Summary of Lecture 13:
The Linear Electrooptic Effect
(Boyd - Chapter 11)

•The electrooptic effect is the change in refractive index of a material induced by the presence of a static electric field.

•In birefringent crystals materials, the change in refractive index depends linearly on the strength of the applied electric field. This change is known as the linear electrooptic

•effect or Pockels effect.

•A second order effect (in terms of nonlinear polarization):

$$P_i(\omega) = 2\epsilon_0 \sum_{jk} \chi_{ijk}^{(2)}(\omega = \omega + 0) E_j(\omega) E_k(0).$$

•The constitutive relation between D and E:

$$D_i = \epsilon_0 \sum_j \epsilon_{ij} E_j$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \epsilon_0 \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}.$$

•For a lossless, non-optically active material, the dielectric permeability tensor is represented by a real symmetric matrix

•Any real, symmetric matrix can be expressed in diagonal form by means of an orthogonal transformation

$$\begin{bmatrix} D_X \\ D_Y \\ D_Z \end{bmatrix} = \epsilon_0 \begin{bmatrix} \epsilon_{XX} & 0 & 0 \\ 0 & \epsilon_{YY} & 0 \\ 0 & 0 & \epsilon_{ZZ} \end{bmatrix} \begin{bmatrix} E_X \\ E_Y \\ E_Z \end{bmatrix}.$$

- Consider energy density:

$$U = \tfrac{1}{2}\mathbf{D}\cdot\mathbf{E} = \tfrac{1}{2}\epsilon_0 \sum_{ij} \epsilon_{ij} E_i E_j,$$

$$U = \frac{1}{2\epsilon_0}\left[\frac{D_X^2}{\epsilon_{XX}} + \frac{D_Y^2}{\epsilon_{YY}} + \frac{D_Z^2}{\epsilon_{ZZ}}\right].$$

- With the following substitution:

$$X = \left(\frac{1}{2\epsilon_0 U}\right)^{1/2} D_X, \qquad Y = \left(\frac{1}{2\epsilon_0 U}\right)^{1/2} D_Y, \qquad Z = \left(\frac{1}{2\epsilon_0}\right)^{1/2} D_Z,$$

INDEX ELLIPSOID:

$$\frac{X^2}{\epsilon_{XX}} + \frac{Y^2}{\epsilon_{YY}} + \frac{Z^2}{\epsilon_{ZZ}} = 1.$$

- In other coordinate systems, in general form:

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + 2\left(\frac{1}{n^2}\right)_4 yz$$
$$+ 2\left(\frac{1}{n^2}\right)_5 xz + 2\left(\frac{1}{n^2}\right)_6 xy = 1.$$

- Next, we consider the effect of applied DC field. First, we introduced *impermeability tensor*

$$E_i = \frac{1}{\epsilon_0}\sum_j \eta_{ij} D_j .$$

- Then, again from energy density expression, we get:

$$1 = \eta_{11}x^2 + \eta_{22}y^2 + \eta_{33}z^2 + 2\eta_{12}xy + 2\eta_{23}yz + 2\eta_{13}xz.$$

$$\left(\frac{1}{n^2}\right)_1 = \eta_{11}, \quad \left(\frac{1}{n^2}\right)_2 = \eta_{22}, \quad \left(\frac{1}{n^2}\right)_3 = \eta_{33},$$
$$\left(\frac{1}{n^2}\right)_4 = \eta_{23} = \eta_{32}, \quad \left(\frac{1}{n^2}\right)_5 = \eta_{13} = \eta_{31}, \quad \left(\frac{1}{n^2}\right)_6 = \eta_{12} = \eta_{21}.$$

• Next we define:

$$\eta_{ij} = \eta_{ij}^{(0)} + \sum_k r_{ijk} E_k + \sum_{kl} s_{ijkl} E_k E_l + \cdots .$$

the electrooptic tensor must be symmetric in its first two indices. For this reason, it is often convenient to represent as a two-dimensional matrix using contracted notation. In terms of this contracted notation, we can express the lowest-order modification of the optical constants:

$$\Delta\left(\frac{1}{n^2}\right)_i = \sum_j r_{ij} E_j,$$

$$\begin{bmatrix} \Delta(1/n^2)_1 \\ \Delta(1/n^2)_2 \\ \Delta(1/n^2)_3 \\ \Delta(1/n^2)_4 \\ \Delta(1/n^2)_5 \\ \Delta(1/n^2)_6 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} .$$

- KDP Example:

$r_{41} = r_{52}$, r_{63} are non-zero

$E = 0$, $$\frac{X^2}{n_o^2} + \frac{Y^2}{n_o^2} + \frac{Z^2}{n_e^2} = 1$$

$E \neq 0$, $$\frac{X^2}{n_o^2} + \frac{Y^2}{n_o^2} + \frac{Z^2}{n_e^2} + 2r_{41}E_x YZ + 2r_{41}E_Y XZ + 2r_{63}E_Z XY = 1$$

For only $E_Z = \frac{V}{L}$, $E_X = E_Y = 0$

Re-orient the ellipsoid along new coordinates

$$X = \frac{x-y}{\sqrt{2}} \qquad Y = \frac{x+y}{\sqrt{2}} \qquad Z = z$$

$$\Rightarrow \frac{1}{2n_o^2}(x^2+y^2-2xy) + \frac{1}{2n_o^2}(x^2+y^2+2xy) + \frac{z^2}{n_e^2} + 2r_{63}E_z\frac{(x-y)(x+y)}{2} = 1$$

$$\Rightarrow \left(\frac{1}{n_o^2} + r_{63}E_z\right)x^2 + \left(\frac{1}{n_o^2} - r_{63}E_z\right)y^2 + \frac{z^2}{n_e^2} = 1$$

(We desire $1 = \frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2}$)

Thus:

$$n_z = n_e$$
$$n_x = n_o - \frac{1}{2}n_o^3 r_{63} E_z$$
$$n_y = n_o + \frac{1}{2}n_o^3 r_{63} E_z$$

} 2 optical axes to 3 optical axes

Phase of light $$\Gamma = (n_y - n_x)\frac{\omega L}{c} = \frac{n_o^3 r_{63}\,\omega V}{c}$$

Voltage controllable phase ϕ