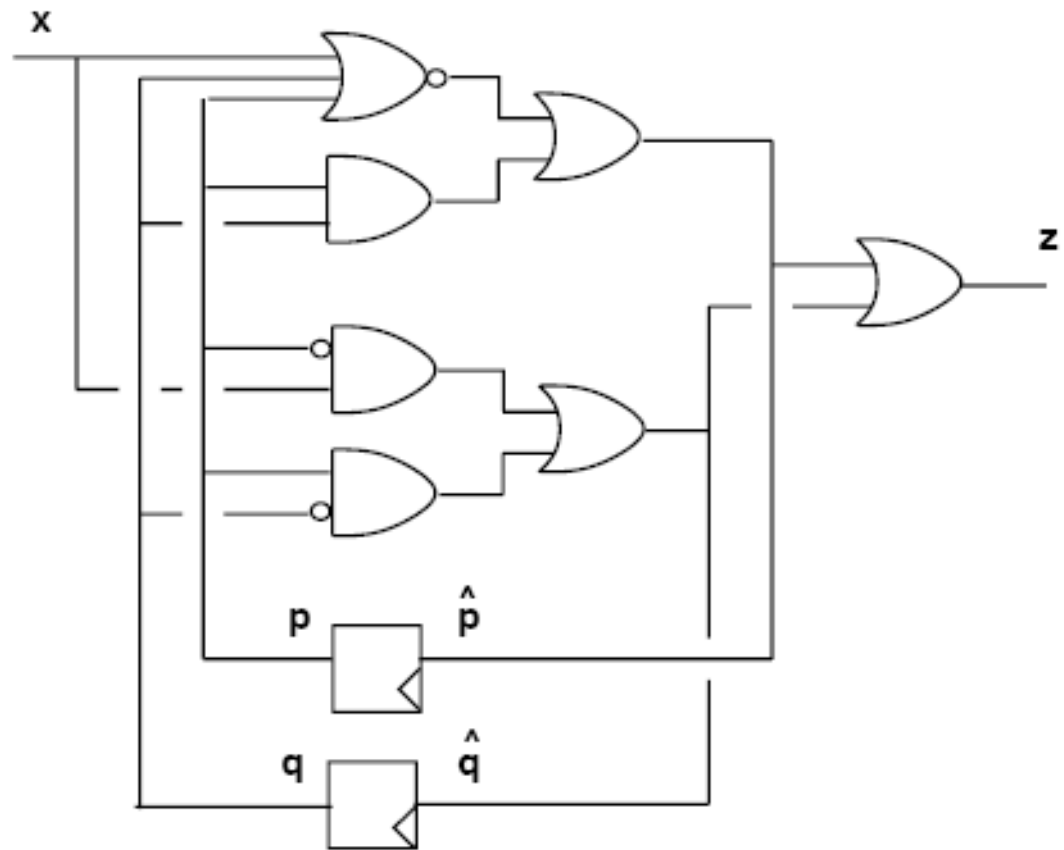
- ▲ Recovers the state information from a structural model

## ◆ Remark

- ▲ A circuit with  $n$  registers may have  $2^n$  states
- ▲ Unreachable states

# State extraction

- ◆ State variables:  $p, q$
- ◆ Initial state  $p=0; q=0$ ;
- ◆ Four possible states



# State extraction

---

## ◆ Reachability analysis

- ▲ Given a state, determine which states are reachable for some inputs
- ▲ Given a state subset, determine the reachable state subset
- ▲ Start from an initial state
- ▲ Stop when convergence is reached

## ◆ Notation:

- ▲ A state (or a state subset) is represented by an expression over the state variables
- ▲ Implicit representation

# Reachability analysis

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◆ State transition function:  $f$

