Fundamentals of Traffic Operations and Control

Introduction to Logistics

École Polytechnique Fédérale de Lausanne
Urban Transport Systems Laboratory - LUTS

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Week 12
December 2018

Outline

- What is Logistics?
- What type of activities involve logistic operations?
- Intro to logistics through examples and in-class exercises

Acknowledgements: Some of the presented material was kindly provided by Prof. *Nikolas Geroliminis (EPFL)*, Prof. *Mor Kaspi (Tel-Aviv University)* and Prof. *Dorit Hochbaum (University of California Berkeley)*

What is “Logistics”?

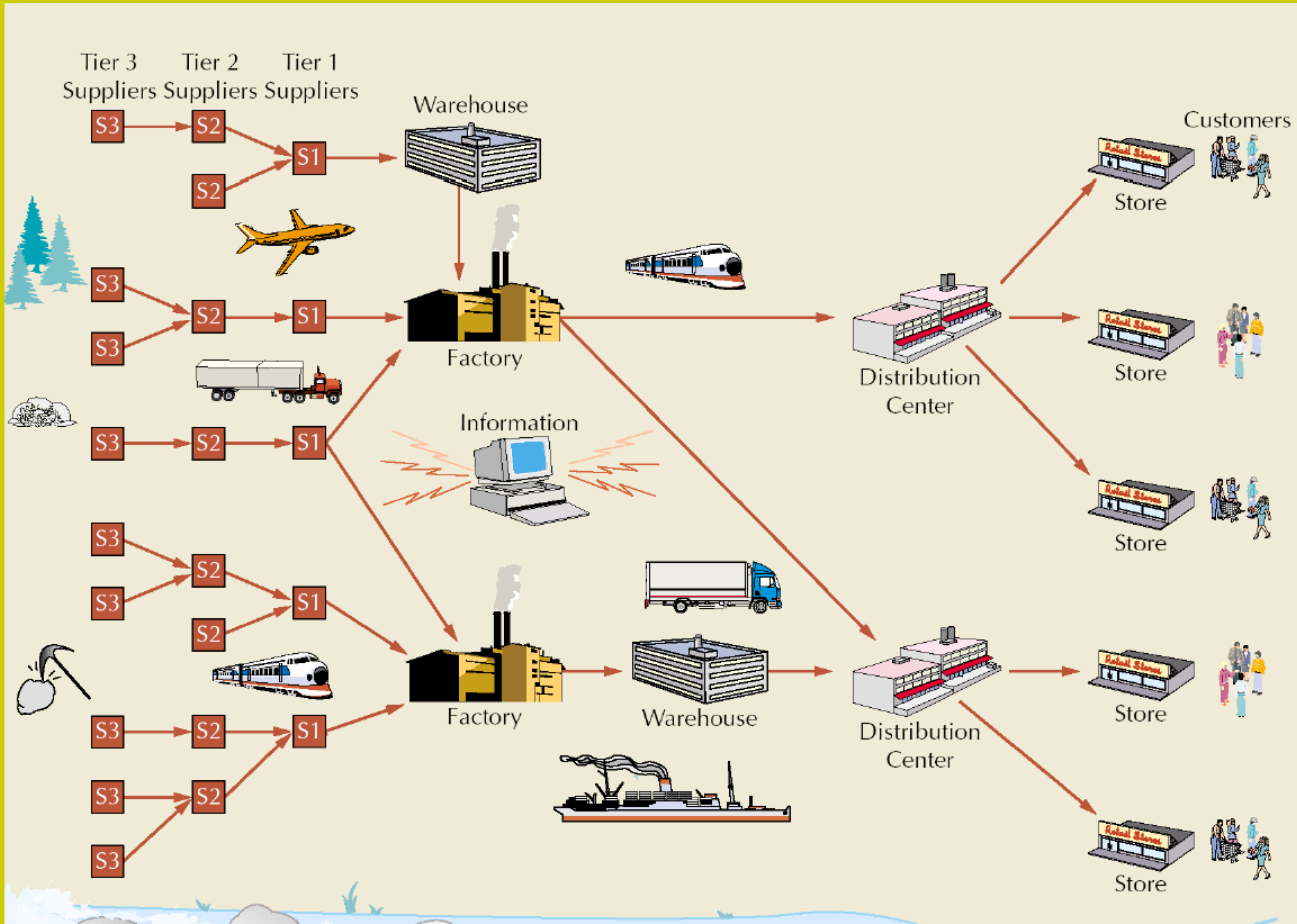
From Wikitionary:

*(**Operations**): The process of **planning**, implementing and controlling the **efficient and effective flow and storage of goods, services and related information** from their point of **origin** to **point of consumption** for the purpose of **satisfying customer** requirements*

- The word originated from the **Greek word** “logistikós” or “logízomai” (I reason, I calculate)
- Initially used by **Greek generals** (**Leon the wise, Alexander the great**) to describe all **procedures for the army’s procurement** on *food, clothing, ammunition, etc.*
- Coined in 1838 by the **Swiss officer *Antoine-Henri Jomini* (From Vaud!)** who served as a general at the French service. In 1814, he withdrew from the Allied Army and returned to Switzerland to prevent the Allies’ violation of **Swiss Neutrality**, study and **teach the art of war** in his own country.
- During **WWII** the Allied made clear use of advanced logistic strategies that contributed to their victory.
- Following WWII there was an increased interest/recognition in “Logistics” as a scientific field of study.

Logistics Today

- **Space Utility**
  - Addition of economic value to goods by moving them from production surplus to points where demand exists
  - Essentially extends the physical boundaries of the market area
- **Time Utility**
  - Addition of economic value to goods (at a demand point at a specific time)
  - Economies of scale



Supply Chain Illustration

Logistic Activities and Decisions

- Activities types:
  - Production
  - Storage and Inventory management
  - Distribution
- Decision types:
  - STRATEGIC (~many years): Facility location, Production rates…
  - TACTICAL (~annually, semi-annually): Leasing space, Product variety, Inventory levels, Shipping frequency…
  - OPERATIONAL (~short-term, daily, hourly): Distribution amount, Number of vehicles for distribution, Vehicle routes, Employees…

STRATEGIC DECISIONS

The (Dr. Gab's) production problem

You are the owner of the Dr. Gab's brewery, which produces the following beers (among others): Pépite and Tempête. You have limited resources as given in the table. You want to maximise your profit for the operation of the brewery

	Pépite	Tempête	Resource Limit
Profit	10	11	N/A
Corn (kg)	2.3	6.8	217
Hops (kg)	0.11	0.11	4.5
Malt (kg)	15	9	540

Which is the best production plan that you can come up with?

	Pépite	Tempête	Resource Limit
Profit	10	16	N/A
Corn (kg)	2.3	6.7	217
Hops (kg)	0.11	0.11	4.5
Malt (kg)	15	9	540

Options:

`Devote all resources to the Pepite:` ~36 barrels — 360 CHF

`Devote all resources to the Tempête:` ~32 barrels — 512 CHF

`7.5 barrels of Pepite, 29.5 barrels of Tempête:` 547 CHF

`12 barrels of Pepite, 28 barrels of Tempête:` 568 CHF

Any other solution?

The (Dr. Gab's) Production Problem as a Linear Program

What is a linear program (LP)?

A method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements and objectives are represented by linear relationships.

An LP typically consists of:

`Parameters:` Input data (the data reported in the table)

`Decision variables:` What do we want to decide upon? (in our case: how much should you produce of each type of beer?)

`Objective Function:` What would we like to achieve through the decision variables? (in our case: maximise profit)

`Constraints:` Is there some "rule" we should respect? (in our case we need to respect resource limitations)

A LP for the (Dr. Gab's)production problem

$$\max \quad 10x + 16y$$ **Profit**

$$s.t. \quad 2.3x + 6.7y \leq 217$$ **Corn**

$$0.11x + 0.11y \leq 4.5$$ **Hops**

$$15x + 9y \leq 540$$ **Malt**

$$x \, , \, y \geq 0$$

	Pépite	Tempête	Resource Limit
Profit	10	16	N/A
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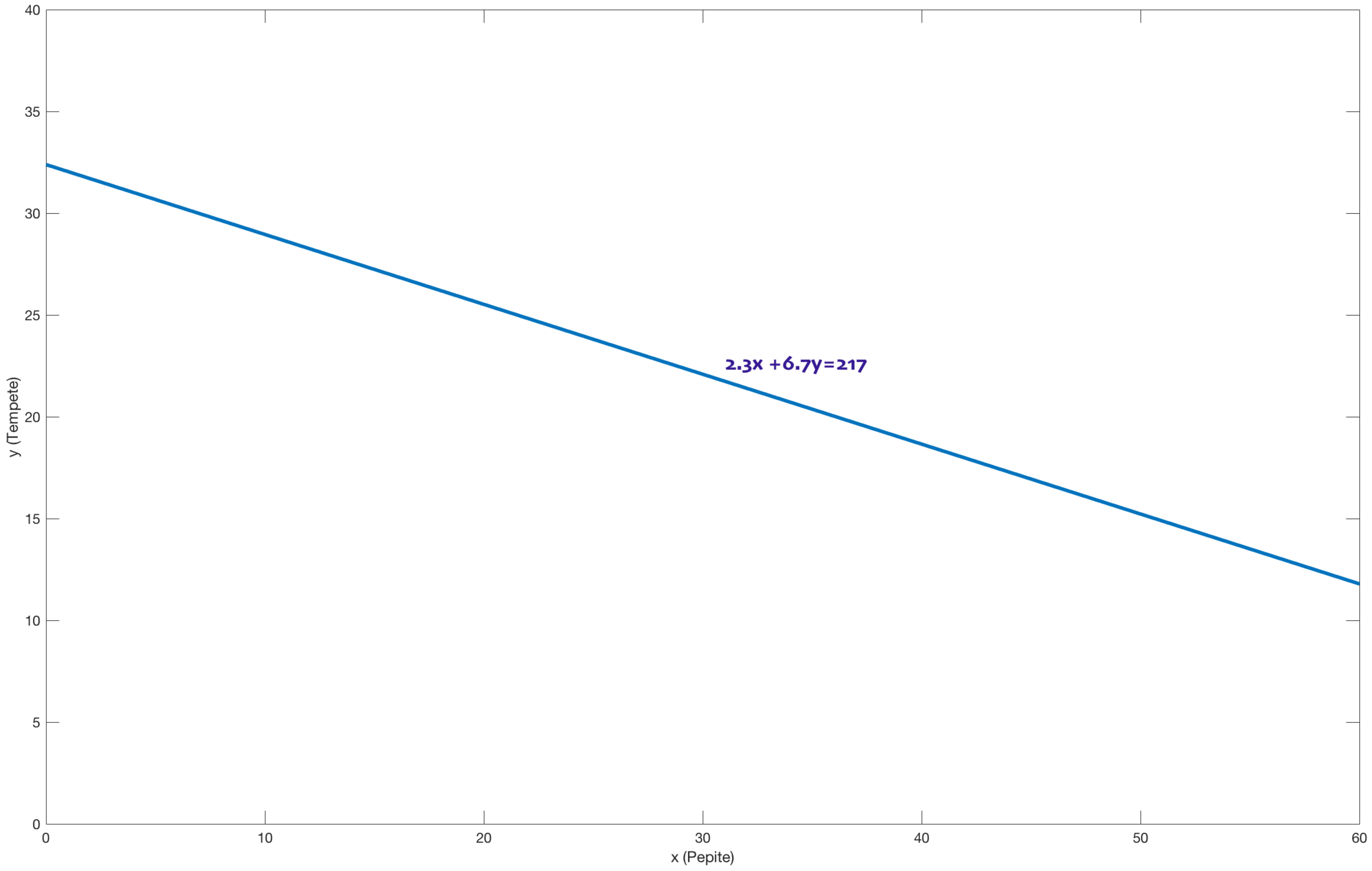
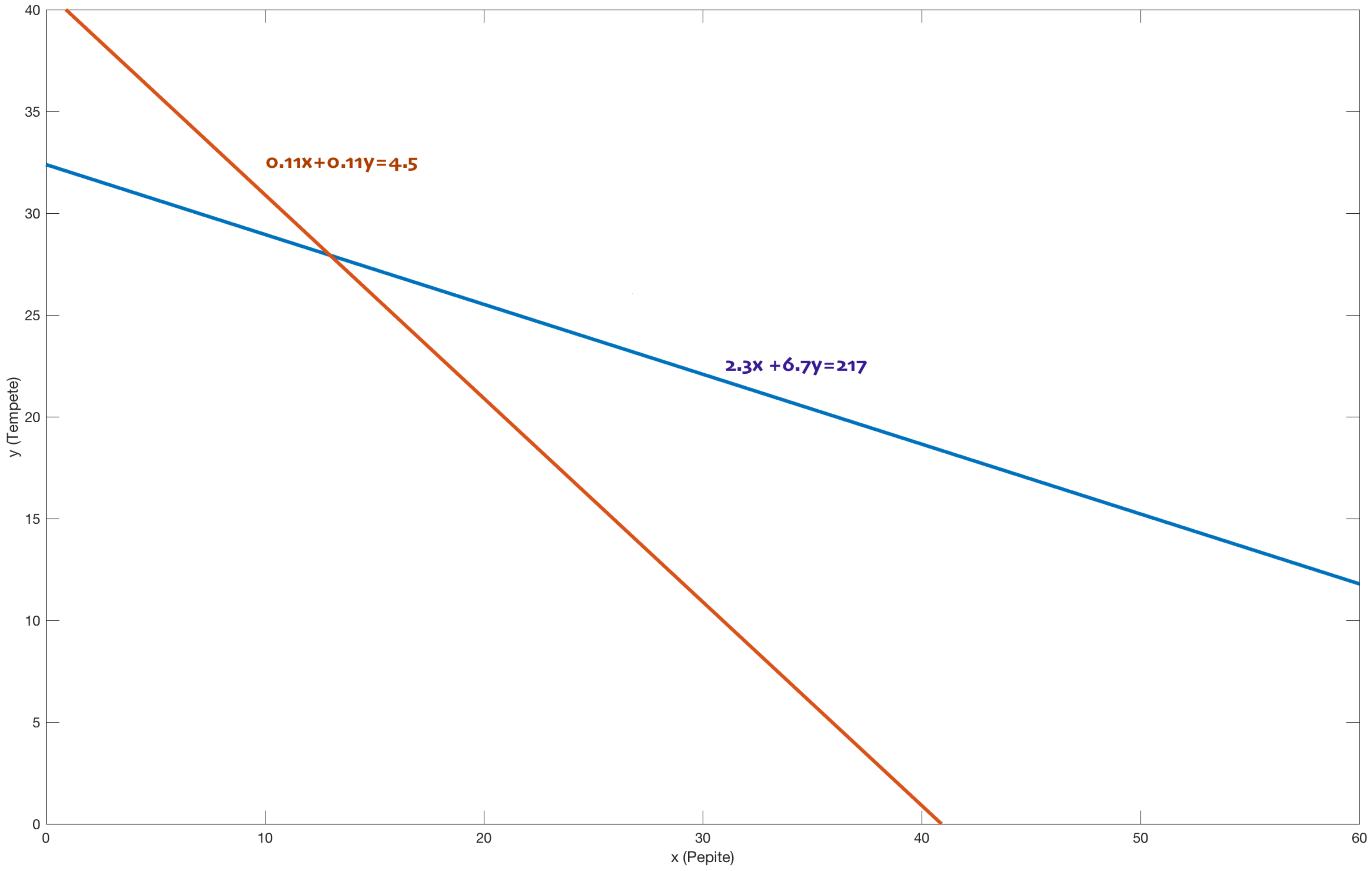
Graphical Method to solve LPs



