Fundamentals of Traffic Operations and Control
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Exercise solutions
Bus priority strategy

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In order to make the problem simple, we need to standardize all the units to meter and second. Therefore, the characteristics of the land and intersection become:

- $v_f = 36\ km/h = 10\ m/s$
- $C = 1800\ veh/h = 0.5\ veh/s = 1\ veh/2s$
- $q = 360\ veh/h = 0.1\ veh/s = 1\ veh/10s$

With this information, we can draw time-space diagram as shown in Figure 1. Since the green signal duration is $40s$, the first two vehicles pass the intersection without any delays. However, next two vehicles (the vehicles which arrive at $20s$ and $30s$ in the figure) are stopped on the end of the lane. The que is discharged by the capacity of the intersection ($1\ veh/2s$), and the following bus is delayed if the bus arrives before the que is discharged completely (red-colored area on the figure).

The bus spend 25 seconds to travel from the beginning to end of the lane and d seconds for dwelling which follows a uniform distribution in the interval $[10s, 20s]$. In order to decide the red signal duration enabling that the bus pass the intersection with zero delay 80% of the times, we can build following equation (Pr stands for 'probability'):

$$Pr\left(R + 4s \leq 25s + d\right) = 0.8. \tag{1}$$

The left side of the inequality in the parenthesis of equation (1) represents the time boundary. If the bus spends less time than the time boundary for travelling (including dwell time), the bus will be delayed on the end of the lane. The right side of the inequality represents the travel time of the bus including the dwell time. Therefore, the red signal duration should be decided by setting the probability that the bus spends more time than the time boundary is 0.8.

Equation (1) is rewritten as follows by substituting $25s$ for both side in the parenthesis:

$$Pr\left(R - 21s \leq d\right) = 0.8. \tag{2}$$

Since the random variable d follows a uniform distribution of the interval $[10s, 20s]$, the probability that d is greater than $12s$ is 0.8. Therefore,

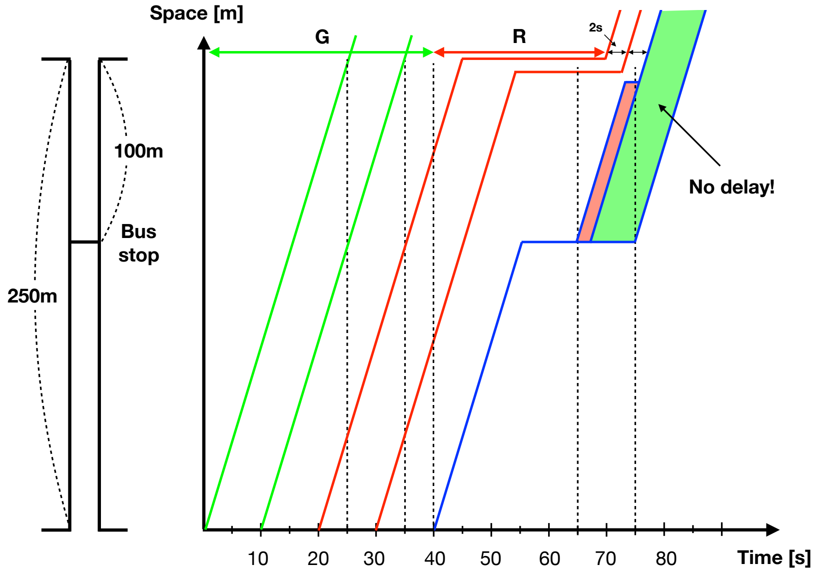

Figure 1: Vehicles and the bus trajectories in a cycle.

$$R - 21s = 12s \tag{3}$$

and finally,

$$R = 33s. \tag{4}$$