Exercise 12

12a. Nonlinear and linear binding forces cancel out:

$$m\frac{d^2\tilde{x}}{dt^2} = F_{binding} + F_{driving} + F_{damping}$$

$$\downarrow$$

$$-m\omega_0^2\tilde{x} + mb\tilde{x}^3 = 0 \quad \text{for } x = d = \text{atomic radius}$$

$$bd^2 = \omega_0^2 \qquad b = \frac{\omega_0^2}{d^2}$$

$$\omega_4, \omega_3, \omega_2, \omega_1 \to \omega, \omega, \omega, -\omega$$

$$\chi^{(3)}_{ijkl}(\omega;\omega,\omega,-\omega) = \frac{Nbe^4\{\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\}}{3\epsilon_0 m^3 \omega_0^8} \quad {}^{=3}$$

$$D(\omega) = \omega_0^2 - \omega^2 - 2i\omega\gamma \Big|_{\omega \ll \omega_0^2} = \omega_0^2$$

$$\chi^{(3)} = \frac{Ne^4}{\epsilon_0 m^3 \omega_0^6 d^2}$$

and there is only one independent element (χ or 3χ).

b. $n_2 = n_0 + n_2 I \qquad n_2 = \frac{3\chi^{(3)}}{4n_0^2 \epsilon_0 c}$

For H_2O:

$N = 55$ mol/L $\to 3.3 \cdot 10^{28}$ H_2O/m^3

$e = 1.6 \cdot 10^{-19}$ C

$\epsilon_0 = 8.85 \cdot 10^{-12}$ C/V.m

$m = 9.1 \cdot 10^{-31}$ kg

ω_0: $\lambda_0 = 180$ nm $\quad \omega_0 = \frac{2\pi c}{\lambda_0} = 10^{16}$ rad/s

$d = 0.14$ nm or 0.28 nm (diameter)

$n_0 = 1.33$

$$\chi^{(3)} = \frac{3.3 \cdot 10^{28} \cdot (1.6 \cdot 10^{-19})^4}{8.85 \cdot 10^{-12} (9.1 \cdot 10^{-31})^3 (1 \cdot 10^{16})^6 (2.8 \cdot 10^{-10})^2} \quad (2.8 \text{ or } 1.4)$$

$= 42$ pm²/V² (2.8)

or 168 pm²/V² (1.4)

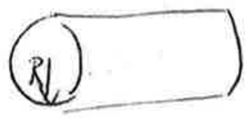
(we compute the solutions for the two possible d values)

$$n_2 = \frac{3 \cdot 4.2 \cdot 10^{-23}}{4 \cdot (1.33)^2 \cdot 8.85 \cdot 10^{-12} \cdot 3 \cdot 10^8} = 6.6 \cdot 10^{-21} \text{ m}^2/\text{W} \quad \text{or} \quad 26.4 \cdot 10^{-21} \text{ m}^2/\text{W}$$

$$6.6 \cdot 10^{-17} \text{ cm}^2/\text{W} \qquad 26.4 \cdot 10^{-17} \text{ cm}^2/\text{W}$$

c. $$\underbrace{\underbrace{\rho_0 C}_{\substack{\text{heat capacity} \\ J/K\cdot cm^3}} \underbrace{\frac{\partial \tilde{T}}{\partial t}}_{K/s}}_{\substack{\text{transformation of energy into} \\ \text{excited thermal motion.}}} - \underbrace{\underbrace{K}_{\substack{W/m\cdot K \\ \text{thermal conductivity}}} \underbrace{\nabla^2}_{1/m} \underbrace{\tilde{T}_1}_{K}}_{\substack{\text{heat transport out of focus} \\ \text{by diffusion}}} = \underbrace{\underbrace{\alpha}_{\substack{\text{abs coeff.} \\ m^{-1}}} I(r)}_{\substack{\text{energy that enters} \\ \text{the system}}} \leftarrow \text{int } J/m^2 \cdot s$$