Graphical Method to solve LPs



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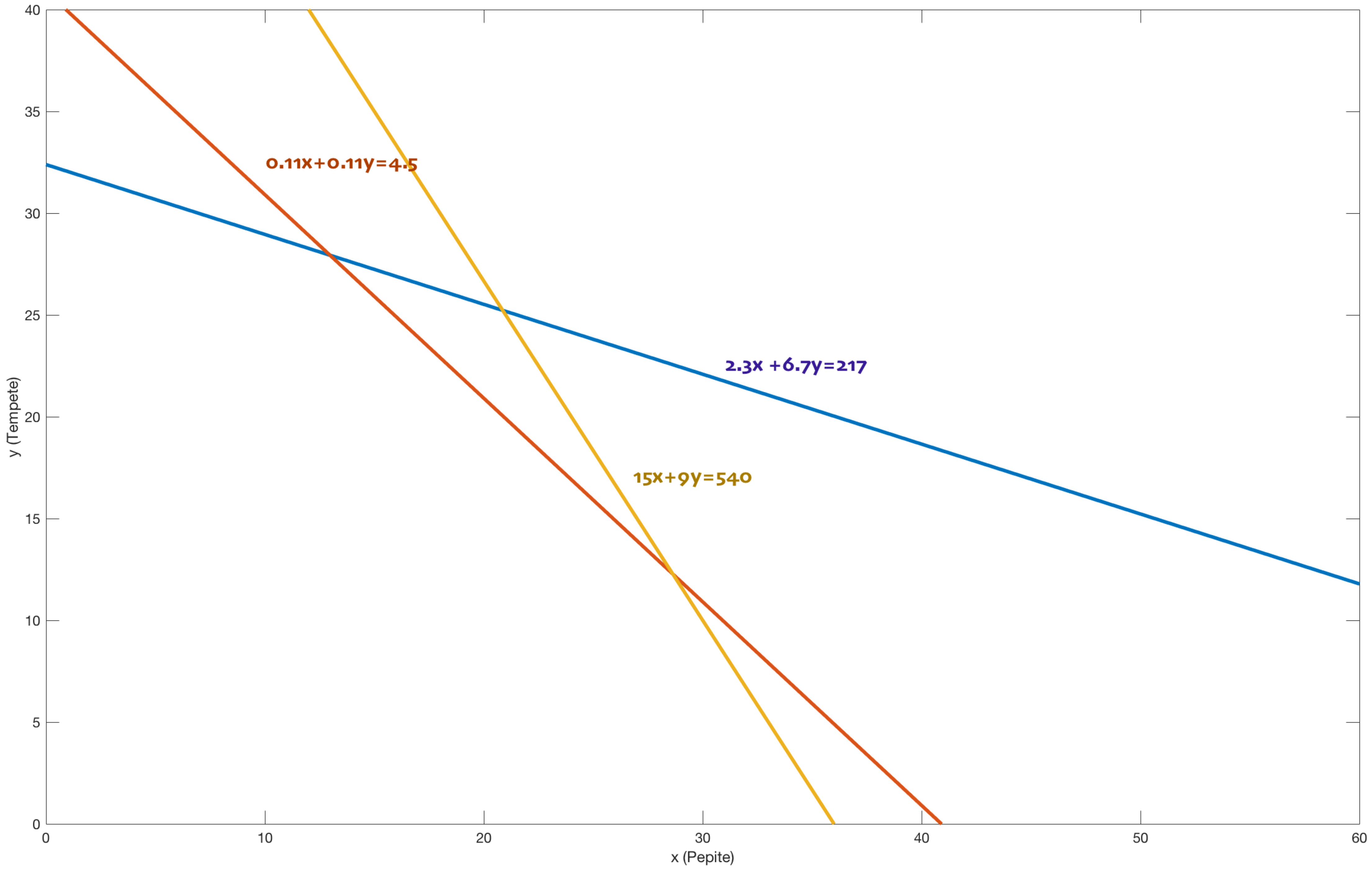
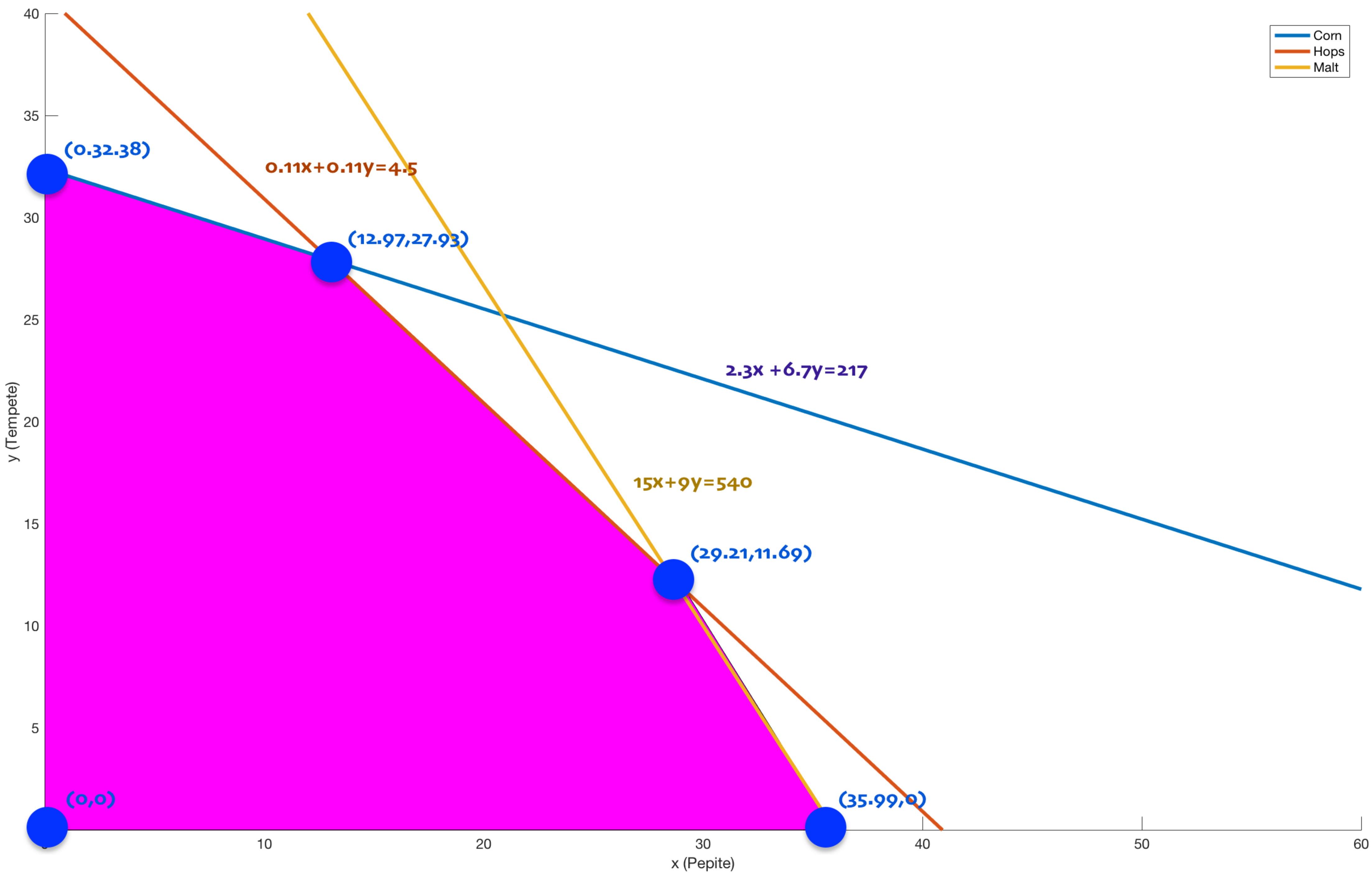
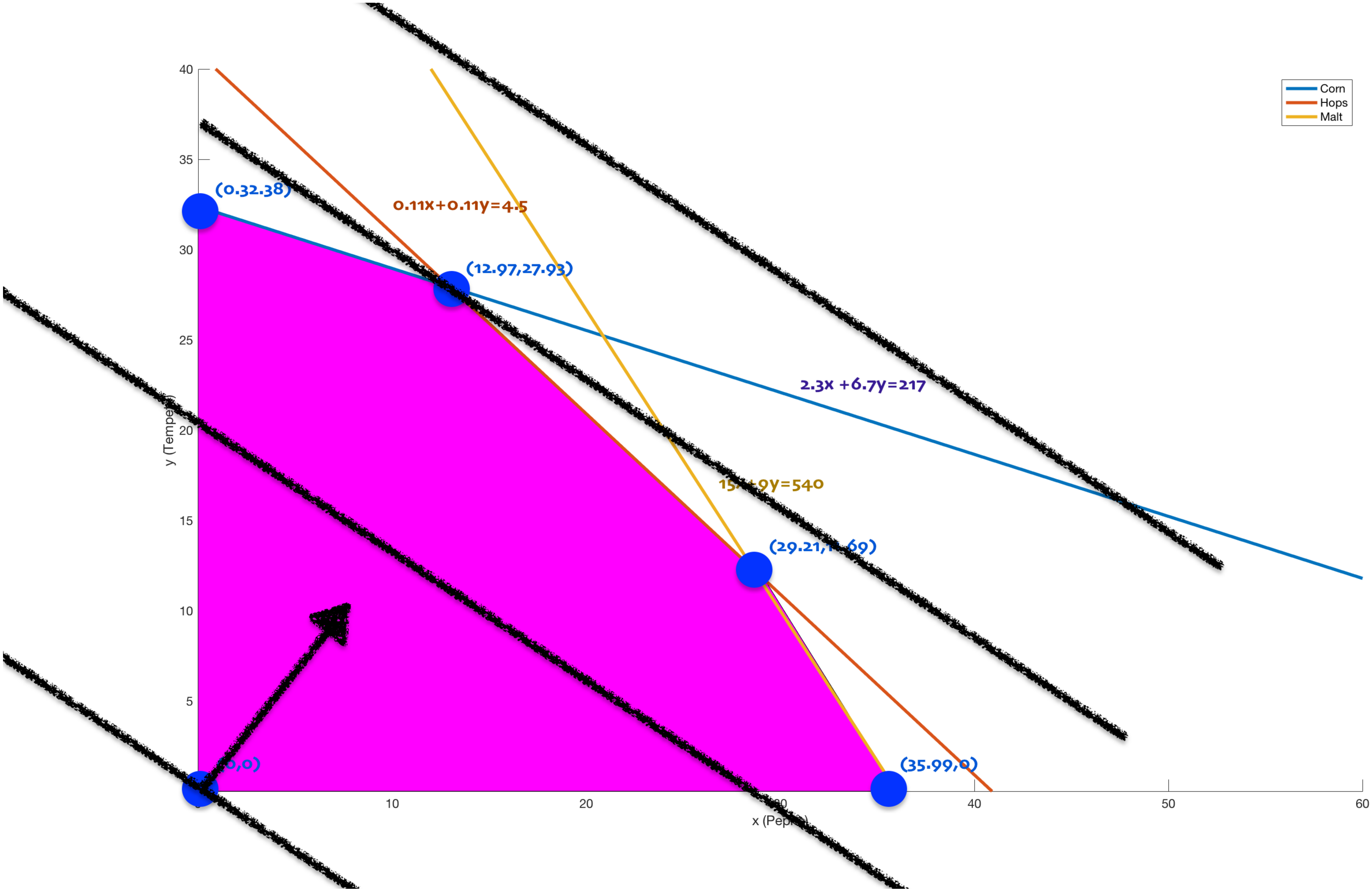


Graphical Method to solve LPs *

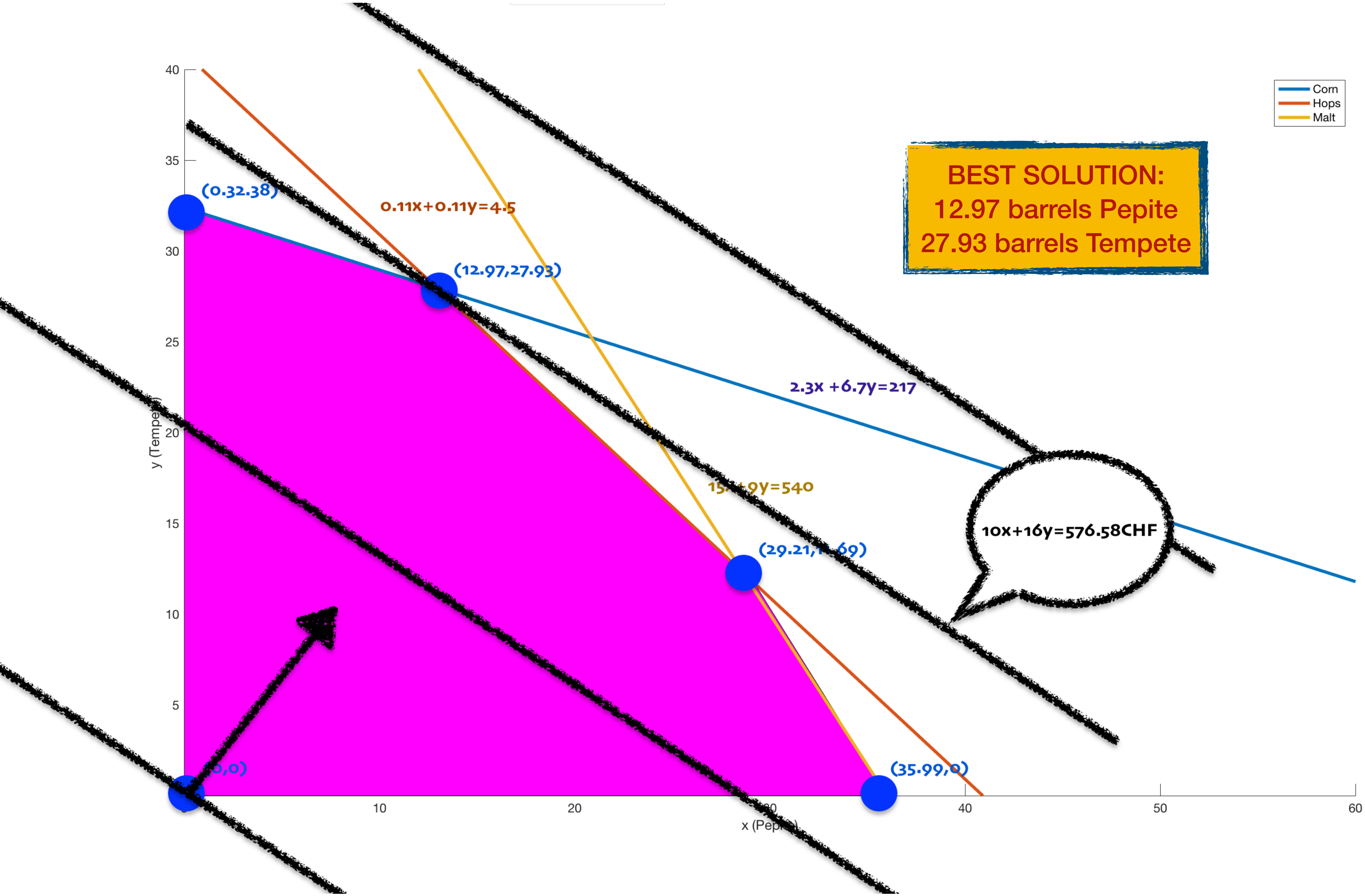


*** For 2 decision variables. For more variables you need Dantzig's Simplex algorithm**

Graphical Method to solve LPs



Graphical Method to solve LPs



What if the owner of Dr. Gab’s want an “integer” number of barrels?

