Fundamentals of Traffic Operations and Control
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Lab 2
Feedback control of bus operations

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Consider a single-line bus network with K_b buses and K_s stops. When cruising between stops, the buses move according to the simple linear dynamics:

$$x_i(t+1) = x_i(t) + T \cdot v_i(t), \tag{1}$$

where $x_i(t)$ (m) and $v_i(t)$ (m/s) is the position and speed of bus i, respectively, while T is the timestep in seconds. The buses move only on one direction (i.e., $0 \leq v_i(t)$), and their position is reset to the stop position when they arrive at a stop (if it is the first stop, position is reset to 0).

Speed of the buses are bounded as follows:

$$\tilde{v}_{\min}(t) \leq v_i(t) \leq \tilde{v}_{\max}(t), \tag{2}$$

where $\tilde{v}_{\min}(t)$ (m/s) and $\tilde{v}_{\max}(t)$ are time-varying lower and upper bounds on bus speeds (modeling the effect of traffic congestion on buses), which are random scalars drawn from a truncated normal distribution, within the intervals $[\tilde{v}_{\min,l}, \tilde{v}_{\min,u}]$ for $\tilde{v}_{\min}(t)$, and $[\tilde{v}_{\max,l}, \tilde{v}_{\max,u}]$ for $\tilde{v}_{\max}(t)$.

When a bus arrives at a stop, it needs to wait for a certain amount of time (called *dwell time*) to allow for passenger transfers, which is modeled as a random scalar drawn from a truncated normal distribution, within the interval $[\mathrm{dt}_{\min}, \mathrm{dt}_{\max}]$, with $\mathrm{dt}_{\min}$ (s) and $\mathrm{dt}_{\max}$ (s) the lower and upper bounds on the dwell time, respectively. The bus is allowed to leave the stop after the dwell time has passed, and if there is no holding command being applied (holding means forcing a bus to stay at a stop although it has finished passenger transfers).

Feedback control schemes can be designed to improve performance of bus operations. One category of bus control methods involve actuating the bus speeds in real time based on spacing errors, which is defined for a single bus as follows:

$$e_i(t) = s_{i,f}(t) - s_{i,r}(t), \tag{3}$$

where $e_i(t)$ (m) is the spacing error of bus i, while $s_{i,f}(t)$ (m) and $s_{i,r}(t)$ (m) are the front and rear spacings of bus i (see fig. 1), respectively, which are defined as:

$$s_{i,f}(t) = x_{i,f}(t) - x_i(t) \tag{4}$$

$$s_{i,r}(t) = x_i(t) - x_{i,r}(t), \tag{5}$$

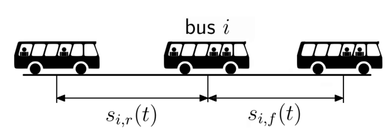

Figure 1: Visualization of front and rear spacings of bus i.

where $x_{i,f}(t)$ (m) and $x_{i,r}(t)$ (m) are the positions of the buses in front of and behind bus i, respectively.

Based on the spacing error, an integral controller (I-controller) can be designed, which involves updating bus speeds as follows:

$$v_i(t) = v_i(t-1) + K_I \cdot e_i(t), \tag{6}$$

where K_I is the integral gain expressing how strongly the I-controller reacts to spacing errors.

A MATLAB implementation of the above bus network model, together with the cases of no control (where $v_i(t) = \tilde{v}_{\max}(t)$) and the I-controller is given together with this lab assignment as the function `run_lab_2.m`. It can be run by executing `d=run_lab_2(ctrl)` (replace `ctrl` with 1 for no control, and 2 for I-controller) from the MATLAB command window (can take around 15-20 seconds to finish), which should create a structure called "`d`" in the workspace. To test if the code works correctly, run once with `d_1=run_lab_2(1)`, and once with `d_2=run_lab_2(2)`, after which there appears two structures called "`d_1`" and "`d_2`" in the workspace. After this, running the supplied function `plot_hws_2.m` by executing `plot_hws_2(d_1,d_2)` from the MATLAB command window should produce the figure seen in fig. 2.

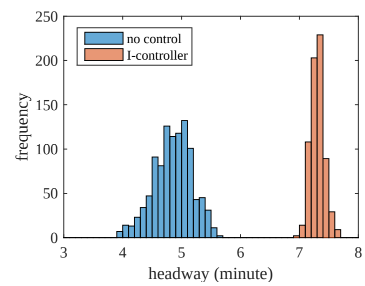

Figure 2: Headway distributions of the no control (blue) and I-controller (red) cases (with $K_I = 1$).

Using (and, when necessary, modifying) the given code, answer the following questions.

a) Explain the reasons of the performance difference between the no control case and the I-controller (as shown in the headway distributions given in fig. 2).

b) A straightforward improvement over the I-controller is the proportional-integral (PI) controller, which involves updating bus speeds as follows:

$$v_i(t) = v_i(t-1) + K_P \cdot (e_i(t) - e_i(t-1)) + K_I \cdot e_i(t), \tag{7}$$

where K_P is the proportional gain expressing how strongly the PI-controller reacts to changes in spacing errors.

Write the MATLAB code implementing the PI-controller (given in eq. (7)) by filling inside the function `compute_PI_control.m`. Run the simulation for the PI-controller using the gain values $K_I = 1$ and $K_P = 10$. Plot a new headway distributions figure including the three cases so far (i.e., no control, I-controller, and PI-controller). Explain the reasons of the performances difference between the I-controller and the PI-controller.

c) Another method of actuation is the holding method, where a bus is forced to hold at a stop for some time even after the passenger transfers are finished. If a bus arrives at a stop earlier than desired, it can be beneficial to make it hold at the stop so as to regularize headways. Bus speeds are not actuated with the holding method, so they move at their maximum speed allowed by traffic.

Propose a simple holding controller that can improve headway regularity and describe its working principle. Write a MATLAB code implementing the holding controller you proposed by filling inside the function `compute_H_control.m`. Plot a new headway distributions figure including the four cases (i.e., no control, I-controller, PI-controller, and the holding controller). Explain the reasons of the performance differences between the holding controller and the other cases.

d) (bonus) The I- and PI-controllers given in eqs. (6) and (7) operate without considering coordination among the buses (i.e., the speed of each bus is updated individually without regarding that the others are also updated). Considering simple feedback control methods from literature, describe only in words (without any coding/implementation) a method of your choice that can achieve coordination.

Note: Part d) is optional. The groups that do it correctly will get an extra 10% added to their final lab 2 grade.