12d. $\frac{\partial T_l}{dt} = \frac{\tilde{T}_l}{\tau}$ $\nabla^2 \tilde{T}_l \cong \frac{\partial^2 \tilde{T}_l}{\partial r^2} \sim \frac{T_l}{R}$



$$\rho_0 C \frac{T_l}{\tau} - \frac{\kappa T_l}{R^2} = \alpha I(r)$$

$R = \phi/2$

When the beam is off: $\frac{\rho_0 C T_l}{\tau} = \frac{\kappa T_l}{R^2}$

$$\tau = \frac{\rho_0 C R^2}{\kappa} = \frac{\rho_0 C \phi^2}{4\kappa}$$

e. H_2O $\rho_0 C$: $4.2\ J/cm^3 K$

$\kappa = 0.56\ W/m \cdot K = 5.6 \cdot 10^{-3}\ W/cm \cdot K$

$\tau = 0.75 \cdot 10^3 \cdot R^2$ [cm]

τ	R (μm)	ϕ
1 μs	0.36	720 nm
1 ms	11.5	23 μm
1 s	36.5	720 μm

f. Neglecting diffusion we get:

+g.

$$\rho_0 C \underbrace{\frac{\partial \tilde{T}}{\partial t}}_{\tilde{T}_l / \tau_p} = \alpha \tilde{I}(r) \Rightarrow \tilde{T}_l \sim \frac{\overbrace{\alpha I(r) \cdot \tau_p}^{F\ \text{fluence}}}{\rho_0 C}$$

$\alpha = 0.6\ m^{-1}$; $6 \cdot 10^{-3}\ cm^{-1}$

$\rho_0 C = 4.2\ J/cm^3 K$

$I = 10\ mJ/cm^2$ @ 100 fs

single pulse

$$\tilde{T}_l = \frac{6 \cdot 10^{-3} \cdot 10 \cdot 10^{-3}}{4.2} = 14.3\ \mu K$$

h. We need to sum $\tilde{T}_l$

τ 1 μs relaxation; ϕ 720 nm

1 MHz : 1 pulse --> 14 μK

1 GHz 1000 pulses --> 14 mK

1 ms:
$\phi = 23\ \mu m$
$4.2 \cdot 10^{-6}\ cm^2$
41 nJ/pulse

~~1 MHz~~ 1 kHz --> 14 μK

1 MHz 14 mK

1 GHz 14 K

1 s
$\phi = 720\ \mu m$
$4.1 \cdot 10^{-3}\ cm^2$ area
41 μJ/pulse @ 1 GHz impossible

1 kHz 14 mK

1 MHz 14 K

1 GHz 14,000 K ← 2 W @ 1 GHz this is impossible

i) $|\Delta n| = \left|\left(\frac{dn}{dT}\right)\right| T_{L\,max} \geq \frac{3\chi^{(3)}}{4\varepsilon_0 c n_0^2}$ ← given at first page of exercises

$$|\alpha| 1 - e^{-\frac{T-T_0}{T_K}} | T_{L\,max} \geq \frac{3\chi^{(3)}}{4\varepsilon_0 c n_0^2}$$

1,33 = n_0

$$T_{L\,max} \geq \frac{3\chi^{(3)} I}{4\varepsilon_0 c n_0^2 |a(1-e^{-\frac{T-T_0}{T_K}})|}$$

$$\tau_p \geq \frac{3\chi^{(3)} \cancel{I} \rho_0 C}{\cancel{I}\alpha_0 \, 4\varepsilon_0 c n_0^2 |a(1-e^{-\frac{T-T_0}{T_K}})|}$$

with $n_0 = 1{,}33$
$T_0 = 2°C$
$T = 25°C$
$T_K = 48{,}5°C$
$a = 2{,}62 \cdot 10^{-5}\ K^{-1}$
$\alpha_0 = 0{,}6\ m^{-1}$

$\underline{\tau_p \geq 155\ ps}$ thermal effects become important