Options for an "integer" solution

$$\begin{array}{lcccl} \max & 5x_1 & + & 8x_2 & \\ s.t. & x_1 & + & x_2 & \leq 6 \\ & 5x_1 & + & 9x_2 & \leq 45 \\ & x_1 & , & x_2 & \geq 0 \\ & x_1 & , & x_2 & \in Z \end{array}$$

LP optimum: **(2.25, 3.75)** → *Objective function value:* **41.25**

How many rounding possibilities? **$2^n = 2^2 = 4$**

(2,4) : Infeasible!

(3,4) : Infeasible!

(2,3) : 34

(3,3) : 39

Options for an "integer" solution

$$
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LP optimum: **(2.25, 3.75)** $\longrightarrow$ *Objective function value:* **41.25**

How many rounding possibilities? **$2^n = 2^2 = 4$**

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(2,3) : 34

(3,3) : 39 $\longrightarrow$ **The actual optimum is (0,5) with z=40!**

So why is rounding a "bad" decision?

1. *May lead to (only) infeasible solutions!*
2. *Even if it is possible to operate rounding, the solution might not be optimal*
3. *Suppose you have a problem with 2000 variables. Then you would result with 2^{2000} possible rounding! This number is prohibitive in terms of complexity*

An Integer Linear Program for the (Dr. Gab's)production problem

$$\max \quad 10x + 16y$$ **Profit**

$$s.t. \quad 2.3x + 6.7y \leq 217$$ **Corn**

$$0.11x + 0.11y \leq 4.5$$ **Hops**

$$15x + 9y \leq 540$$ **Malt**

$$x, \; y \geq 0$$

$$x, \; y \in Z$$

	Pépite	Tempête	Resource Limit
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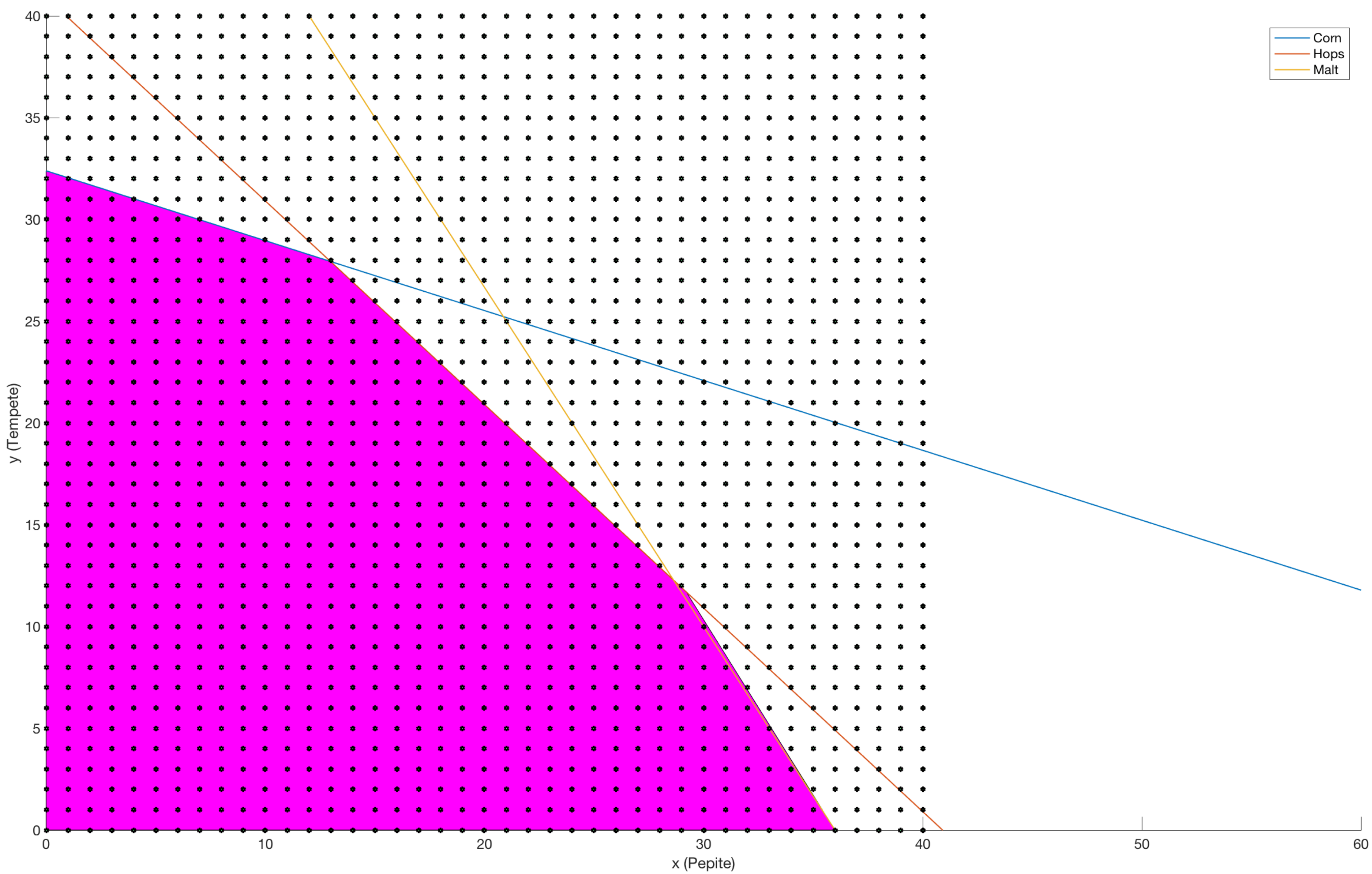
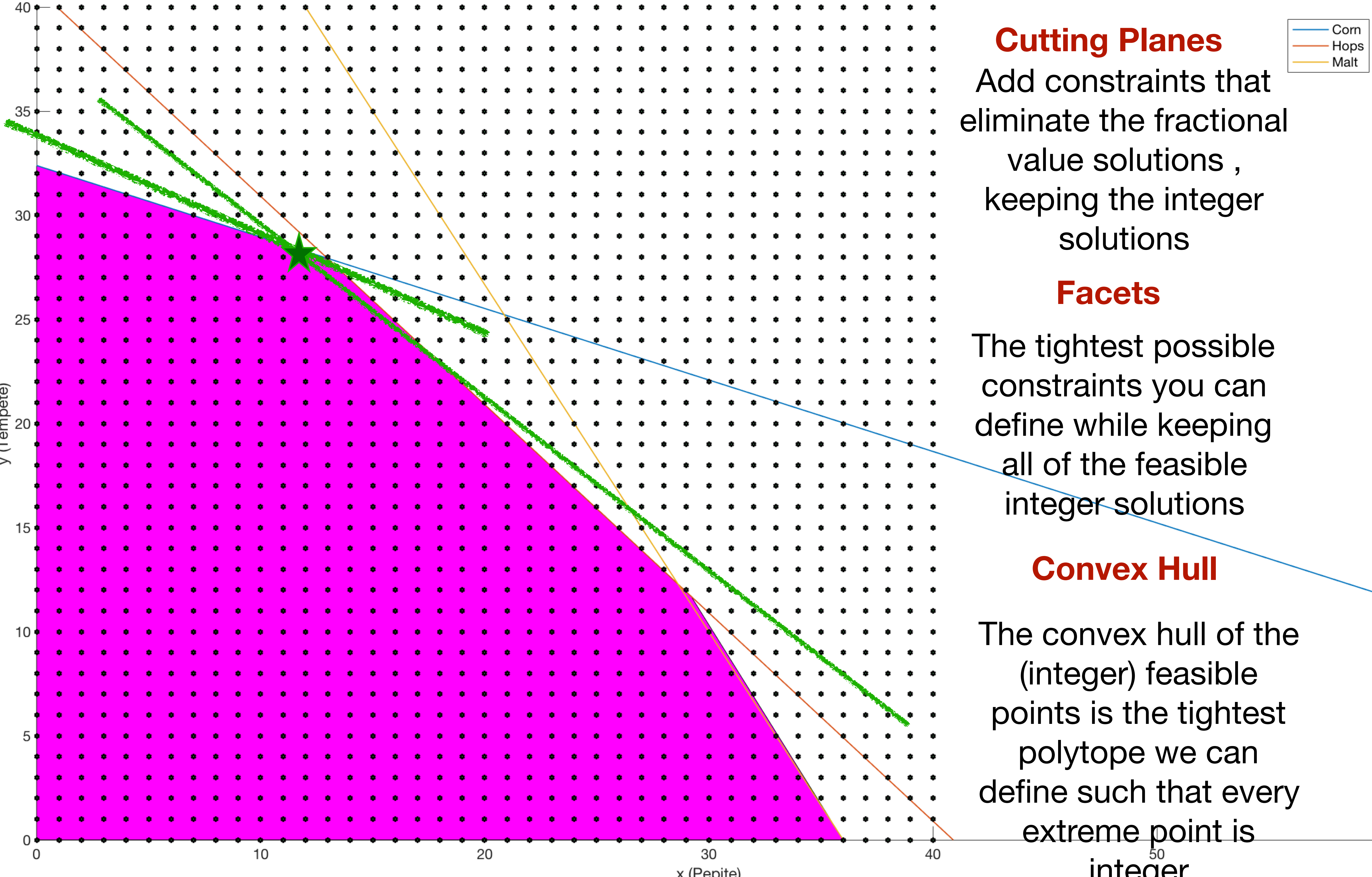
What does it mean to have an ILP?

An ILP is a mathematical optimisation formulation in which *all* of the decision variables are restricted to be *integer*

When some decision variables are *integer and* other *continuous*, we have a *Mixed Integer Linear Program (MILP)*



Cutting Planes



Cutting Planes

Add constraints that eliminate the fractional value solutions , keeping the integer solutions

Facets

The tightest possible constraints you can define while keeping all of the feasible integer solutions

Convex Hull

The convex hull of the (integer) feasible points is the tightest polytope we can define such that every extreme point is integer

Branch-and-Bound Method (B&B)

$$
\begin{array}{llcllll}
\max & 10x & + & 16y & & \\
s.t. & 2.3x & + & 6.7y & \leq & 217 \\
 & 0.11x & + & 0.11y & \leq & 4.5 \\
 & 15x & + & 9y & \leq & 540 \\
 & x & , & y & \geq & 0 \\
 & x & , & y & \in & Z
\end{array}
$$

BEST (LP) SOLUTION:
12.97 barrels Pepite
27.93 barrels Tempete

Subproblem 0

$Z_u = 576.58$
$x=12.97$ $y=27.93$

$y \leq 27$

Subproblem 1

$Z_u = 571.11$
$x=13.91$
$y=27$

$y \geq 28$

Subproblem 2

$Z_u = 575.82$
$x=12.782$
$y=28$

Branch-and-Bound Method (B&B)

$$
\begin{array}{lrcrcr}
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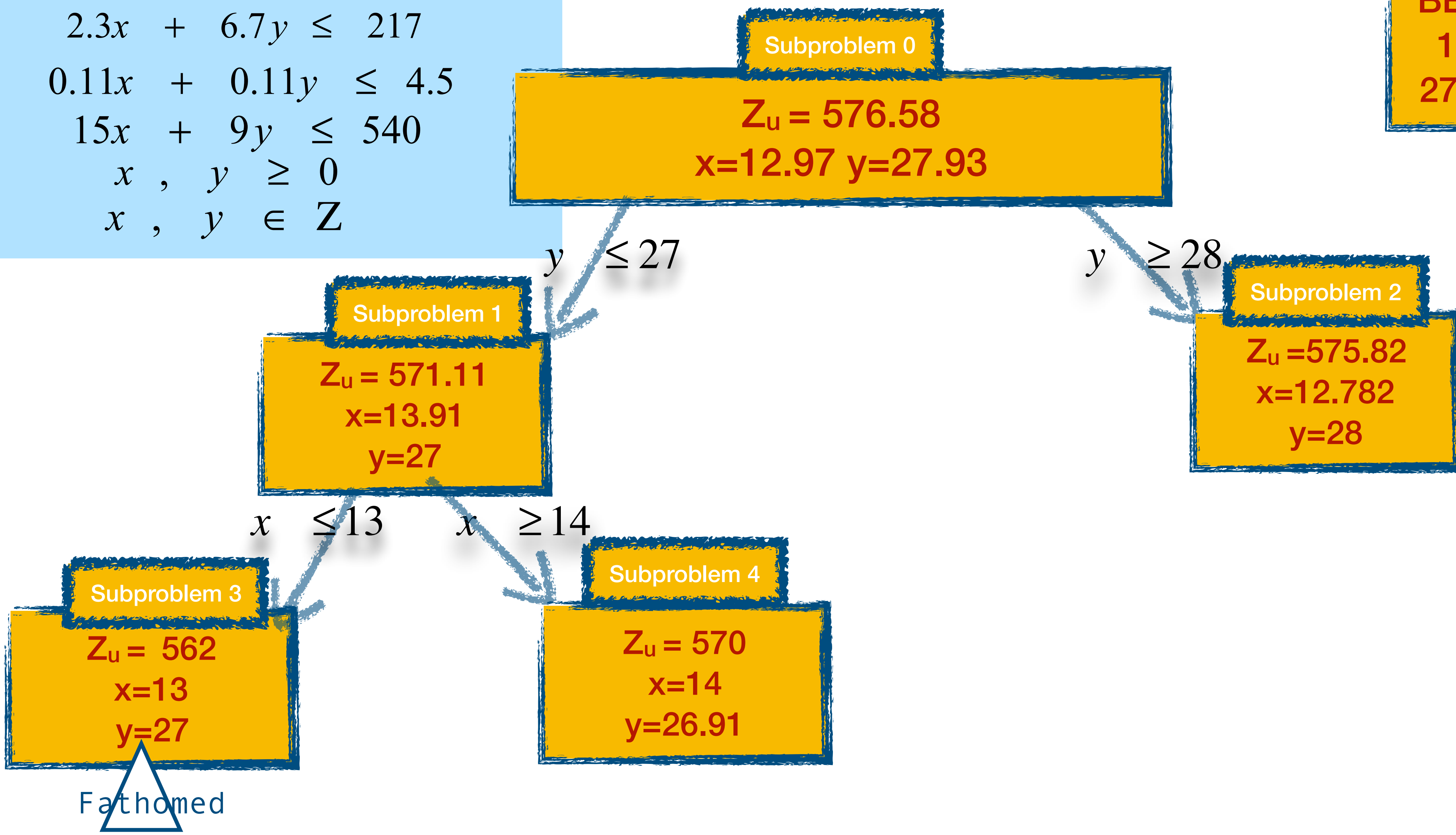