◆ Initial state:  $r_0$

◆ States reachable from  $r_0$

▲ Image of  $r_0$  under  $f$

◆ States reachable from set  $r_k$

▲ Image of  $r_k$  under  $f$

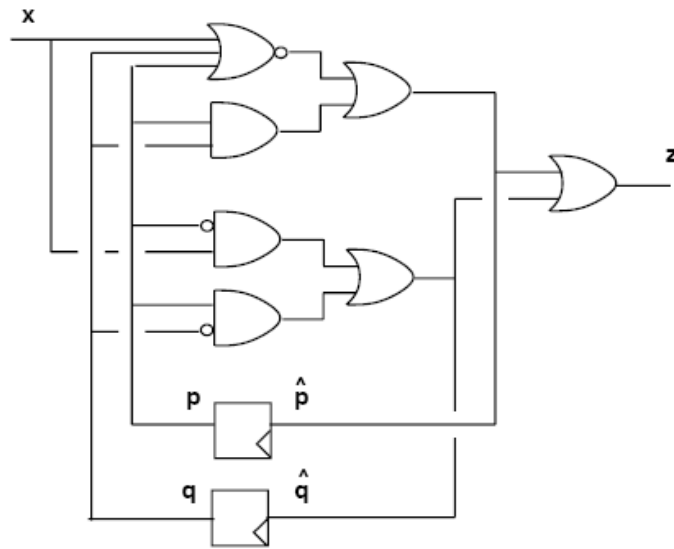
◆ Iteration

▲  $r_{k+1} = r_k \cup (\text{image of } r_k \text{ under } f)$

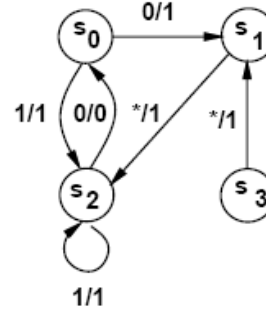
◆ Convergence

▲  $r_{k+1} = r_k$  for some  $k$

# Example



(a)



(b)

- Initial state  $r_0 = p'q'$ .
- The state transition function  $\mathbf{f} = \begin{bmatrix} x'p'q' + pq \\ xp' + pq' \end{bmatrix}$

# Example

## ◆ Image of $p' q'$ under $f$ :

▲ When ( $p = 0$  and  $q = 0$ ),  $f$  reduces to  $[x' \ x]^T$

▲ Image is  $[0 \ 1]^T \cup [1 \ 0]^T$

## ◆ States reachable from the reset state:

▲ ( $p = 1; q = 0$ ) and ( $p = 0; q = 1$ )

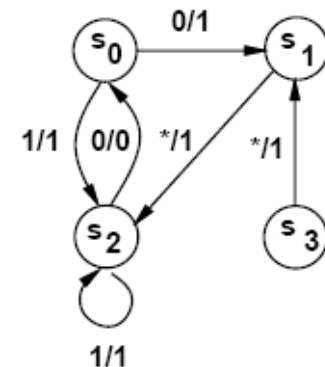
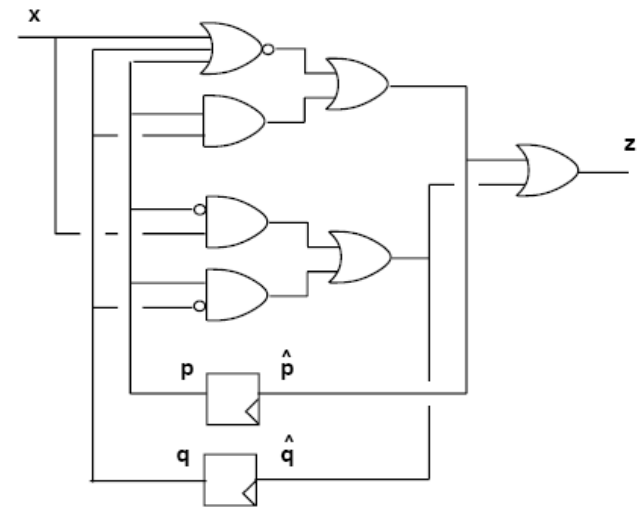
▲  $r_1 = p' q' + pq' + p' q = p' + q'$

## ◆ States reachable from $r_1$ :

▲  $[0 \ 0]^T \cup [0 \ 1]^T \cup [1 \ 0]^T$

## ◆ Convergence:

▲  $s_0 = p' q'$ ;  $s_1 = pq'$ ;  $s_2 = p' q$ ;