Branch-and-Bound Method (B&B)

$$
\begin{aligned}
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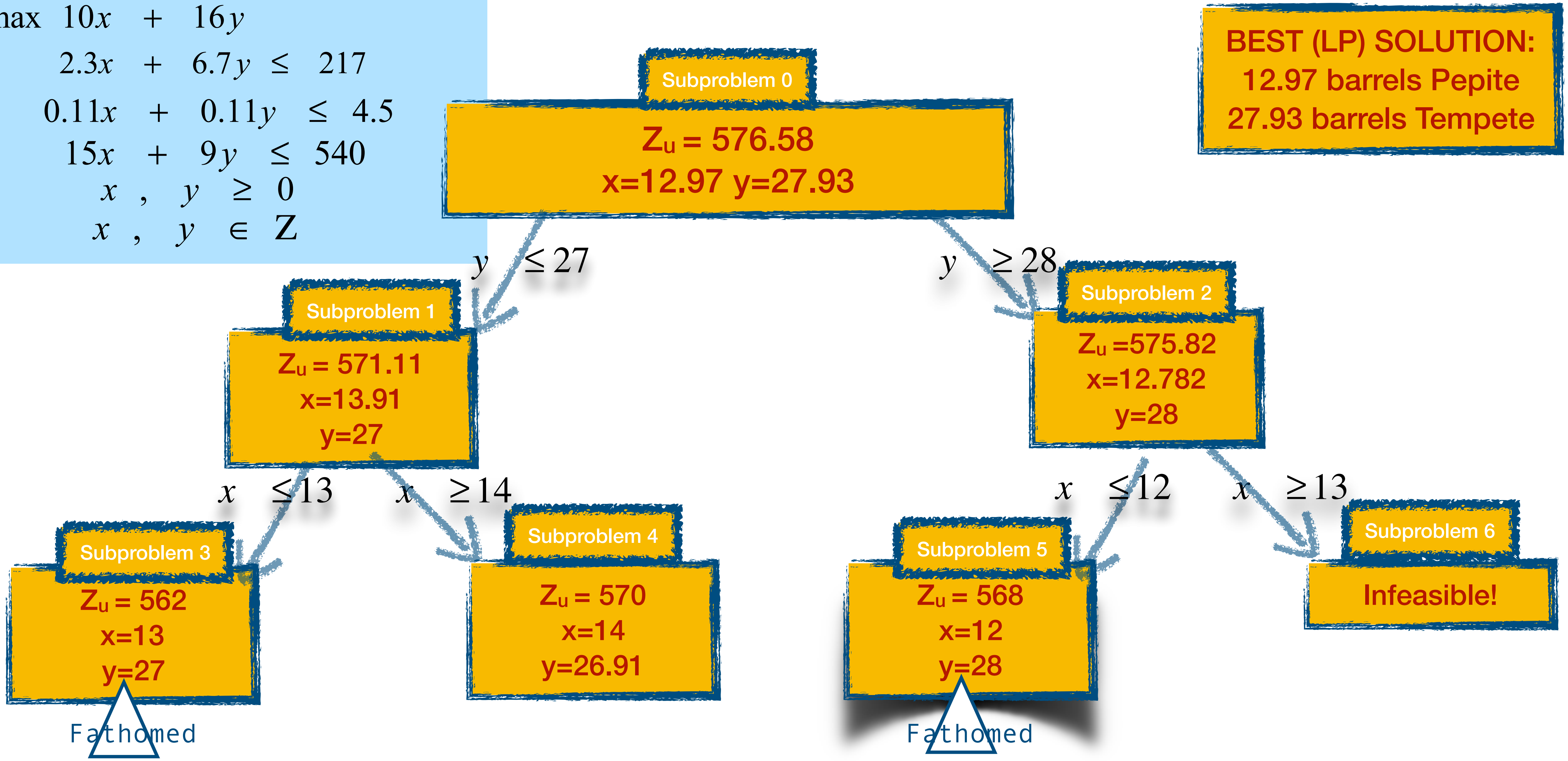

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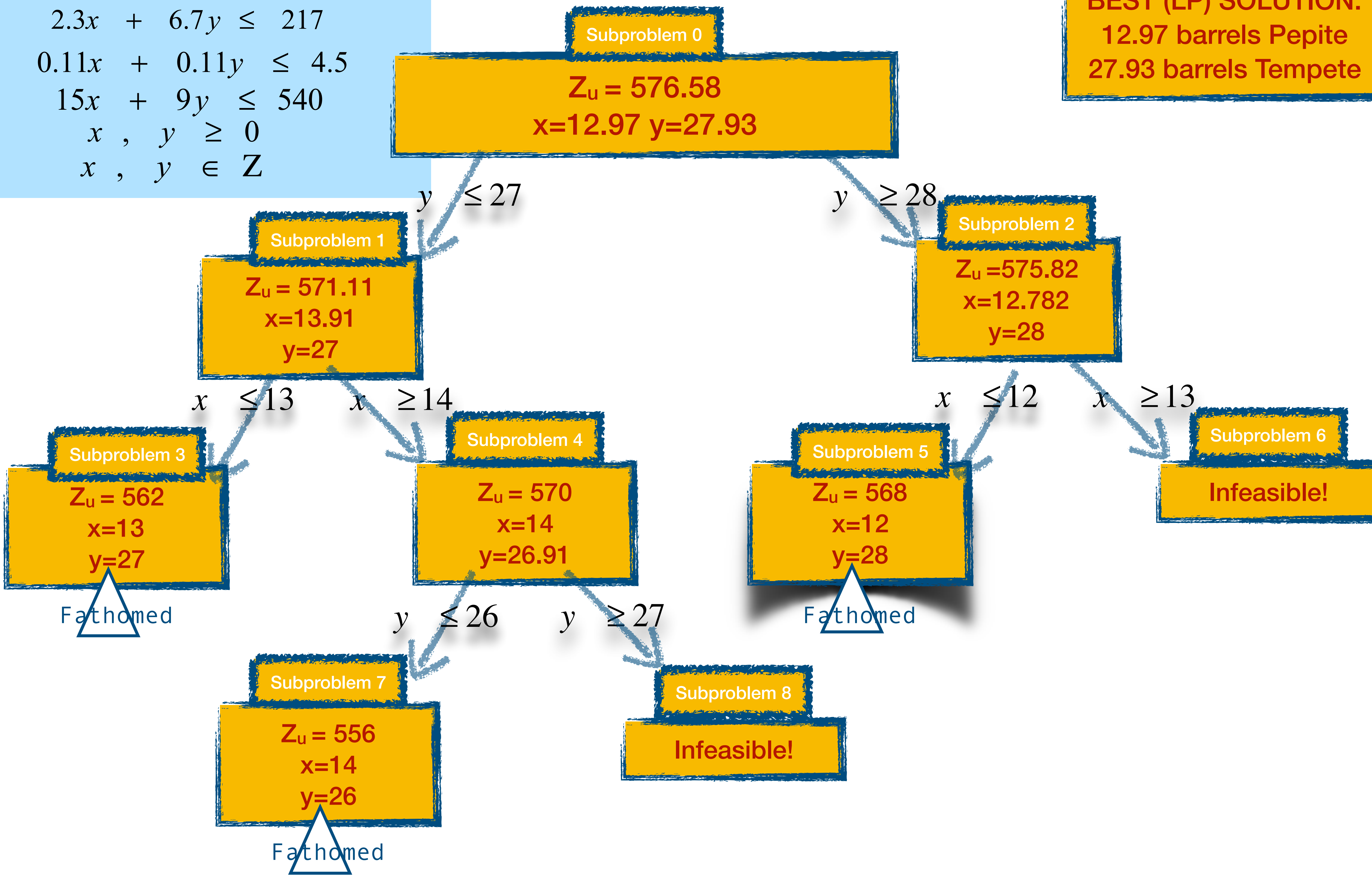


“Rules” for the B&B Method

1. *Solve the Linear Relaxation problem (subproblem 0)*
2. *The next subproblems to be solved always start from the “left” of the tree*
3. *Always branch on the most fractional decision variable*
4. *In a maximization/minimization problem the objective value can only decrease/increase while going down a branch of the B&B tree*
5. *Each branching decision acts as an additional constraints on the “root node” (i.e. the LP relaxation)*
6. *A node can be fathomed because: 1) A feasible integer solution has been found; 2) The problem is infeasible; 3) The value of the upper bound Z_u is lower or equal than the incumbent solution Z_L*

TACTICAL DECISIONS

(Satellite) Storage and Inventory Management

The owner of the bar "Satellite" at EPFL would like to know how many boxes of "Tempête" should be ordered from the Dr. Gab's factory, as well as the frequency of such orders.

Over the past year, demand has averaged 10 boxes per day. Although the demand is not the same each day, it rarely exceeds or falls below the average by much. Consequently, the owner of Sat feels confident using an **annual demand of 3600 boxes considering there are 360 days per year**.

Because of its long-standing relationship with the Dr. Gab's brewery, Sat receives an excellent delivery service, almost always receiving an order five days after placing it. In cooperation with its accountants, Sat estimates that **an order of Q boxes per year from the vendor costs 75+16Q CHF**. Sat also estimates that **the holding cost would be CHF4 for all boxes during a cycle time, 25% of a box's unit cost.**

Sat's **current policy** calls for an order of **600 boxes to arrive every 60 days**. Since daily demand is ~10 boxes per day, the arrival of 600 boxes every 60 days has led to very few stockouts. However, Sat is ensure whether its current policy is optimal. Perhaps, it should order **less frequently but in larger quantities (e.g. 1200 boxes every 120 days) or more frequently is smaller quantities (e.g. 150 boxes every 15 days)**. Sat is willing to change its current ordering policy but wants to ensure that stockout under the new policy are also infrequent.

Can you help Sat's owner making his decision? - Exercise session!

Economic Order Quantity Problem

Assumptions:

1. *Single Product*

2. *Constant Deterministic Demand*

3. *Instantaneous Arrivals (no lead time)*

4. *No backorders (no negative inventory — as soon as your inventory is empty, you need to re-order)*

Economic Order Quantity Problem

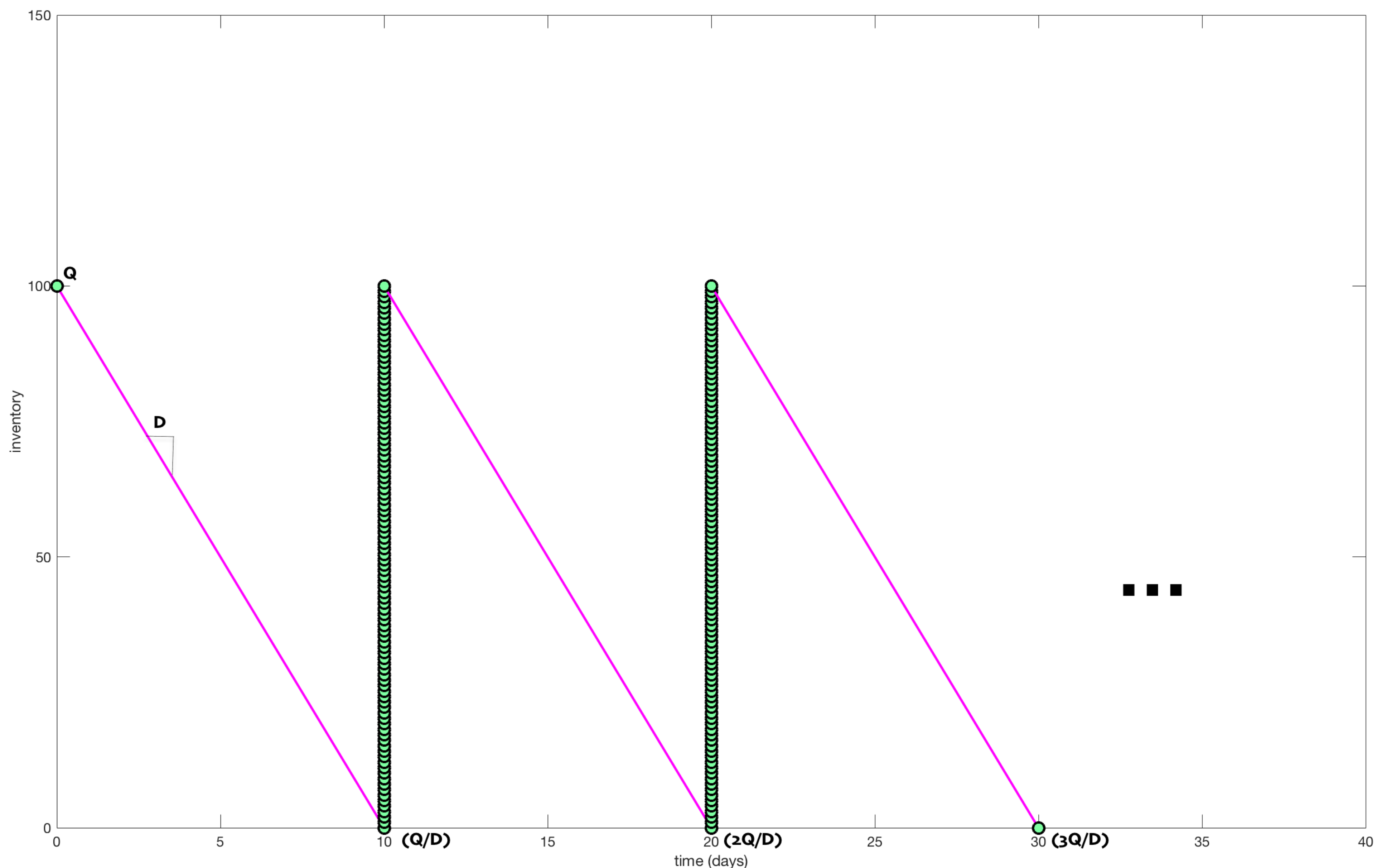
D - demand (e.g. boxes per year)

C -unit cost (CHF per box)

K -fixed cost (CHF)

h - holding cost (CHF per box per year)

Q - order quantity (boxes)



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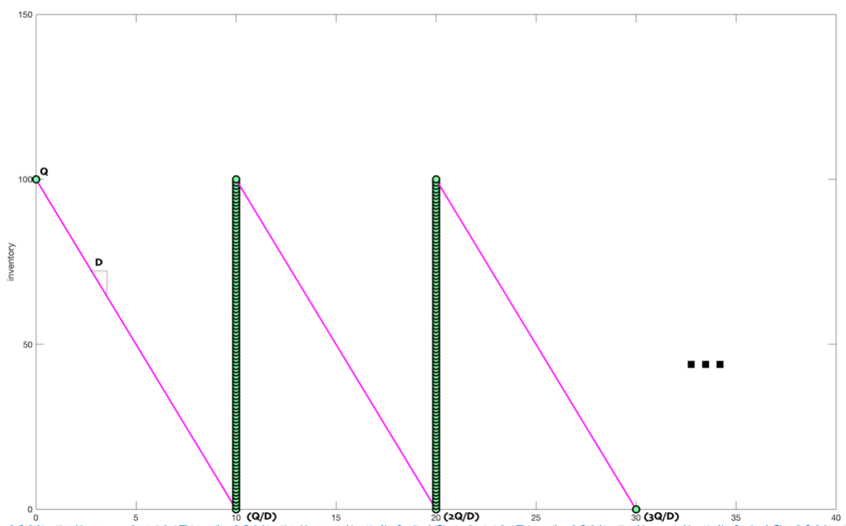

TOTAL COST= Fixed Order Cost + Unit Cost + Holding Cost

$$TC_{Cycle} = K + cQ + h\frac{Q^2}{2D}$$

Area under the “triangles”

Note that this is a convex function but if you try to minimise it, it will make your inventory negative! Divide by the cycle length!

$$TC_{time} = K\frac{D}{Q} + cD + h\frac{Q}{2}$$

Economic Order Quantity Problem

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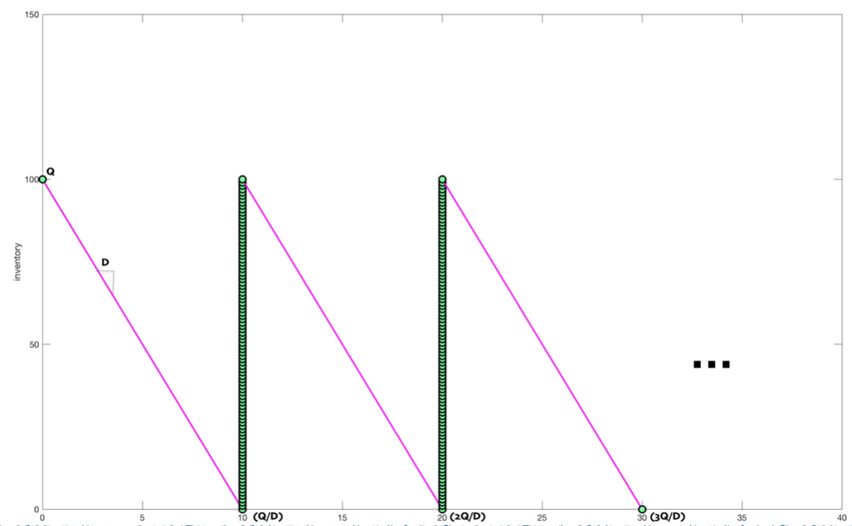


This function is convex
Minimize something = Take the 1st derivative!

$$TC_{time} = K\frac{D}{Q} + cD + h\frac{Q}{2}$$

$$TC'_{time} = -K\frac{D}{Q^2} + \frac{h}{2}$$

$$Q^* = \sqrt{\frac{2KD}{h}}$$

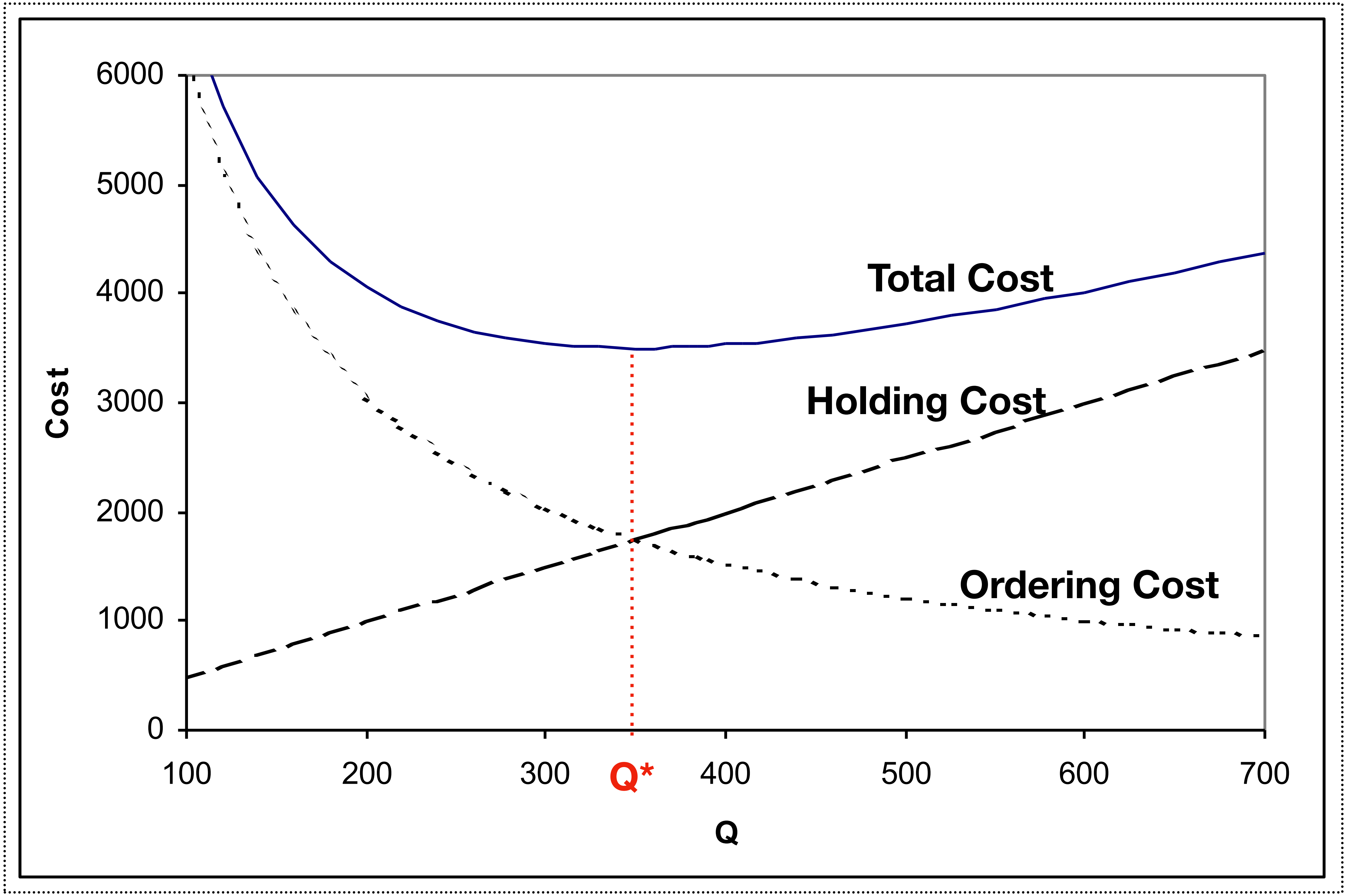
C disappears!
It does not matter what the unit cost is. No matter what the demand is, you are always going to pay the cost of total demand

If K=0, then Q=0 : No order cost, you might order every day!

If h=Inf, then Q=0 : You want to minimise the inventory you hold

If h=0, then Q=Inf : Because there is no holding cost, we might order everything all together!

Graphical Representation



(Satellite) Storage and Inventory Management

SATELLITE
bar · concerts · cafés-théâtres

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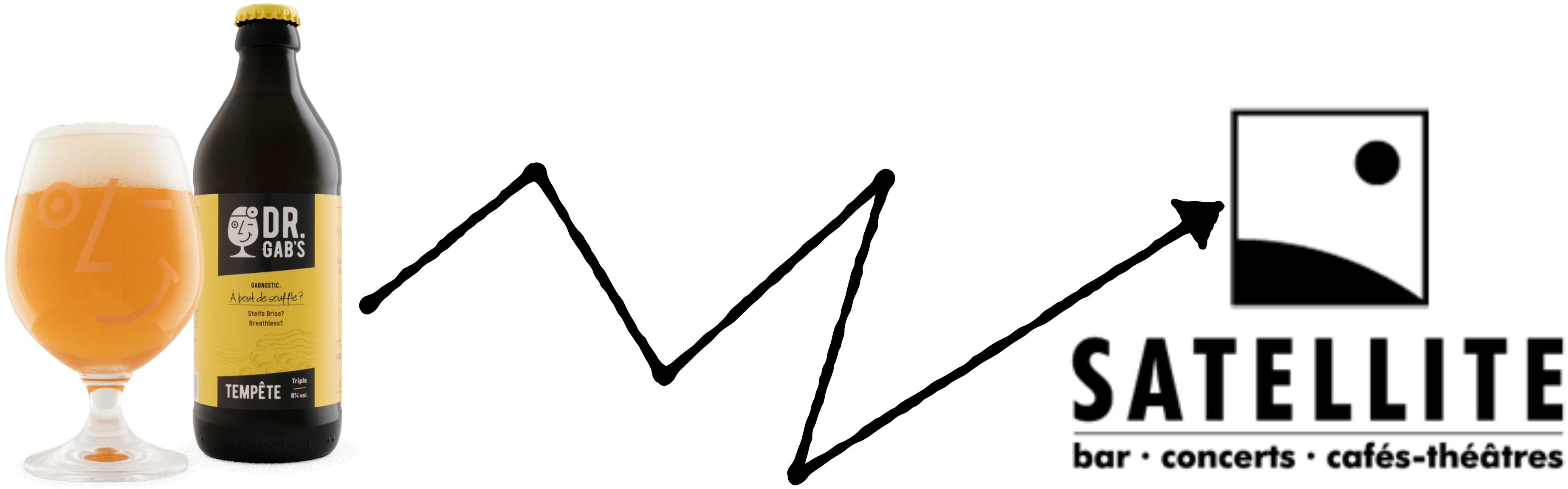
OPERATIONAL DECISIONS

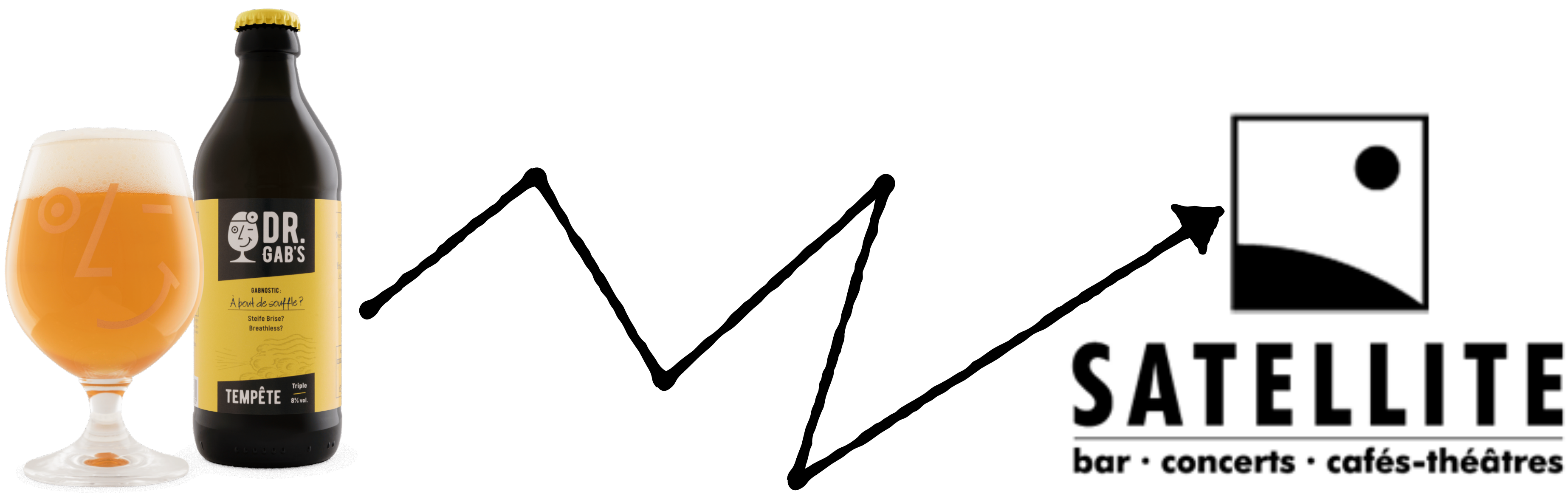
The Shortest Path Problem



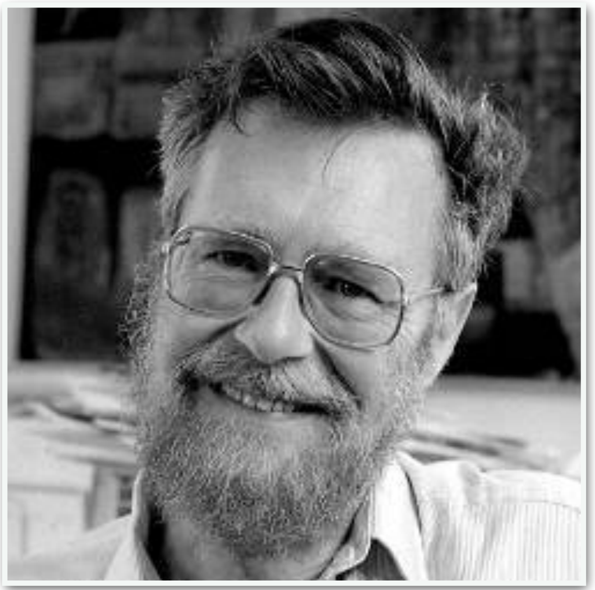
Dr. Gab’s need to **transport** the ordered **“Tempête” beers** from its factory in **Puidoux to EPFL** in the quantity and frequency found in the previous problem. The driver of Dr. Gab’s is ready to leave with a full truck, and needs to **follow the shortest path** from Puidoux to EPFL.