# Completing the extraction

---

- ◆ **Determine state set**

  - ▲ **Vertex set**

- ◆ **Determine transitions and I/O labels**

  - ▲ **Edge set**

  - ▲ **Inverse image computation**

  - ▲ **Look at conditions that lead into a given state**



# Example

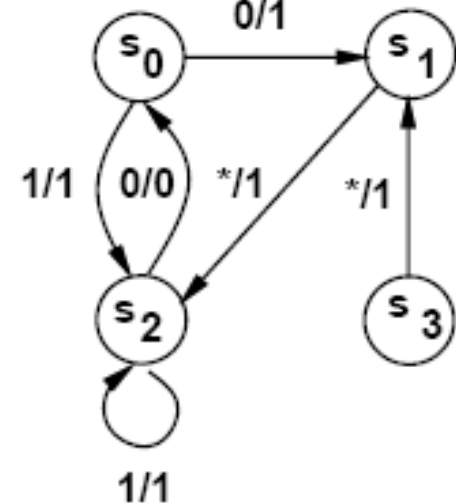
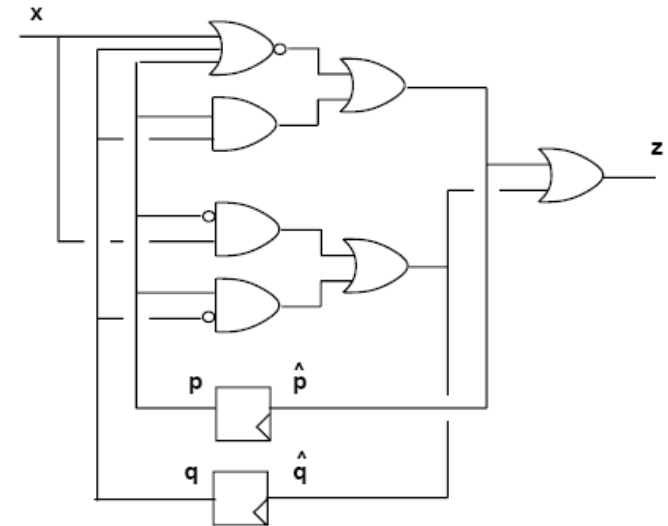
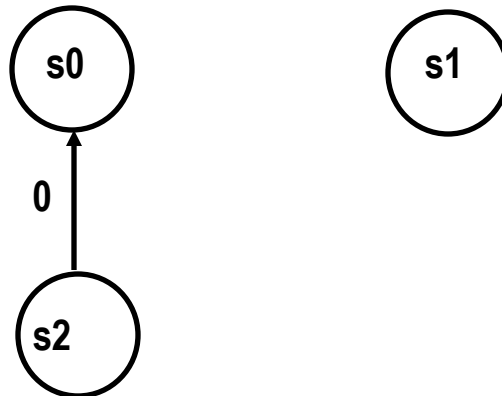
## ◆ Transition into $s_0 = p' q'$

▲ Patterns that make  $f = [0\ 0]^T$  are:

$$(x' p' q' + pq)' (xp' + pq')' = x' p' q$$

▲ Transition from state  $s_2 = p' q$  under input  $x'$

▲ And so on ...



# Remarks

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- ◆ Extraction is performed efficiently with implicit methods
- ◆ Model transition relation  $\chi(i, x, y)$  with BDDs
  - ▲ This function relates possible triples:
    - ▼ ( input, current\_state, next\_state )
  - ▲ Image of  $r_k$ :
    - ▼  $S_{i,x} ( \chi(i,x,y) r_k(x) )$
    - ▼ Where  $r_k$  depends on inputs  $x$
  - ▲ Smoothing on BDDs can be achieved efficiently

# Summary

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- ◆ **State extraction can be performed efficiently to:**
  - ▲ Apply state-based optimization techniques
  - ▲ Apply verification techniques
- ◆ **State extraction is based on forward and backward state space traversal:**
  - ▲ Represent state space implicitly with BDDs
  - ▲ Important to manage the space size, which grows exponentially with the number of registers