Can you give the Dr. Gab’s driver instructions to reach Sat as soon as possible?

The Shortest Path Problem



Single-Source Shortest Path Problem - The problem of finding shortest paths from a source vertex to all other vertices in a graph



Many algorithms based on dynamic programming:

Dijkstra's algorithm*

Bellman-Ford

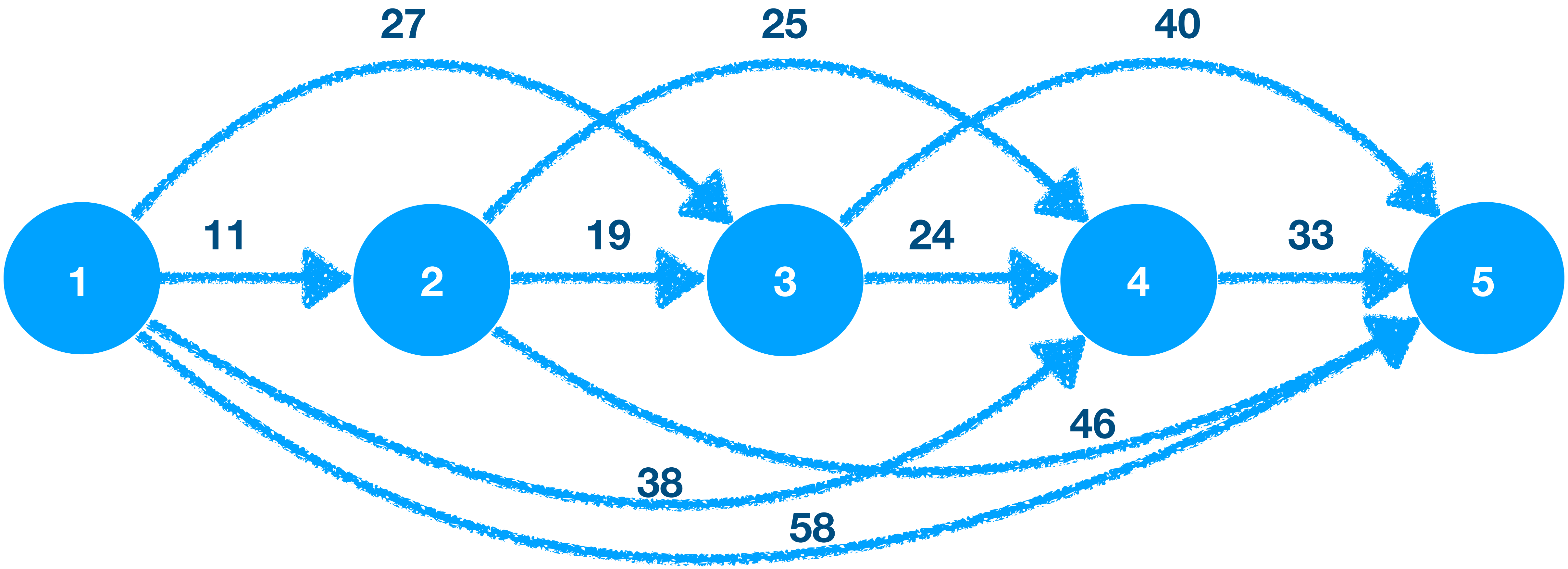
A*

Floyd-Warshall

…

*E.W. Dijsktra (1959) A Note on Two Problems in Connection with Graphs. Numerische Mathematik,1. 269-271

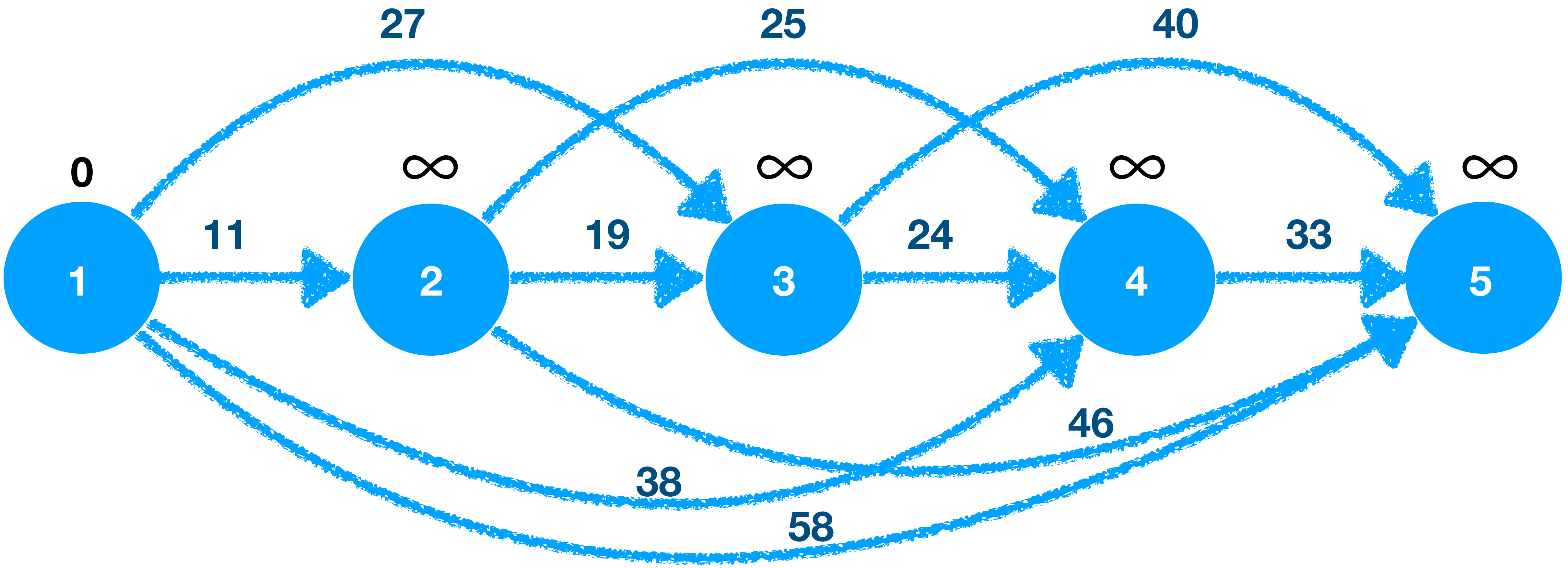
Dijkstra's Algorithm 1



1. Runs on a weighted graph (all arcs with non-negative weights)

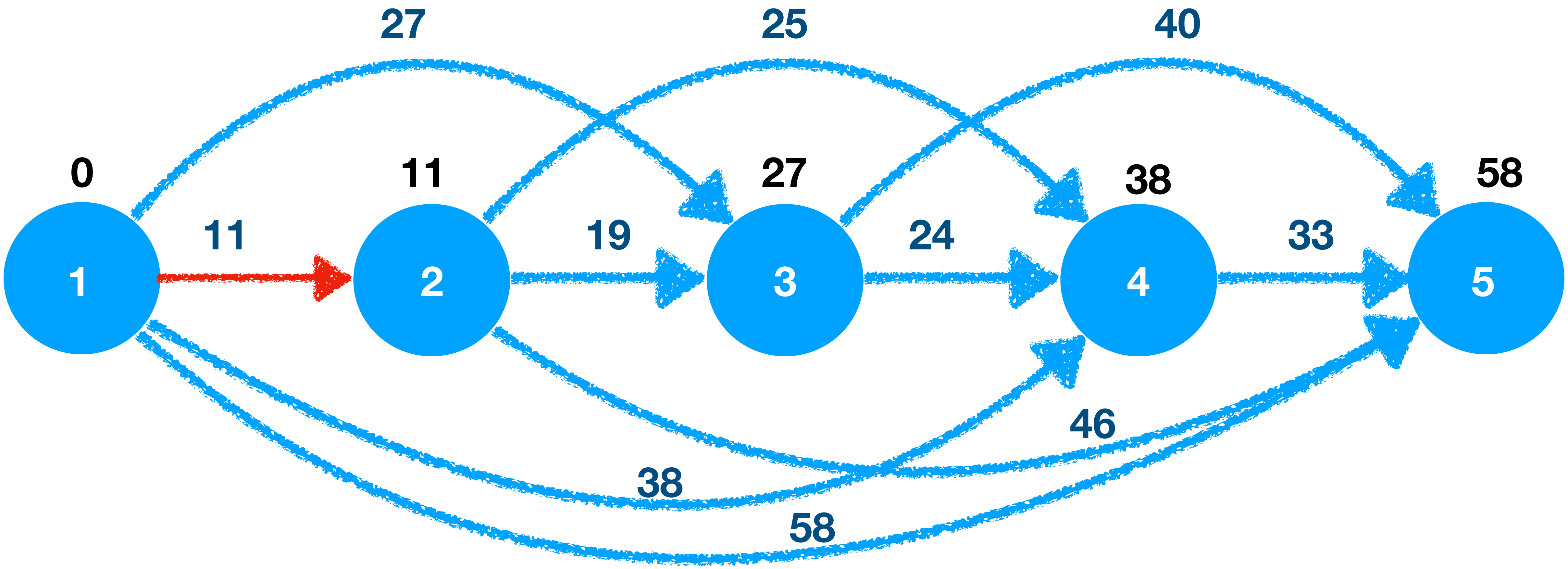
3. Finds the least cost path from a "source" node to all other nodes in the graph, producing a shortest path tree

Dijkstra's Algorithm 1



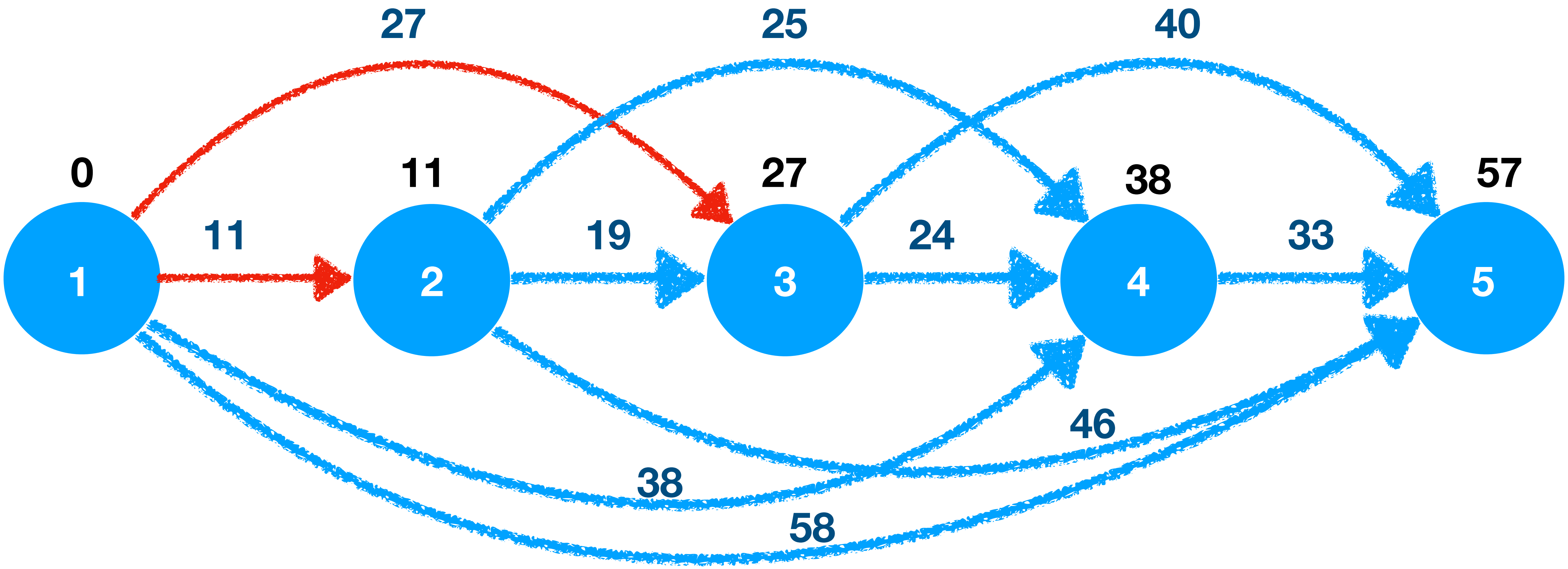
Node / Iteration	1	2	3	4	5
1	0	∞	∞	∞	∞
2					
3					
4					
5					

Dijkstra's Algorithm 1



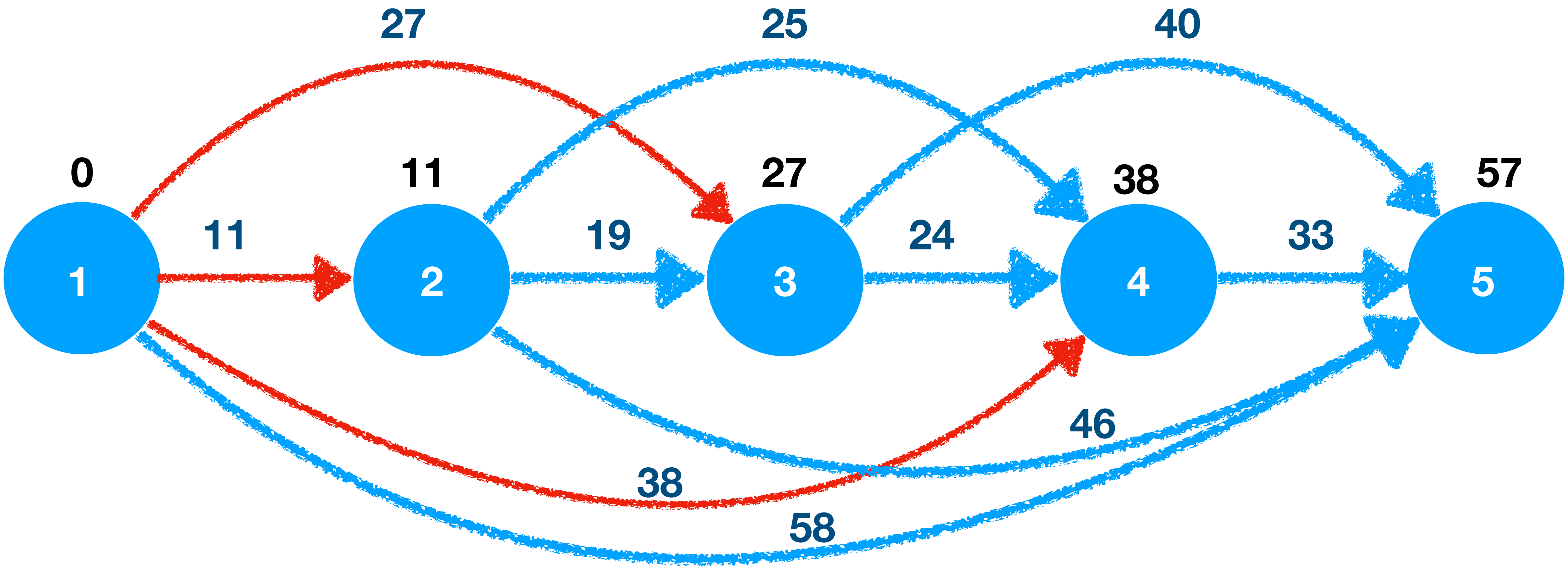
Node Iteration	1	2	3	4	5
1	0	∞	∞	∞	∞
2	0	11^{1}	27^{1}	38^{1}	58^{1}
3					
4					
5					

Dijkstra's Algorithm 1



Node / Iteration	1	2	3	4	5
1	0	∞	∞	∞	∞
2	0	11^1	27^1	38^1	58^1
3	0	11^1	27^1	38^1	57^2
4					
5					

Dijkstra's Algorithm 1



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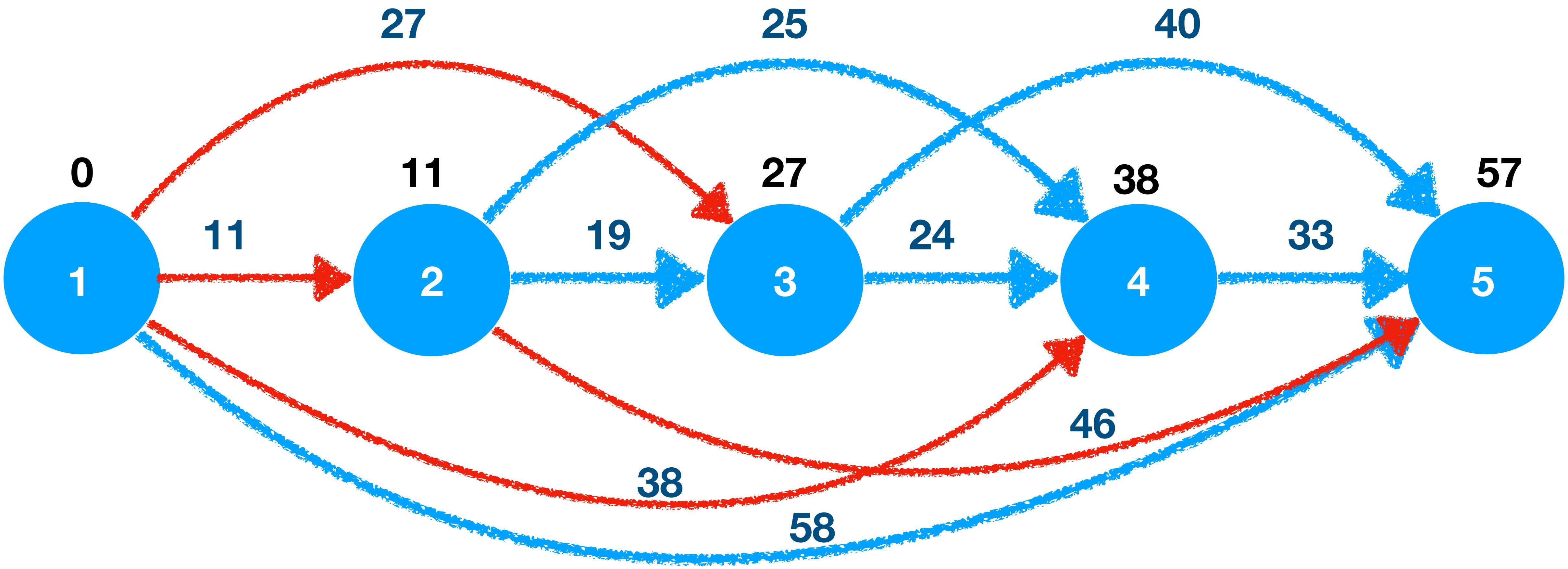
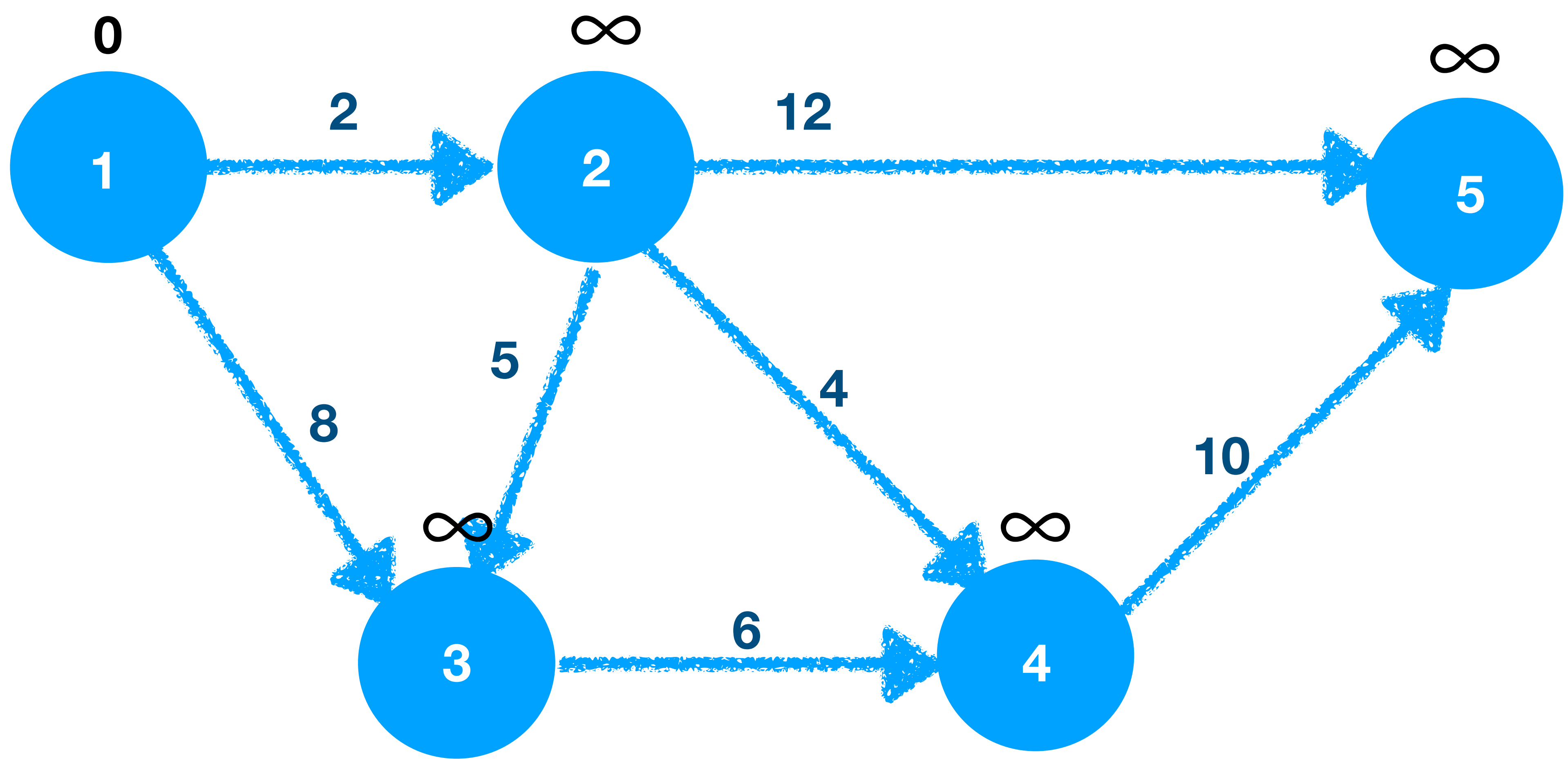
5					

Dijkstra's Algorithm 1



Node / Iteration	1	2	3	4	5
1	0	∞	∞	∞	∞
2	0	11^{1}	27^{1}	38^{1}	58^{1}
3	0	11^{1}	27^{1}	38^{1}	57^{2}
4	0	11^{1}	27^{1}	38^{1}	57^{2}
5	0	11^{1}	27^{1}	38^{1}	57^{2}

Dijkstra's Algorithm 2



Node / Iteration	1	2	3	4	5
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2					
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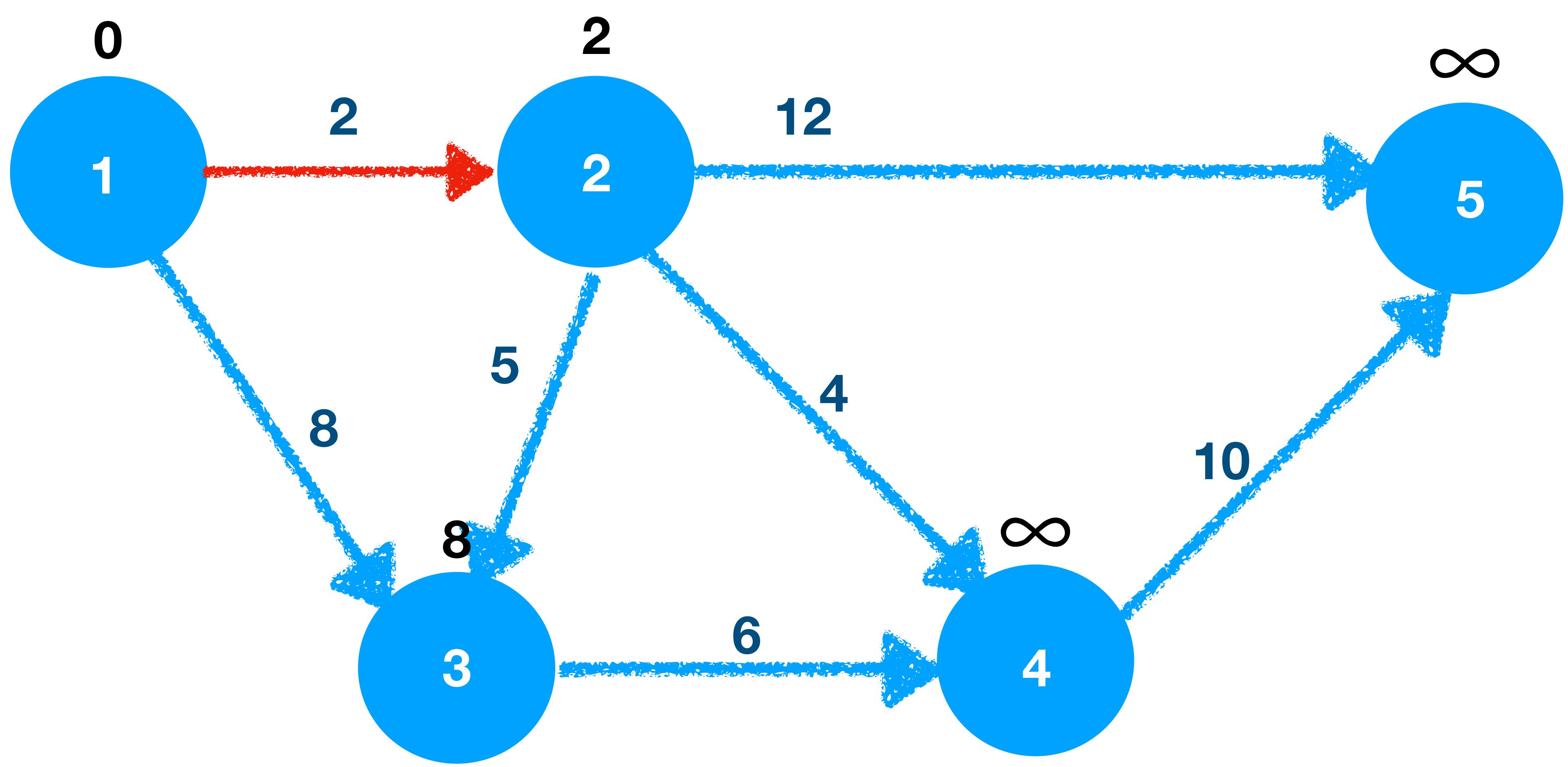
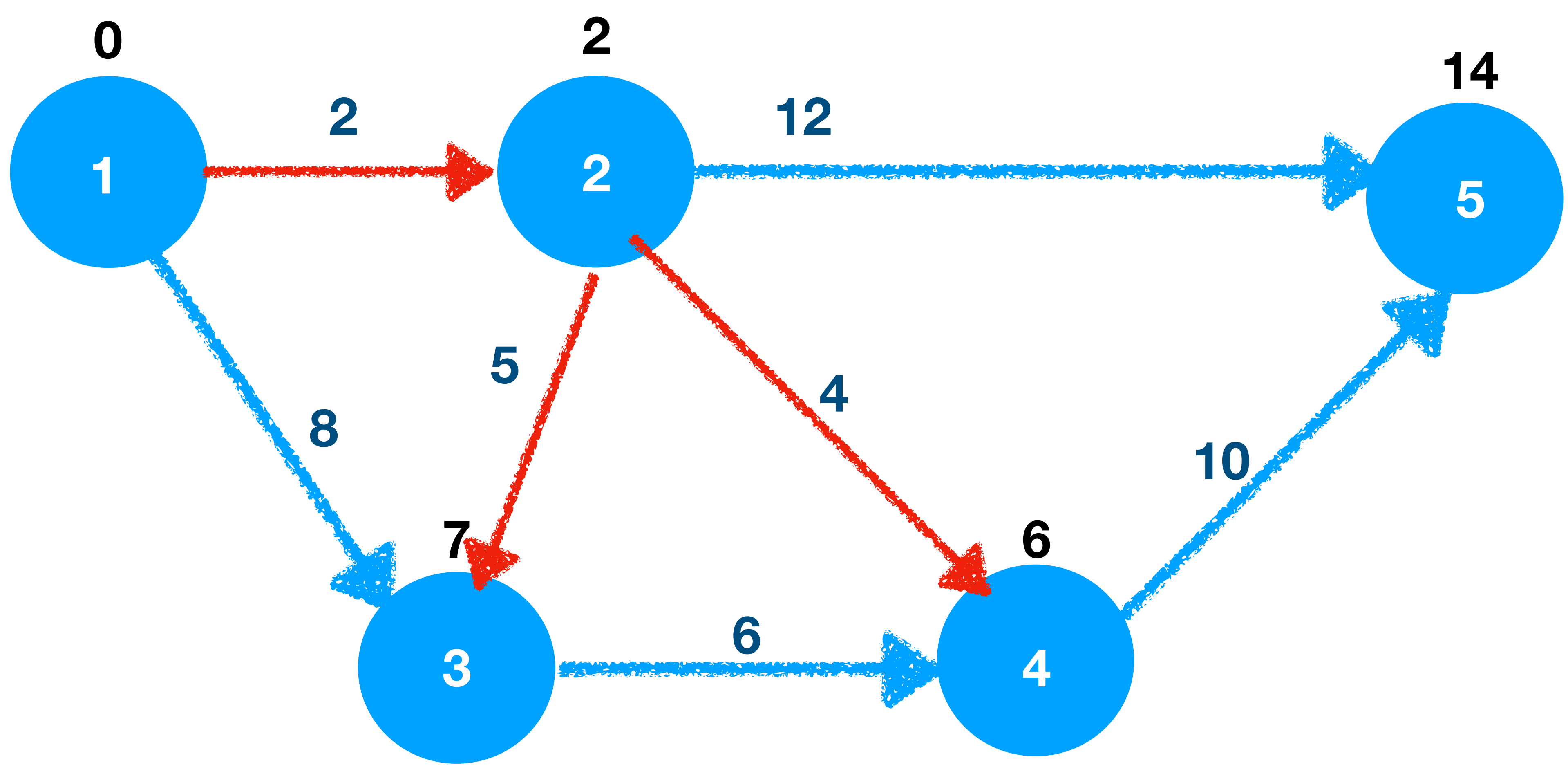
Dijkstra's Algorithm 2



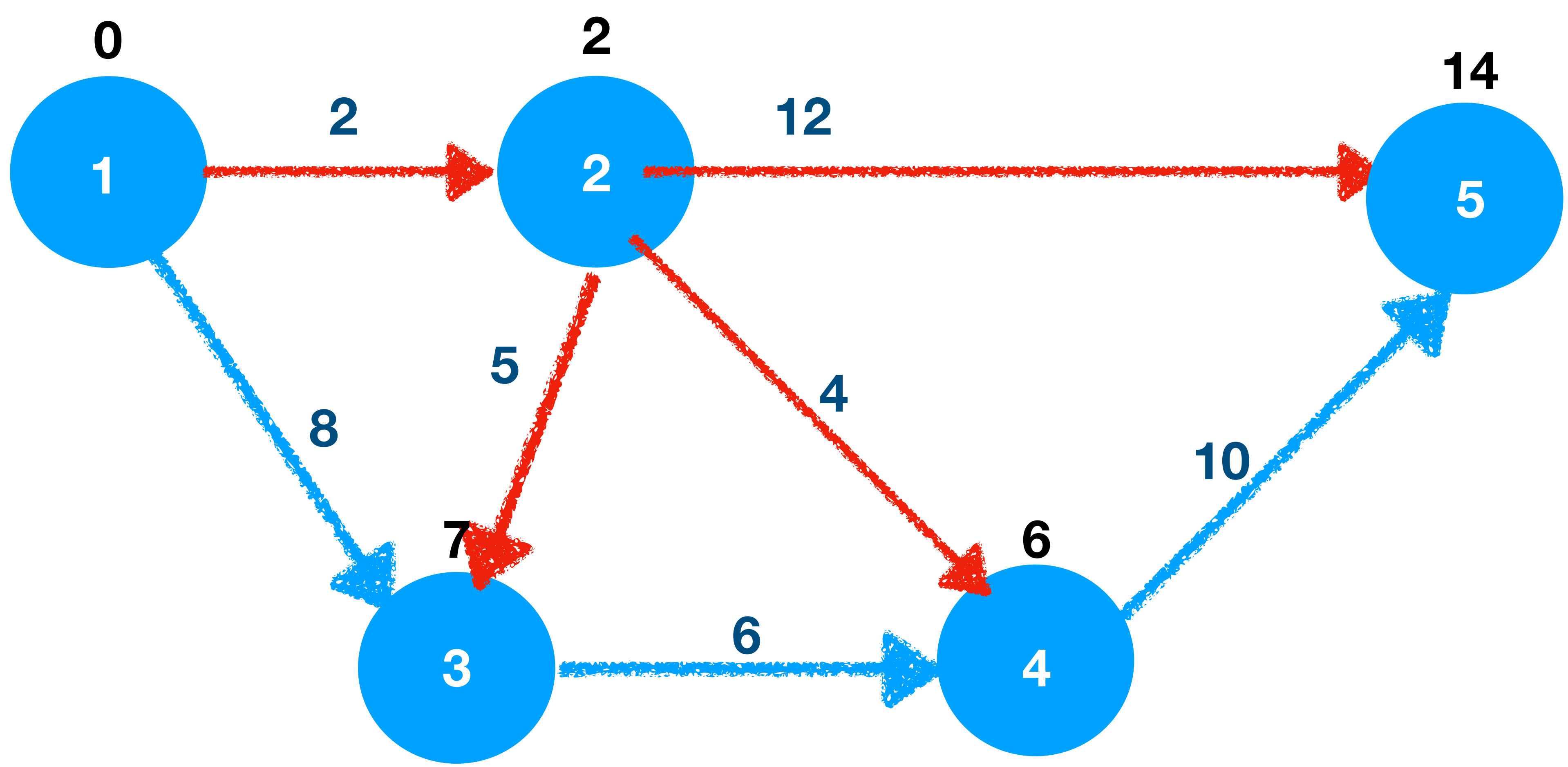
Node Iteration	1	2	3	4	5
1	0	∞	∞	∞	∞
2	0	2^1	8^1	∞	∞
3					
4					
5					

Dijkstra's Algorithm 2



Node Iteration	1	2	3	4	5
1	0	∞	∞	∞	∞
2	0	2^1	8^1	∞	∞
3	0	2^1	7^2	6^2	14^2
4					
5					

Dijkstra's Algorithm 2



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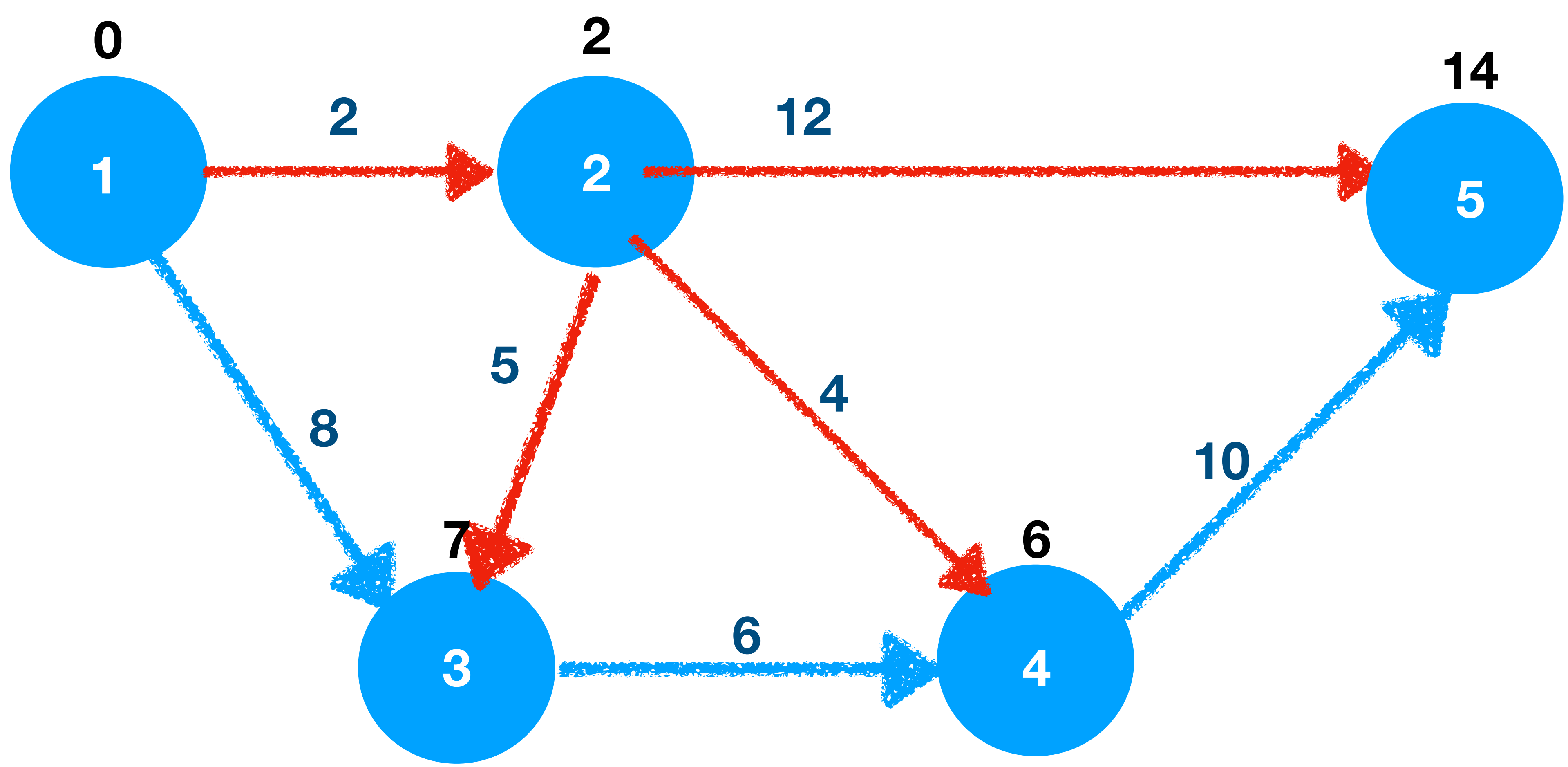
2	0	2^1	8^1	∞	∞
3	0	2^1	7^2	6^2	14^2
4	0	2^1	7^2	6^2	14^2
5	0	2^1	7^2	6^2	14^2

Dijkstra's Algorithm 2



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“Steps” for Dijkstra’s algorithm

1. *Mark all nodes unvisited. Create a set of all the unvisited nodes called the unvisited set.*

2. *Assign to every node a tentative distance value: set it to zero for our initial node and to infinity for all other nodes. Set the initial node as current*

3. *For the current node, consider all of tis unvisited neighbours and calculate their tentative distances through the current node.*

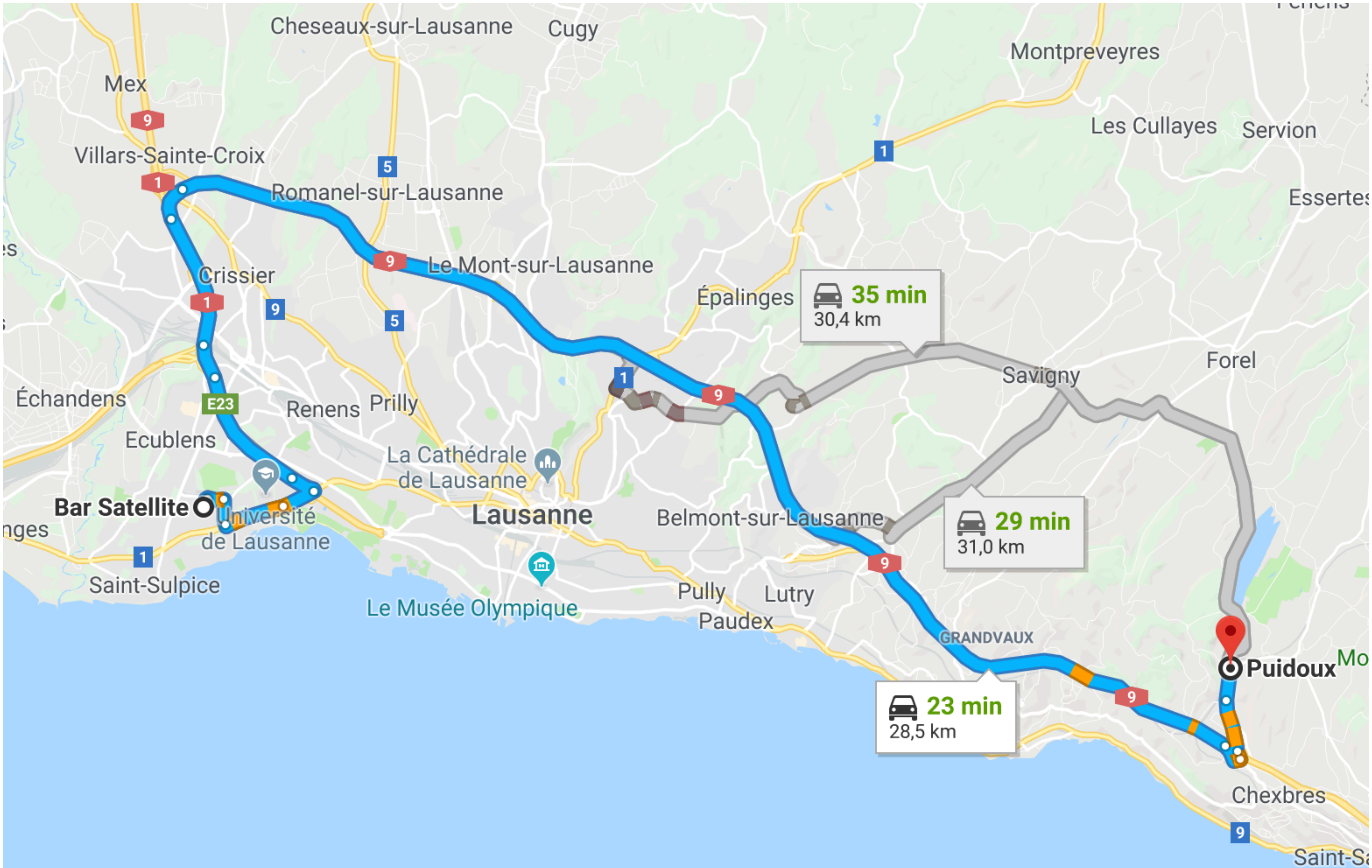
4. *Compare the newly calculated tentative distance to the current assigned value and assign the smaller one. For example, if the current node A is marked with distance 6 and the edge connecting it with a neighbour B has length 2, then the distance to B through A will be 6+2=8. If B was previously marked with distance greater than 8 then change it to 8. Otherwise, keep the current value.*

5. *When we are done considering all of the unvisited neighbours of the current node, mark the current node as visited and remove it from the unvisited set. A visited node will never be checked again.*

6. *If the destination node has been marked visited (when planning a route between two specific nodes) or if the smallest tentative distance among the nodes in the unvisited set is infinity (when planning a complete traversal, it might occur when there is no connection between the initial node and remaining unvisited nodes), then stop. The algorithm has finished.*

7. *Otherwise, select the unvisited node that is marked with the smallest tentative distance, set it as the new “current node” and go back to step 3*

Sat’s Problem



Discussion

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